

# Crisis in Semileptonic Heavy Meson Decays

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# What constitutes a “crisis” in physics

- A friend who needs help?
- An unexpected result? (this can be a good kind of crisis to have)
- Contradictory results?
- An unexplainable result?
- All of the above
- None of the above

(In this talk, we may see hints of the 1<sup>st</sup> four...)

# In 2002 FOCUS published a result...

- But it sort of disagreed with a result that was published at almost the same time
- But FOCUS found something in the decay that other experiments only saw hints of
- So FOCUS took a little more time in publishing everything about this decay
- But the difference remained...
- And there were other things that were interesting

# What is a particle decay?

A transition of a particle from one (*initial*) state to another (*final*) state (consisting of more particles) – as described by Fermi's Golden Rule:

$$\left[ \begin{array}{c} \textit{Transition} \\ \textit{Rate} \end{array} \right] = \left\{ \begin{array}{c} \textit{Universal} \\ \textit{Coupling} \end{array} \right\} \left\{ \begin{array}{c} \textit{Matrix} \\ \textit{Element} \end{array} \right\} \left\{ \begin{array}{c} \textit{Phase} \\ \textit{Space} \end{array} \right\}$$

But a particle *lifetime* is governed by the Total Transition Rate which includes all possible states (i.e. we don't get different lifetimes for the same *initial* particle by measuring different *final* states unless very special physics is involved)

# Another way to look at this:

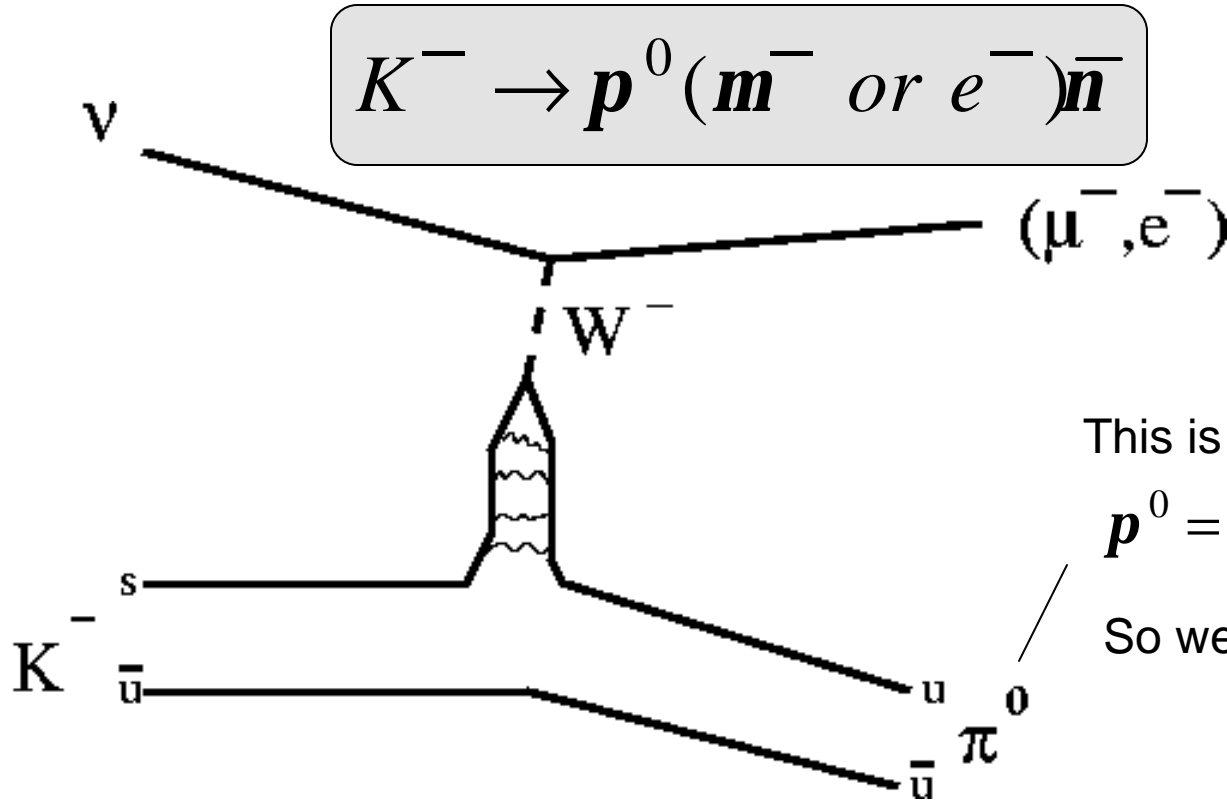
- Even though different initial states (particles) can have different lifetimes, decay rates for individual transitions can be almost the same if the physics (matrix element) and the phase space (“Q”) are similar!

