Welcome to PHYS 225a Lab

Introduction, class rules, error analysis

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Lab objectives

- To introduce you to modern experimental techniques and apparatus.
- Develop your problem solving skills.
- To teach you how to:
  - Document an experiment (Elog – a web-based logbook!)
  - Interpret a measurement (error analysis)
  - Report your result (formal lab report)
- Lab safety:
  - Protect people
  - Protect equipment

Navigating the 225a Lab web page

http://www.hep.vanderbilt.edu/~velkovja/VUteach/PHY225a

A measurement is not very meaningful without an error estimate!

“Error” does NOT mean “blunder” or “mistake”.

No measurement made is ever exact.

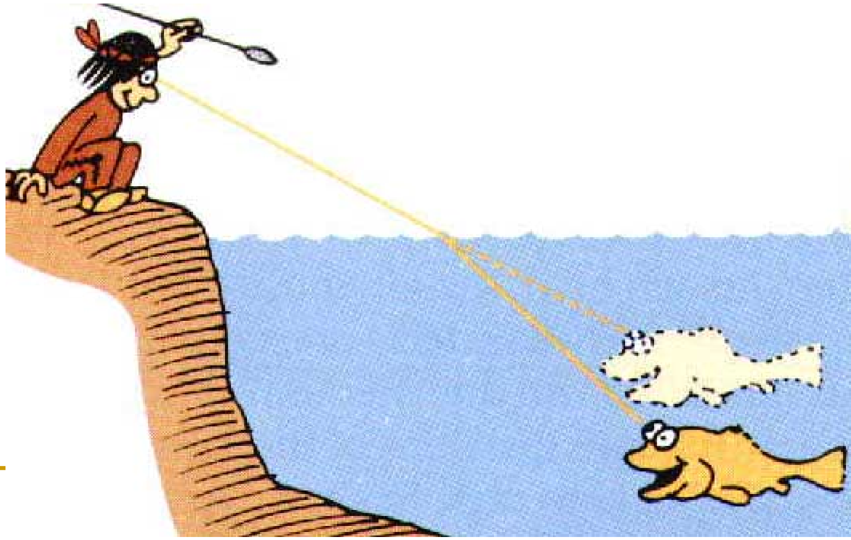
- The **accuracy** (correctness) and **precision** (number of significant figures) of a measurement are always limited by:
  - Apparatus used
  - skill of the observer
  - the basic physics in the experiment and the experimental technique used to access it
- Goal of experimenter: to obtain the best possible value of some quantity or to validate/falsify a theory.
- What comprises a deviation from a theory ?
  - Every measurement MUST give the RANGE of possible values

Types of errors (uncertainties) and how to deal with them:

- Systematic
  - Result from mis-calibrated device
  - Experimental technique that always gives a measurement higher (or lower) than the true value
  - Systematic errors are difficult to assess, because often we don't really understand their source (if we did, we would correct them)
  - One way to estimate the systematic error is to try a different method for the same measurement

- Random
  - Deal with those using statistics

What type of error is the little Indian making ?



Determining Random Errors: if you do just 1 measurement of a quantity of interest

- Instrument limit of error and least count
  - **least count** is the smallest division that is marked on the instrument
  - The **instrument limit of error** is the precision to which a measuring device can be read, and is always equal to or smaller than the least count.
- Estimating uncertainty
  - A volt meter may give you 3 significant digits, but you observe that the last two digits oscillate during the measurement. What is the error ?

