Speed of Light

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1 General Introduction

In this part of the lab we will demonstrate how you can do a fairly sophisticated measurement with a few inexpensive items. You will use your digital scope, a laser pointer, a cheap timer chip, some batteries, cheap photo-diodes, some cable, and a few odds and ends to measure the speed of light. At least that is what this experiment would have measured until a few years ago.

A measurement of the speed of light requires that one already knows what a meter and a second are. Common sense definitions of these quantities began in France about the time of the American Revolution. The year and the circumference of the earth looked like good, stable quantities which would be independent of political turmoil and of the length of any particular kings forearm. Unfortunately neither is very good. The length of the year does vary. As we learned to scale (or frequency divide) very high frequencies, clocks using the periods of atomic transitions as the basic beat became practical. The current, since 1967, definition of the second is based on a Cesium clock maintained by the National Institute of Science and Technology[1] (NIST), formerly the national Bureau of Standards. The formal definition of the second is now the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom. That number[2] is exact. The wavelength is about 3 cm, or microwave.

The history of the meter was even more tumultuous. Measuring the meridian through Paris to better than a few percent was a daunting task. The first serious attempt was in error by 0.02% because the flattening of the earth at the poles was neglected in reducing the data. That meant that the meter was wrong by 0.2 mm, a serious error for any kind of precision machine work. In 1927 the standard was shifted to the distance between two scratches on a platinum-indium bar stored at Paris, France at constant temperature. Of course that meant that anyone not in the good graces of the French government didn't have access to the primary length standard, but it was the development of interference experiments that forced the next step. These interference experiments, the Michaelson interferometer for example, permitted the measurement of the wavelength of light by counting fringes as a mirror was moved through a known distance. Just as for the second, the uncertainty in the standard meter became the dominant error in the determination of

[1] http://www.nist.gov/
[2] http://physics.nist.gov/cuu/Units/current.html

several wavelengths, so in 1960[3] the definition was flipped to make the standard a certain number of wavelengths of one of the spectral lines emitted by krypton-86 and the French bar became a secondary standard. Alas, even that was not good enough to keep ahead of technology. Improvements in atomic clocks soon made it possible to measure time so well that the dominant uncertainty in the measurement of the speed of light became the error in the standard meter. As always, a definition had to go. The standard meter is now the distance[4] light in a vacuum goes in a $1/299\,792\,458$ of a second, or in more primitive terms the standard meter is how far light goes in $9\,192\,631\,770\ /\ 299\,792\,458 = \ 30.663\,318\,988\cdots$ ticks[5] of the standard atomic clock. That means that the speed of light has entered our system of units as a fundamental constant, not as something we can measure.

All of this activity has been the work of an international committee of gray-beard type scientists and has led to our current system of units called the International System of Units[6](SI). Of course we will continue to speak of experiments like this one as measuring the speed of light, but in principal you are measuring the calibration of either your measure stick, your scope timing circuit, or your ability to use them accurately. Which do you think you are doing?

The speed of light is a very large number: around the earth eight times in one second or from the sun to the earth in eight minutes. Clearly one needs either a very large distance or a very fast clock to measure it. Galileo was probably one of the first to try. He stationed a helper on a mountain top a few kilometers away with instructions to uncover his lantern when he saw that Galileo had uncovered his. Galileo would then count his pulse beats between the time he uncovered his lantern and the time he saw that his assistant had uncovered his lantern. They correctly concluded from this experiment that the speed was too large to be measured with such short distances. About a century later Roemer and Huygens measured the speed of light by noting the times when one of Jupiter's moons disappeared behind Jupiter when Earth and Jupiter were on the same side of the sun and trying to predict the times of disappearance when the earth was on the opposite side of the sun. They found that the disappearances when on the opposite side were all about 15 minutes late and correctly interpreted the additional delay to the time it took light to travel the extra distance, the diameter of the earth's orbit. The limitation of their clock was still human reaction time, but they had found a baseline enough longer than the distance between Galileo's two mountains so that human reaction time was not the dominant source of error. Our scopes and a little modern electronics permit us to measure times as short as a few tens of nanoseconds, so we can use distances like the length of the hallway in Stevenson Center. (In really mixed up but convenient units, the speed of light is about a foot per nanosecond.)

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[3]During the preceding decade various other lines had been used as the standard as spectroscopy technology improved and sharper lines were discovered.

[4]*ibid.*

[5]The ratio is exact. It is obviously a rational number but may be a repeating decimal.