$$\frac{\% A \rightarrow ZX}{\textit{Lifetime A}} \approx \frac{\% C \rightarrow YX}{\textit{Lifetime C}} \quad \left\{ \begin{array}{l} C \sim A, Z \sim Y \\ \textit{Lifetime A} \neq C \end{array} \right\}$$

A powerful check for states with great similarities

# A simple case

- Semileptonic Pseudoscalar Meson decay
- Hadronic current tends to be simpler and quasi-free of final state interactions

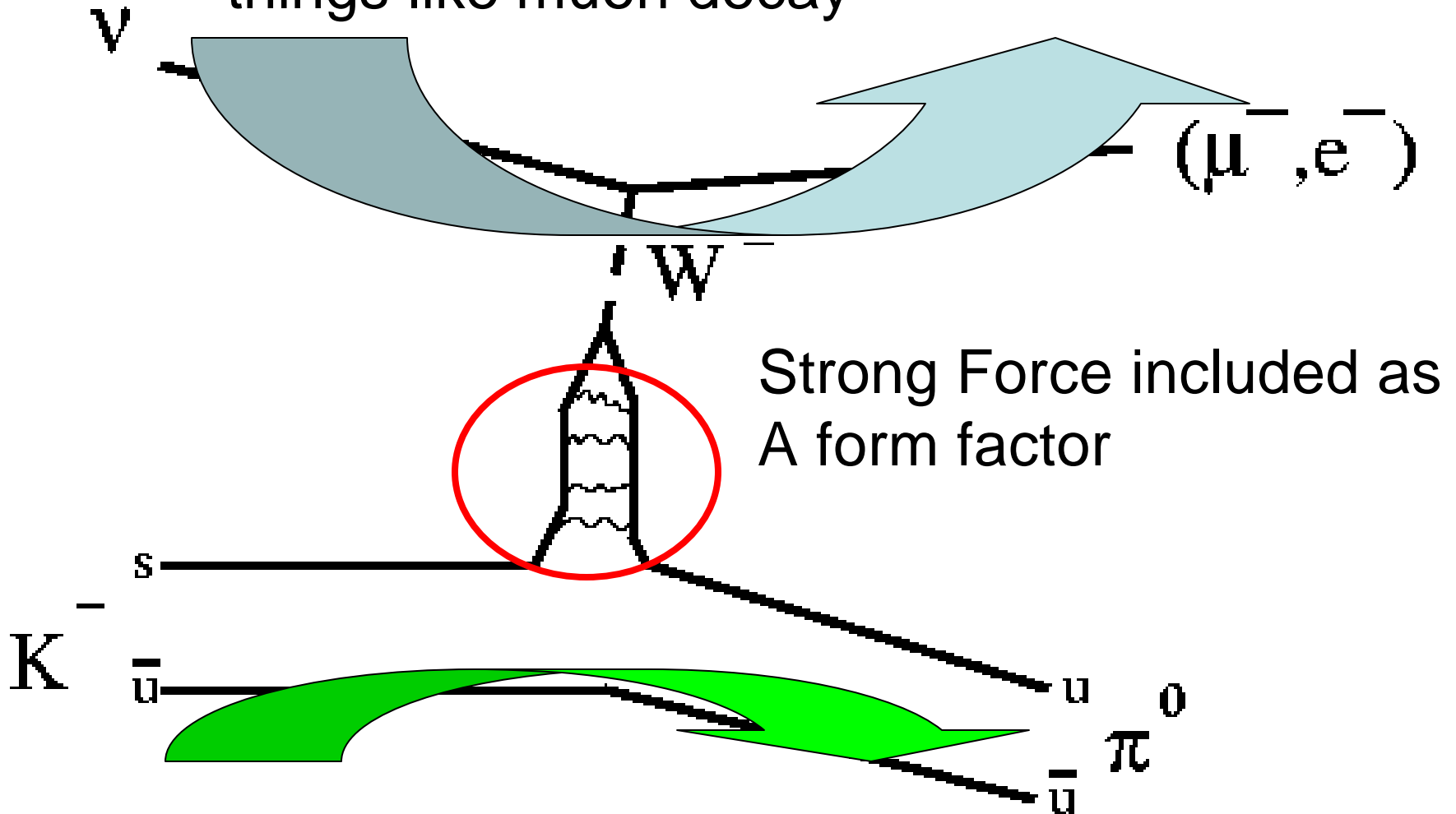


This is actually a combo:

$$p^0 = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d})$$

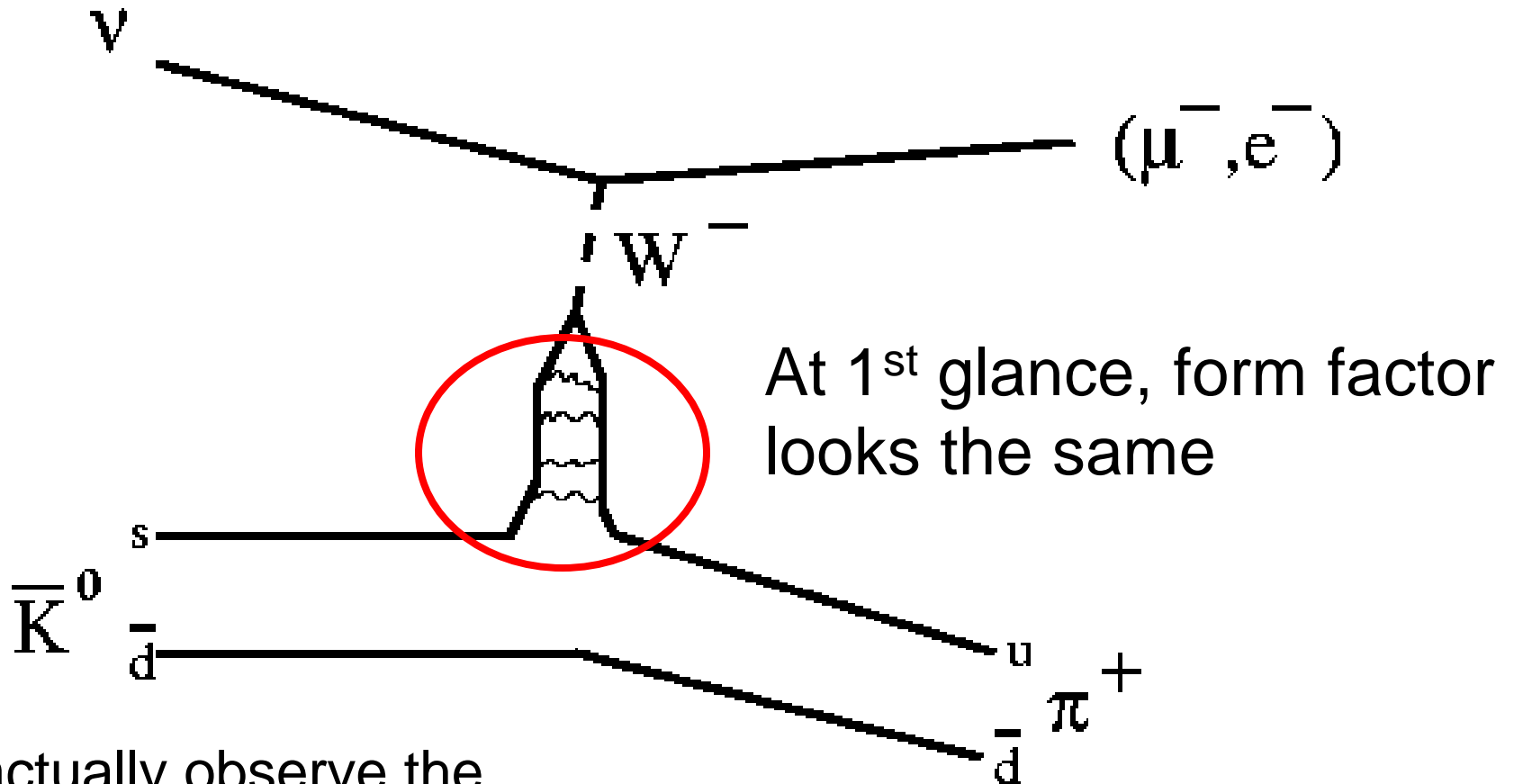
So we only get half...