Example: Determine the Instrument limit of error and least count

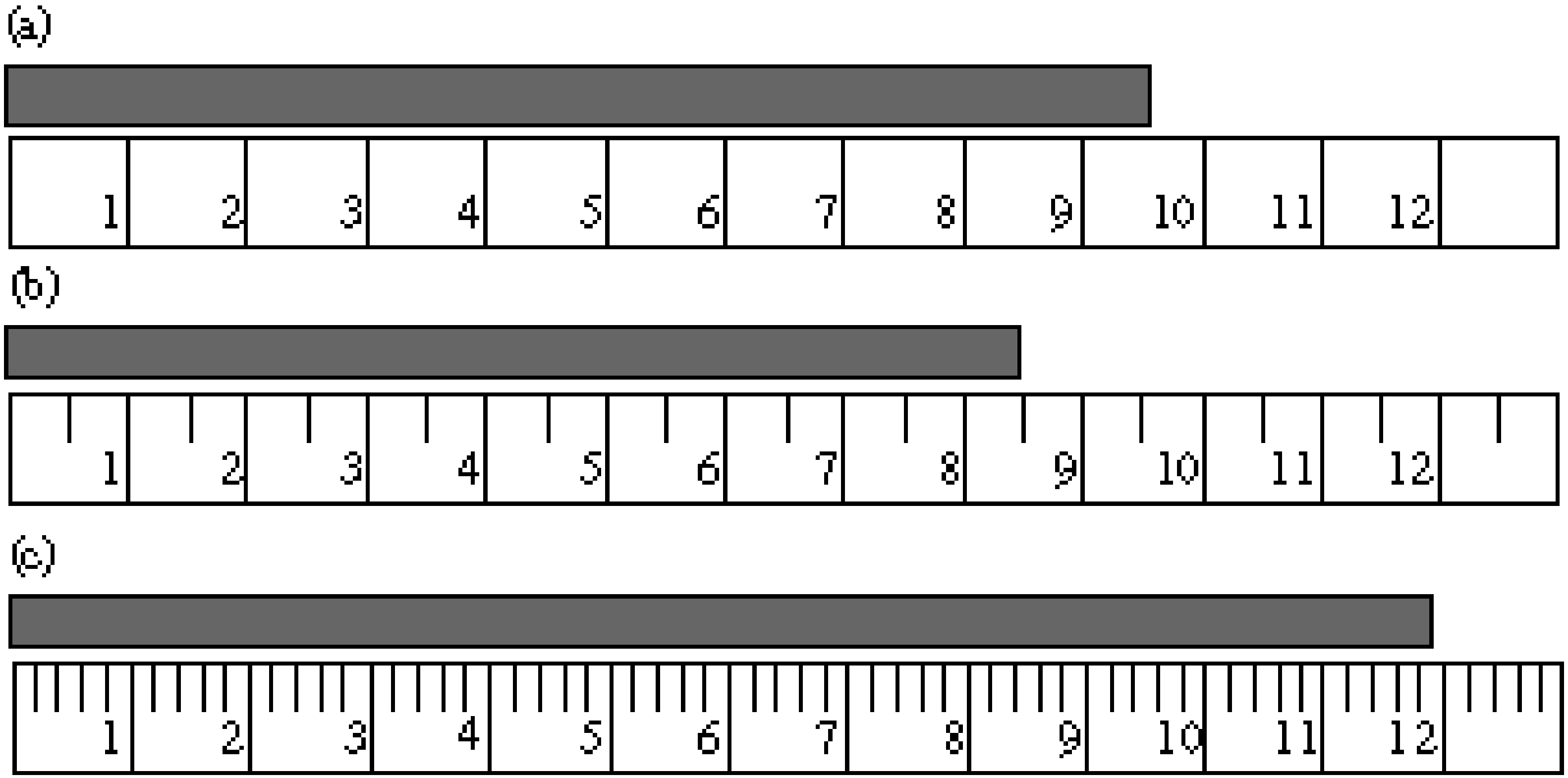
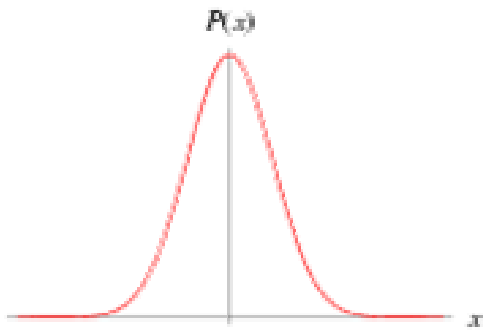


Figure 1 For each object and scale above, determine the least count of the scale, the ILE, and the length of the gray rod. The scales are all in centimeters.