[6]It is a pretty remarkable example of international cooperation, but it went off the deep end when it assigned a factor of $\epsilon_\circ \approx 10^{-11}$ between $\mathbf{D}$ and $\mathbf{E}$ in a vacuum where they are the same physical field. (That's just MSW's prejudice coming through.)

2 Apparatus

Tektronix TDS 210 Digital Scope
laser pointer (from Kroger's)
2 photo diodes
timer chip NE555P
breadboard, resistors, capacitors
batteries (6 V lantern battery)
focusing lens

3 Overview of procedure

It is the development of laser diodes in the visible range which has made this experiment feasible with inexpensive, safe apparatus. The earlier gas lasers generally required dangerous high voltage and showed variation in the time between trigger and flash. The laser used in this experiment was designed for continuous operation as a pointer, but the operating voltage is only six volts and that is easily turned on and off directly with the output of the timer chip. Moreover, there is negligible jitter in the delay between the time power is applied to the laser and the flash. This pulsed (sharply turned on and off) light source is the heart of the experiment. Because the laser produces light in the form of a coherent plane wave, the light beam can be transmitted for tens of meters and then focused to get most of its intensity on a photo-diode a few mm in size. The photo-diode conducts when light hits it, and the voltage across a resistor in series with the photo-diode rises sharply. The time between the laser power pulse and the photo-diode pulse is the sum of two times, the delays in the devices and the time taken by the light to go from the laser to the photo-diode. If the circuit delays are constant (little jitter) and the delay is not too long, the time between the laser pulse and the photo-diode pulse can be measured directly and the speed of light can be determined from the differences in the times measured for two different light path distances. The time taken by the electrical signals to go through the cables to the scope is significant (about 1.3 nanoseconds per foot of cable), so the cabling must not be changed between measurements. That suggests using a mirror and moving only the mirror to change the light path. A picture of the set-up is shown in Figure 1.

You will connect two signals to the scope: the pulse that turns on and off the laser and the pulse from the photo-diode. Then using the time cursors you can measure the time difference between the two pulses, which will give you a measure of the time it takes light to travel from the laser to the mirror and back to the photo-diode.You will also measure the path-length using a tape-measure. In principle, the ratio of these two numbers distance/time should give you the speed of light. However, you can not assume that the delays in the two electrical circuits (pulser and photo-diode) are the same. Thus your measurement of the time will likely have some constant offset (systematic error). You can still make a rather reliable measurement of the speed of light if you use several independent measurements of distance and time and then determine c from the slope of the plot of distance *vs* time. The intercept may not be zero, but we are not interested in this number.

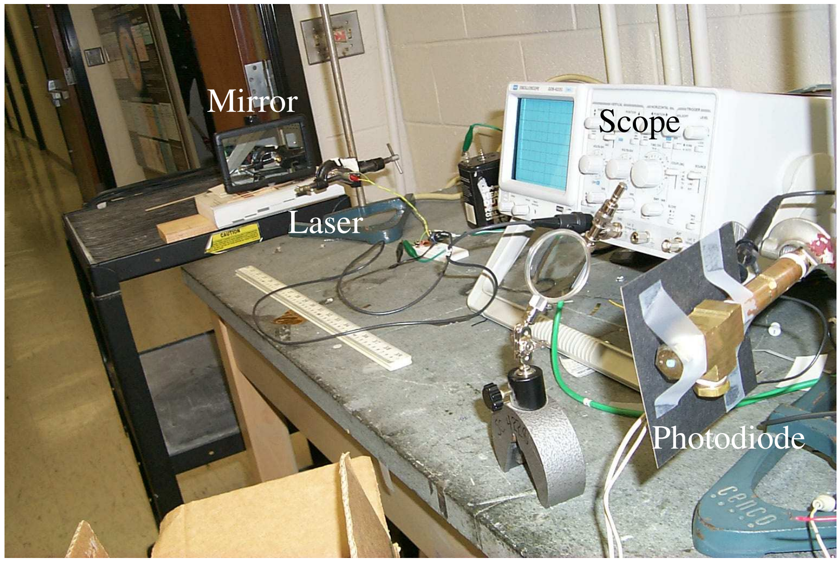

Figure 1: Picture of the set up. You'll notice a different kind of scope in the picture, but otherwise your set-up will be pretty similar.