Weak current is well understood from things like muon decay



Other quark in the decay acts as a "Spectator"

Similar Decay to compare:  $\bar{K}^0 \rightarrow p^+ (m^- \text{ or } e^-) \bar{n}$



We actually observe the neutral kaon as an admixture of particle and anti-particle. With long and sort lifetimes.

But in the bottom line, we've changed a d for a u



# Lets do a comparison

	$K^{+/-}$	$K^0_{\text{long}}$	$\pi^+$	$\pi^0$	$\mu^+$	$e^+$	$\nu$
Mass(MeV)	494	498	140	135	106	0.511	$\sim 0$
Lifetime(ns)	12.4	51.8	26.0	$\sim 0$	2200	stable	'stable'

$$M(K^{\pm}) - M(p^0) = 359 \text{ MeV} \quad M(K_L^0) - M(p^{\pm}) = 358 \text{ MeV}$$

$$\%(K_L^0 \rightarrow p^{\mp} (m^{\pm} \text{ or } e^{\pm}) n) = 38.81 e, 27.19 m$$

$$\%(K^{\pm} \rightarrow p^0 (m^{\pm} \text{ or } e^{\pm}) n) = 4.87 e, 3.27 m$$

Notice % electron modes > % muon modes

-Muons eat Q (expect muons  $\sim 2/3$  electron modes)

-and effects proportional to  $M(\text{lepton})^2$  (few percent effect)

# End up with....

	Experiment	“Theory”
$\frac{\% K_L^0(\mathbf{m})}{\% K_L^0(e)}$	$= 0.70 \pm 0.01$	0.67
$\frac{\% K^+(\mathbf{m})}{\% K^+(e)}$	$= 0.67 \pm 0.01$	0.67
$(1/2) \frac{\% K_L^0(e)t_+}{\% K^+(e)t_L}$	$= 0.953 \pm 0.016$	0.97
$(1/2) \frac{\% K_L^0(\mathbf{m})t_+}{\% K^+(\mathbf{m})t_L}$	$= 0.994 \pm 0.022$	0.97

$$\frac{\% K_L^0(e)t_S}{\% K_S^0(e)t_L} = \frac{(38.81)(0.08958 \text{ ns})}{(0.069)(51.8 \text{ ns})} = 0.97 \pm 0.06$$

This better be close to 1!

# This is complicated stuff

- Lots of model corrections on the order of 1%
- Used to find  $V_{us}$  CKM angle (Part of the coupling)
- Discrepancy appears to be in neutral kaon
- Combine carefully with other:  $V_{ud}$   $V_{ub}$  ... Get

Expect:  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

(2002 *PDG Estimates*) =  $0.9957 \pm 0.0019$  (2002)

(2004 *PDG Estimates*) =  $0.9967 \pm 0.0015$  (2004)

Find a greater than 2 sigma discrepancy!