Determining Random Errors: if you do multiple measurements of a quantity of interest

- Most random errors have a Gaussian distribution (also called normal distribution)



$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/[2\sigma^2]}$$

μ – mean, σ^2 - variance

- This fact is a consequence of a very important theorem: the central limit theorem
  - When you overlay many random distributions, each with an arbitrary probability distribution, different mean value and a finite variance => the resulting distribution is Gaussian

Average, average deviation, standard deviation

- Average: sum the measured values; divide by the number of measurements
- Average deviation: find the absolute value of the difference between each measured value and the AVERAGE, then divide by the number of measurements
- Sample standard deviation: σ (biased: divide by N …or unbiased: divide by N-1) . Use either one in your lab reports.

$$\mu \equiv \overline{x} \equiv \langle x \rangle \equiv \frac{1}{N}\sum_{i=1}^{n} x_i$$

$$\alpha \equiv \frac{1}{N}\sum_{i=1}^{N} |x_i - \mu| = \langle |x_i - \mu| \rangle.$$

$$\sigma \equiv \sqrt{\frac{1}{N}\sum_{i=1}^{N} (x_i - \mu)^2}$$

Example: average, average deviation, standard deviation

Time, t, [sec].	$(t - \langle t \rangle)$, [sec]	$\lvert t - \langle t \rangle \rvert$, [sec]	**$(t-\langle t \rangle)^2$ [sec^2]**
7.4			
8.1			
7.9			
7.0			
$\langle t \rangle = 7.6$ average			

Example: average, average deviation, standard deviation

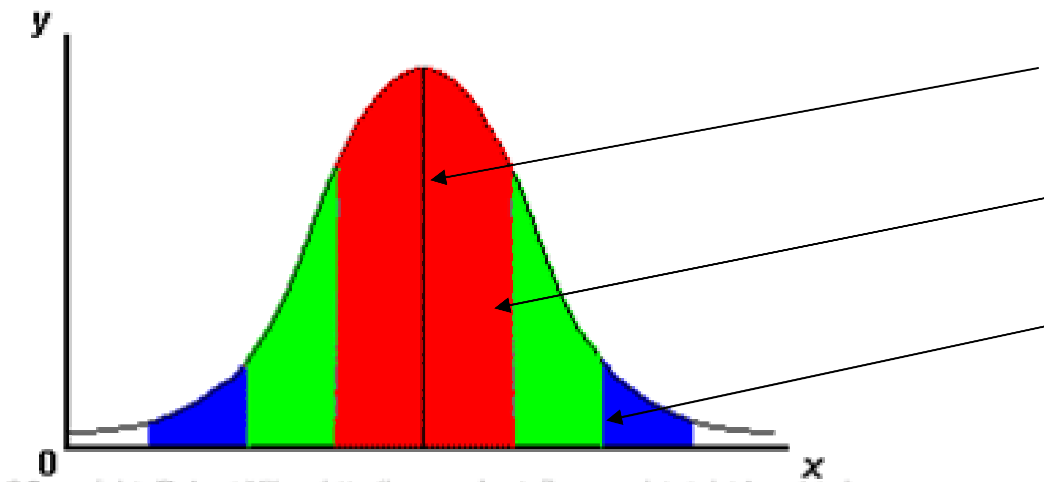
Time, t, [sec].	$(t - \langle t \rangle)$, [sec]	$\lvert t - \langle t \rangle \rvert$, [sec]	**$(t-\langle t \rangle)^2$ [sec^2]**
7.4	-0.2	0.2	0.04
8.1	0.5	0.5	0.25
7.9	0.3	0.3	0.09
7.0	-0.6	0.6	0.36
$\langle t \rangle = 7.6$ average	$\langle t-\langle t \rangle \rangle = 0.0$	$\langle \lvert t-\langle t \rangle \rvert \rangle = 0.4$ Average deviation	(unbiased) Std. dev = 0.50

Some Exel functions

- =SUM(A2:A5) Find the sum of values in the range of cells A2 to A5.
- .=AVERAGE(A2:A5) Find the average of the numbers in the range of cells A2 to A5.
- =AVEDEV(A2:A5) Find the average deviation of the numbers in the range of cells A2 to A5.
- =STDEV(A2:A5) Find the sample standard deviation (unbiased) of the numbers in the range of cells A2 to A5.
- =STDEVP(A2:A5) Find the sample standard deviation (biased) of the numbers in the range of cells A2 to A5.

Range of possible values: confidence intervals

- Suppose you measure the density of calcite as (2.65 ± 0.04) g/cm^3 . The textbook value is 2.71 g/cm^3 . Do the two values agree? Rule of thumb: if the measurements are within 2 σ –they agree with each other. The probability that you will get a value that is outside this interval just by chance is less than 5%..