4 The Pulser Circuit (Figures 2 and 3)

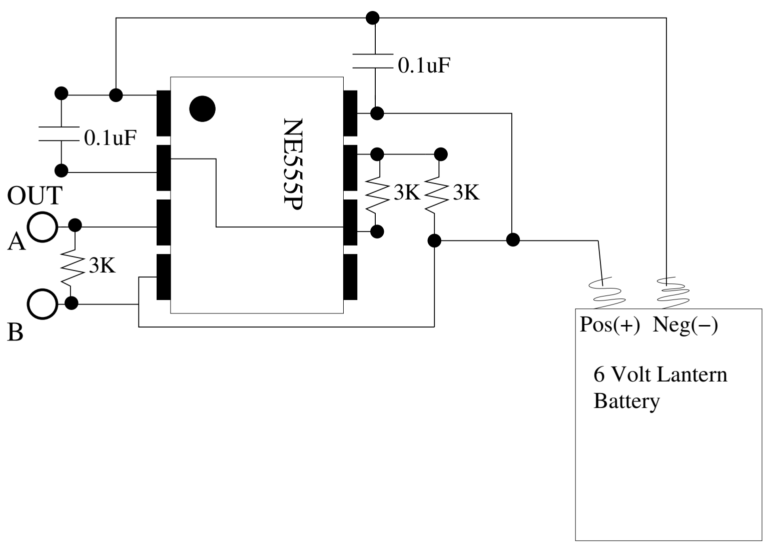

Figure 2: Here is the relevant circuit diagram for the pulser.

The NE555P is an oscillator which is scaled (frequency divided) down to a low frequency and produces a square wave at its output. The output has sufficient power capability to drive (power) the laser. The circuit shown in Figure 2 is constructed on a breadboard and placed in a box which is attached to the battery. Figure 3 shows the circuit which is in the box. To observe the on and off of the pulser circuit you need to hook up the scope probe tip to point A and the ground clip of the probe to point B. In the current set-up (Fall 2006) access to point A is provided by stripping the insulation off a little section of the yellow cable that goes to the inside of the laser pointer. The

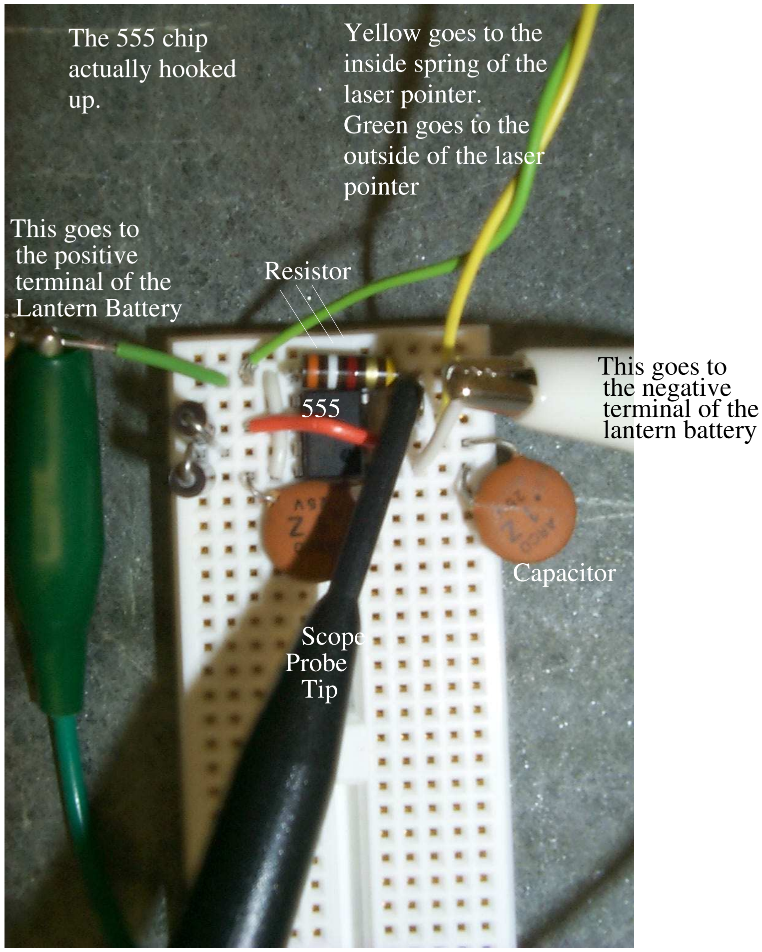

Figure 3: Here is what the circuit looked like after hooking up everything.

ground of the probe is simply connected to the positive terminal of the battery. Now you can connect the negative terminal of the battery and adjust the scope to trigger on the square wave. The on and off parts of the square wave are not of equal duration. Which is on and which is off? Which transition do you want to trigger on? Measure the duration of the on and off parts. Be sure you are triggering on the correct transition and then turn up the sweep speed so that you can see the turn on time of the pulse.