# Other Heavy Mesons decay too

Approximately

$$\Gamma_e \propto M_{Heavy}^5 (1 - 2(M_{Light} / M_{Heavy}))$$

And (at most!)  $dM_{Heavy} \sim dM_{light} \sim 5 \text{ Mev}$

Expect differences to get smaller for similar light/heavy ratio with increasingly heavier mesons when we change an up for a down quark

$$\begin{aligned} (M(D^+) / M(D^0))^5 = 1.01 & \quad \text{And} \quad \Gamma D^+(e, \mathbf{m}) / \Gamma D^0(e, \mathbf{m}) = 1.01 \\ (M(B^+) / M(B^0))^5 = 1.00 & \quad \text{Expect} \quad \Gamma B^+(e, \mathbf{m}) / \Gamma B^0(e, \mathbf{m}) = 1.0 \\ & \quad \Gamma D(\mathbf{m}) / \Gamma D(e) = 0.98 - 0.97 \\ & \quad \Gamma B(\mathbf{m}) / \Gamma B(e) = 0.996 \end{aligned}$$

And the case is fine for B's

$$\Gamma B^0(e, \mathbf{m}) / \Gamma B^+(e, \mathbf{m}) = 1.08 \pm 0.15$$

But Charm mesons are interesting!

$$\Gamma D^+(e) / \Gamma D^0(e) = 0.74 \pm 0.11 \quad \text{Another } >2 \text{ sigma effect!}$$

$$\Gamma D^+(\mathbf{m}) / \Gamma D^0(\mathbf{m}) = 0.87 \pm 0.31$$

$$\Gamma D^0(\mathbf{m}) / \Gamma D^0(e) = 0.89 \pm 0.06$$

$$\Gamma D^+(\mathbf{m}) / \Gamma D^+(e) = 1.0 \pm 0.38$$

Errors are too big to tell

# How do we increase our Charm?

- Where's the biggest error coming from?

$$\%(D^0 \rightarrow K^- e^+ \mathbf{n}) = 3.58 \pm 0.18$$

$$\%(D^0 \rightarrow K^- \mathbf{m}^+ \mathbf{n}) = 3.19 \pm 0.17$$

$$t(D^0) = 410.3 \pm 1.5 \text{ ps}$$

$$\%(D^+ \rightarrow \bar{K}^0 e^+ \mathbf{n}) = 6.7 \pm 0.9$$

$$\%(D^+ \rightarrow \bar{K}^0 \mathbf{m}^+ \mathbf{n}) = 7.0 \pm 2.5$$

$$t(D^+) = 1040 \pm 7 \text{ fs}$$

Best  
Candidates!

# Is there anything else that helps?

- Charm and Beauty mesons have a lot more mass than kaons, so there are more decay possibilities
- In fact, earlier models predicted that:

$$\Gamma(D \rightarrow Ke\mathbf{n}) \approx \Gamma(D \rightarrow K^*e\mathbf{n})$$

Lowest excited Kaon state

-But Experiments throughout the 90's were seeing

$$\frac{\Gamma(D \rightarrow K^*e\mathbf{n})}{\Gamma(D \rightarrow Ke\mathbf{n})} \approx 0.5$$

So in 2002, a new measurement:

$$\frac{\% D^+ \rightarrow \bar{K}^* e^+ n}{\% D^+ \rightarrow \bar{K}^0 e^+ n} = 0.99 \pm 0.06 \pm 0.07$$

- Got me pretty excited
- And a friend of mine got pretty excited because if you compared his measurement to theirs:

$$\frac{\% D^+ \rightarrow \bar{K}^* m^+ n}{\% D^+ \rightarrow \bar{K}^* e^+ n} = 1.23 \pm 0.12$$

And 0.95 was expected



# And I was especially excited...

- Since I was working on a measurement of:

$$\% D^+ \rightarrow \bar{K}^{*0} \mathbf{m}^+ \mathbf{n}$$

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$$\% D^+ \rightarrow \bar{K}^0 \mathbf{m}^+ \mathbf{n}$$

- And about half of the previous experiments that compared rates from  $D^+$  and  $D^0$  just assumed:

$$\Gamma(D^+ \rightarrow \bar{K}^{*0} (\mathbf{m} \text{ or } e)^+ \mathbf{n}) = \Gamma(D^0 \rightarrow K^{*-} (\mathbf{m} \text{ or } e)^+ \mathbf{n}), \quad \text{or}$$

$$\Gamma(D^+ \rightarrow \bar{K}^0 (\mathbf{m} \text{ or } e)^+ \mathbf{n}) = \Gamma(D^0 \rightarrow K^- (\mathbf{m} \text{ or } e)^+ \mathbf{n})$$