Random distributions are typically Gaussian, centered about the mean

range	CI
σ	0.6826895
2σ	0.9544997
3σ	0.9973002
4σ	0.9999366
5σ	0.9999994

Why take many measurements ?

- Note the in the definition of σ, there is a sqrt(N) in the denominator , where N is the number of measurements

Indirect measurements

- You want to know quantity X, but you measure Y and Z
- You know that X is a function of Y and Z
- You estimate the error on Y and Z: How to get the error of X ? The procedure is called "error propagation".
- General rule: f is a function of the independent variables u,v,w ….etc . All of these are measured and their errors are estimated. Then to get the error on f:

$$f(u, v, w...)$$

$$\sigma_f^2 = \sigma_u^2 \left(\frac{\partial f}{\partial u} \right)^2 + \sigma_v^2 \left(\frac{\partial f}{\partial v} \right)^2 + \sigma_w^2 \left(\frac{\partial f}{\partial w} \right)^2 + ...$$

How to propagate the errors: specific examples (proof and examples done on the white board)

- Addition and subtraction: x+y; x-y
  - Add absolute errors
- Multiplication by an exact number: a*x
  - Multiply absolute error by the number
- Multiplication and division
  - Add relative errors

Another common case: determine the variable of interest as the slope of a line

- Linear regression: what does it mean ?
- How do we get the errors on the parameters of the fit ?

Linear regression I

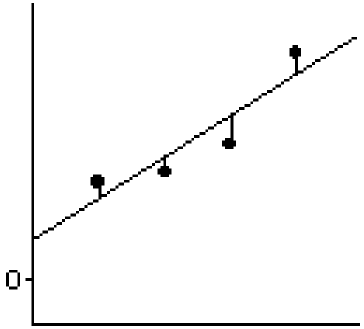
- You want to measure speed
  - You measure distance
  - You measure time
  - Distance/time = speed
- You made 1 measurement : not very accurate
- You made 10 measurements
  - You could determine the speed from each individual measurement, then average them
  - But this assumes that you know the intercept as well as the slope of the line distance/time
  - Many times, you have a systematic error in the intercept
  - Can you avoid that error propagating in your measurement of the slope ?

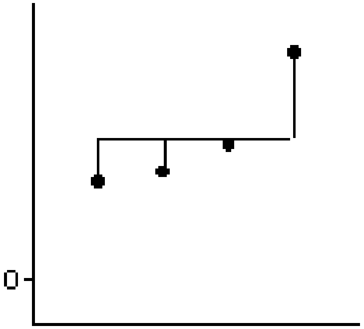
Linear regression: least square fit

- Data points (x_i, y_i) , $i = 1\ldots N$
- Assume that $y = a+bx$: straight line
- Find the line that best fits that collection of points that you measured
- Then you know the slope and the intercept
- You can then predict y for any value of x
- Or you know the slope with accuracy which is better than any individual measurement
- How to obtain that: a least square fit

Residuals:

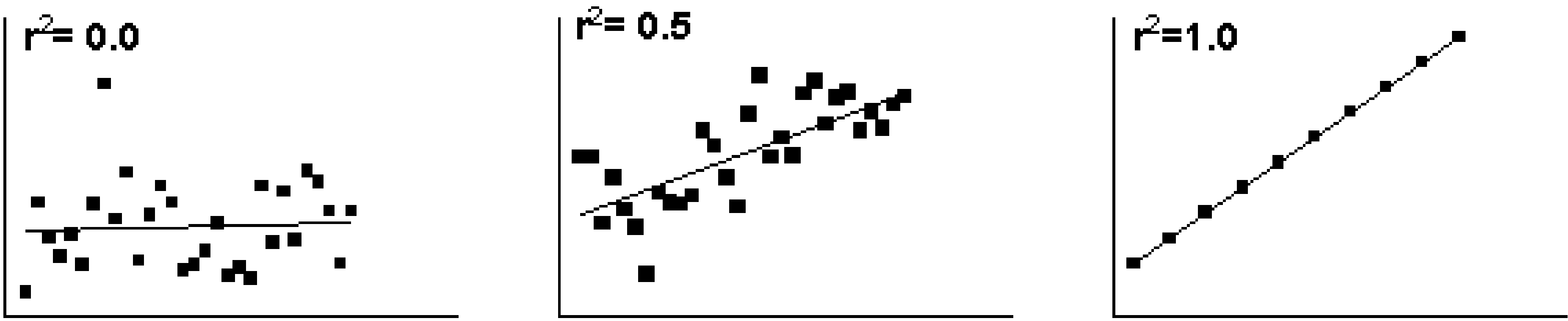
- The vertical distance between the line and the data points
- A linear regression fit finds the line which minimizes the sum of the squares of all residuals