5 Hooking up the Photo-diode (Figures 4, 5)

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Figure 4 is a circuit diagram showing how the 9 Volt battery, photo-diode and 3 kΩ resistor are connected in series to detect the laser pulse. A resistor has been soldered to the side of the photo-diode which should be negative. Thus connecting the negative side of the battery to the resistor as shown in Figure 4 gives the correct polarity. The photo-diode conducts when the laser light is on during the on part of the pulser cycle, the current which flows and that makes an IR drop in the resistor. With the scope connected as shown in Figure 4, you should be able to see the pulse from the photo-diode. Turn the scope time setting to 100 ns/div. Trigger on the pulser (connected in the other channel)

- the signal from the photo-diode should come some time after the pulser signal. This may turn out to be the most tedious part of the set-up. You need to adjust the lens and carefully focus the light onto the dot of the photo-diode. Once you see any signal on the scope, you should be able to adjust the lens to optimize its amplitude. You are (almost) ready to do the measurement. Does the light pulse correspond to the rising or falling edge of the photo-diode pulse? Which one should you use for the time measurement ?

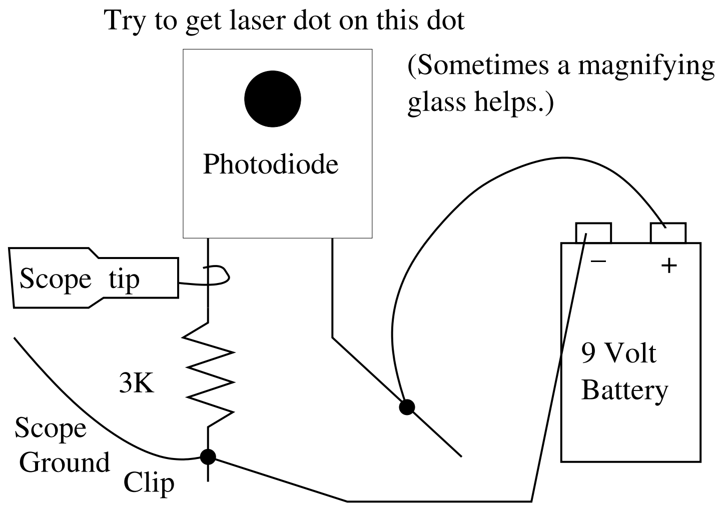

Figure 4: Diagram of the photo-diode hook up.

Figure 5: Picture of the photo-diode hook up. Notice the magnifying glass. This can really help to get the laser to be concentrated on the little button of the photo-diode. If you look closely in a color reproduction, you can see a little red dot on the photo-diode and on the lens. That's the laser beam!.

6 Measurement

Make sure the laser is working and you have both the pulser signal and the photo-diode signal on the scope screen. At this point you have to decide how you are going to measure the time between the two pulses such that you minimize your systematic error. You can put your "start" and "stop" time at the turn on of the pulser and the photo-diode or you can position the cursors at some fraction (let's say half-maximum) of the pulse height. Whatever method you decide to use, you should do all your measurements consistently. It is a good idea to place your start signal as far left on the scope as you can, position time cursor #1 at the "start" time and not touch it any more. Then you take measurements at different positions of the mirror and each time move just cursor #2 to give you the "stop" time. Take 8 – 10 measurements moving the mirror away from the laser in about 10 ft intervals.

7 Data analysis

1. Plot your data: distance vs time. Fit it with a straight line. What parameter of your line is related to the speed of light? Does your line go through (0,0)? If not, what causes the offset ?

2. Now, to see the virtue of using the slope obtained from many points do the following exercise:

   - Choose two of your data points: one with relatively short distance of travel and another - with a long distance of tavel.
   - Assign approximate errors on the distance and the time measurements. Determine Δd, Δt and their errors. Then find the speed of light and its error. How does this result compare to the definition of c ?
   - Go back to your spreadsheet. Use all your data points. "Ask" Excel to give you the error on the linear regression parameters. You will have to use Tools/Data Analysis/Regression. After you select the x and y axes input ranges, you will get several tables and a chart. One of the tables contains the parameters, their errors, the 95% confidence intervals on each parameter and a few other things that we are not currently interested in. Note that Excel incorrectly labels the slope parameter with the name of the X axis. How does the value of c and its error compare to what you got using just two of your data points ? Does it agree with the accepted value (the definition of c) ?

3. Discuss the sources of uncertainty in your measurement, and any improvements you can think of.