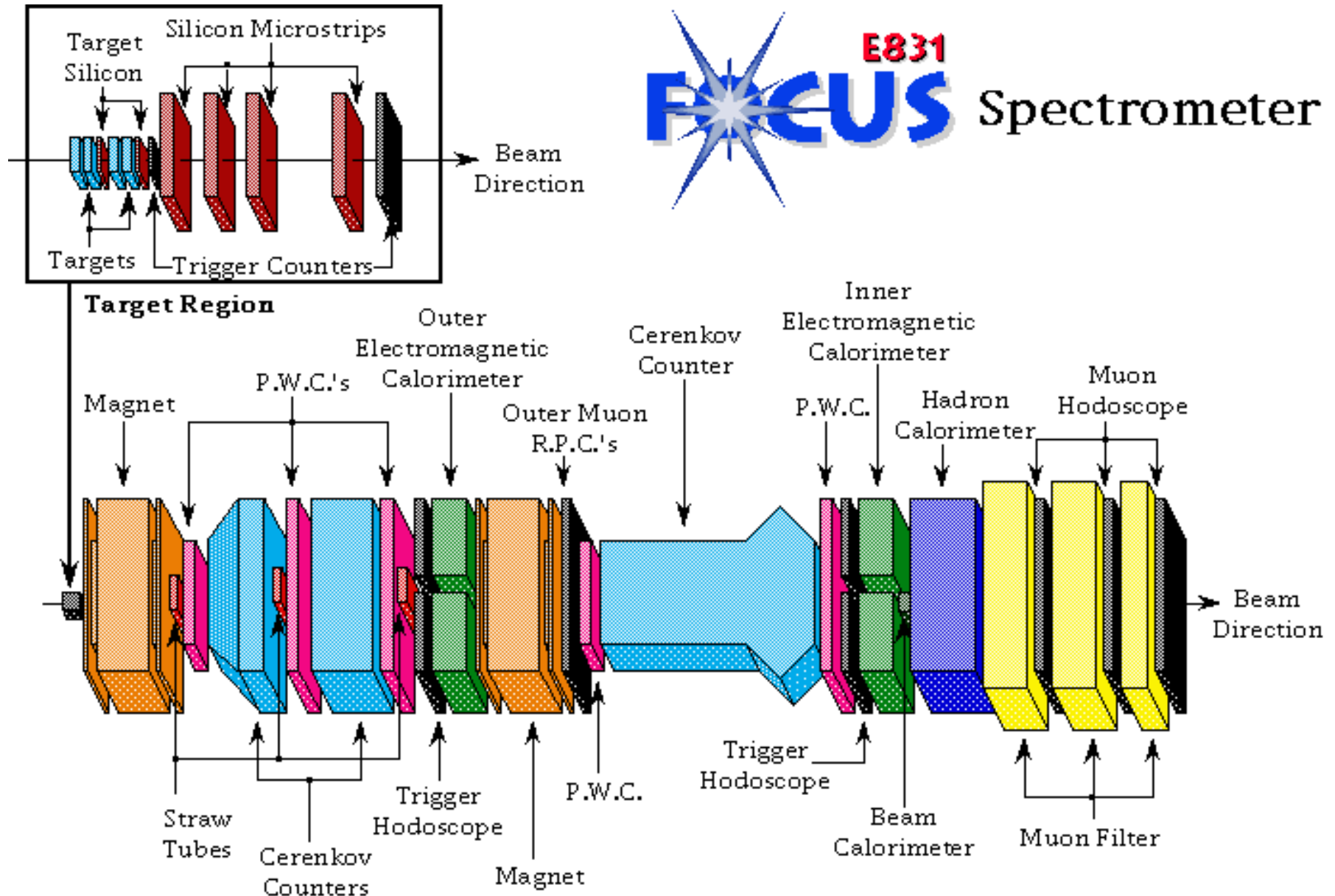
Including me in my thesis ....(This is a crisis!)