How good is the fit? r^2 – the regression parameter

- If there is no correlation between x and y , $r^2 = 0$
- If there is a perfect linear relation between x and y, the $r^2 = 1$



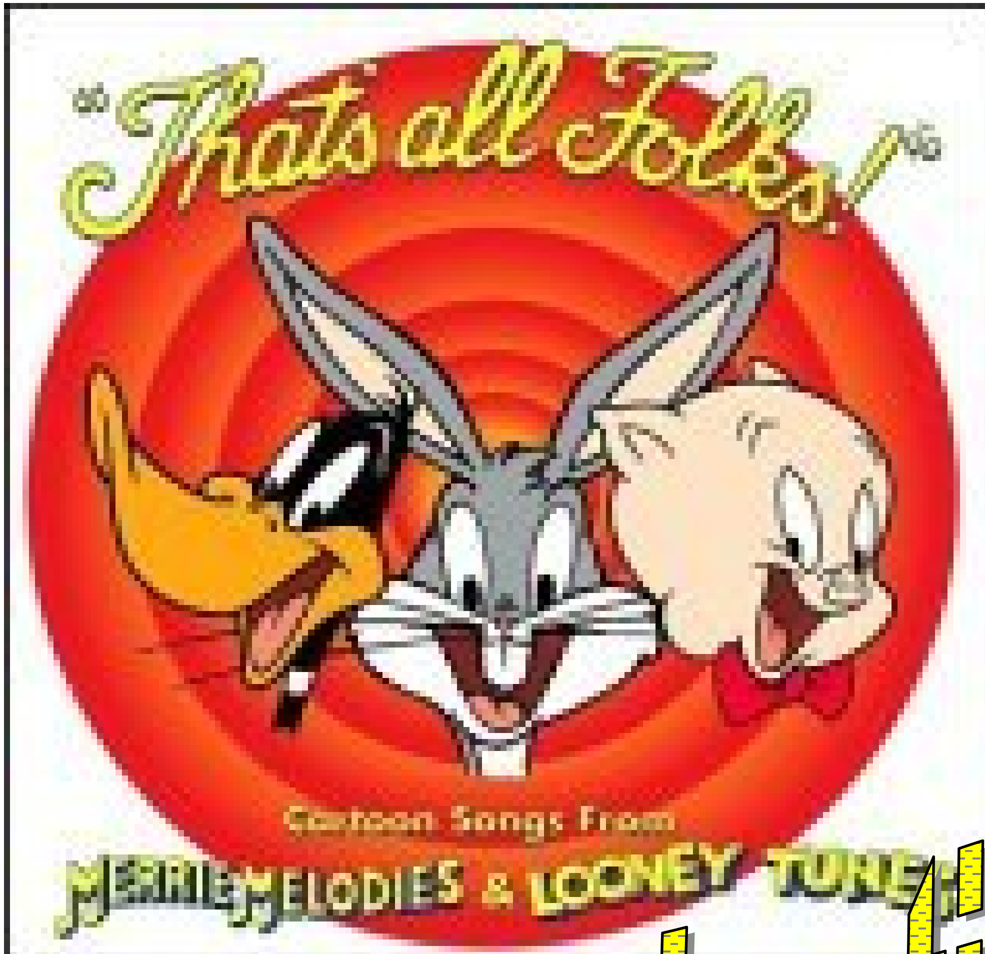
Exel will also give you the error on the slope + a lot more (I won’t go into it)

- Use:Tools/Data analysis/Regression
- You get a table like this:

	X	Y	Z	AA
21		*Coefficients*	*Standard Error*	*t Statistic*
22				
23	Intercept	7.76523109	2.45280031	3.16586355
24	Distance	1.86142516	0.18203112	10.225862

slope

errors



Happy error hunting !