# So how do you measure this?

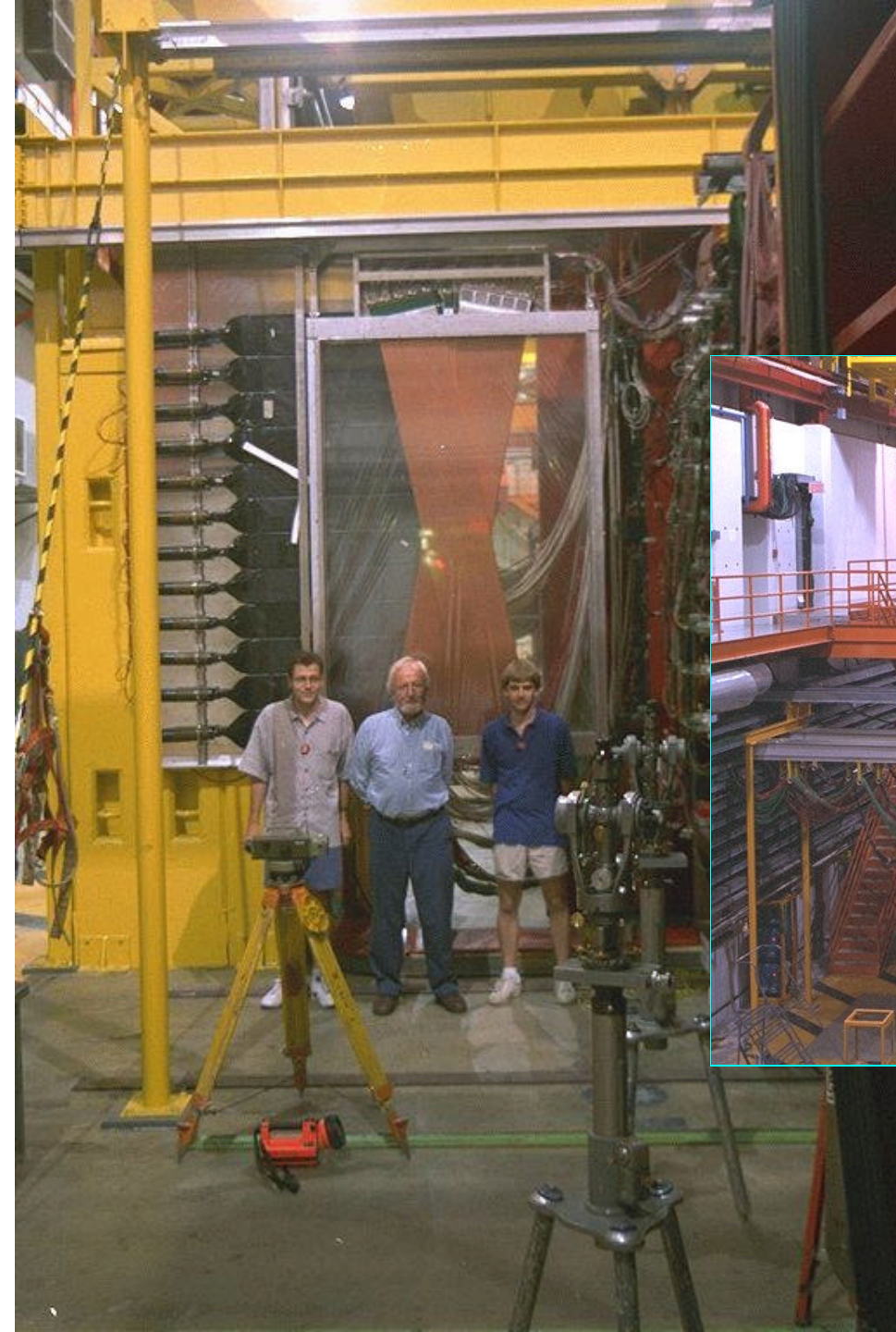
- Since the shorter lived neutral kaon and the excited neutral kaon decay into 2 charged tracks at a very predictable rate, the systematic errors, which are prevalent when comparing decays with different track multiplicities or decays containing reconstructed neutrals in only one state, will tend to be smaller... Blah blah blah

You try to pick decays which have similar topologies and compare them

# Because Experiments are Complex

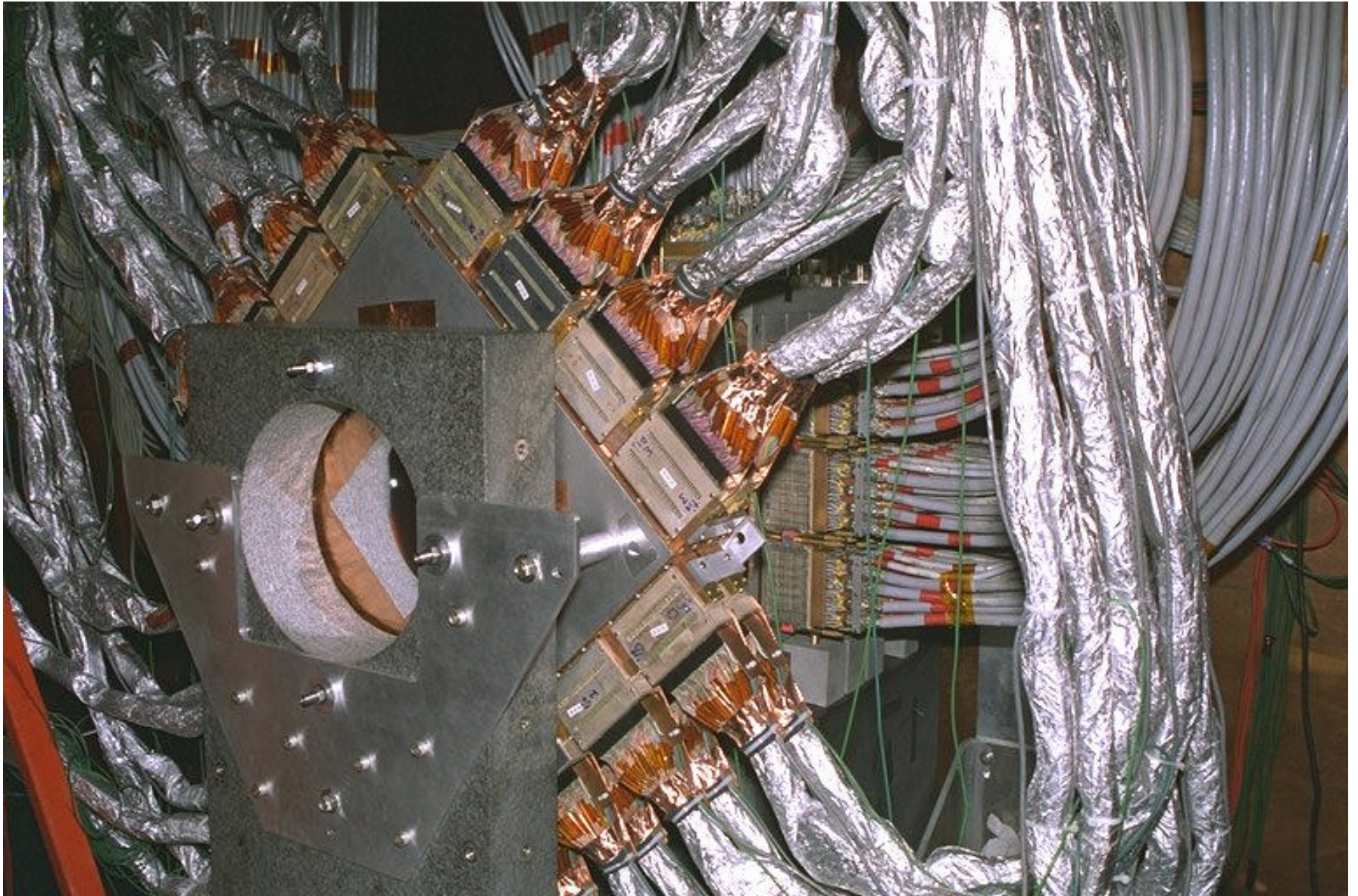


# Big



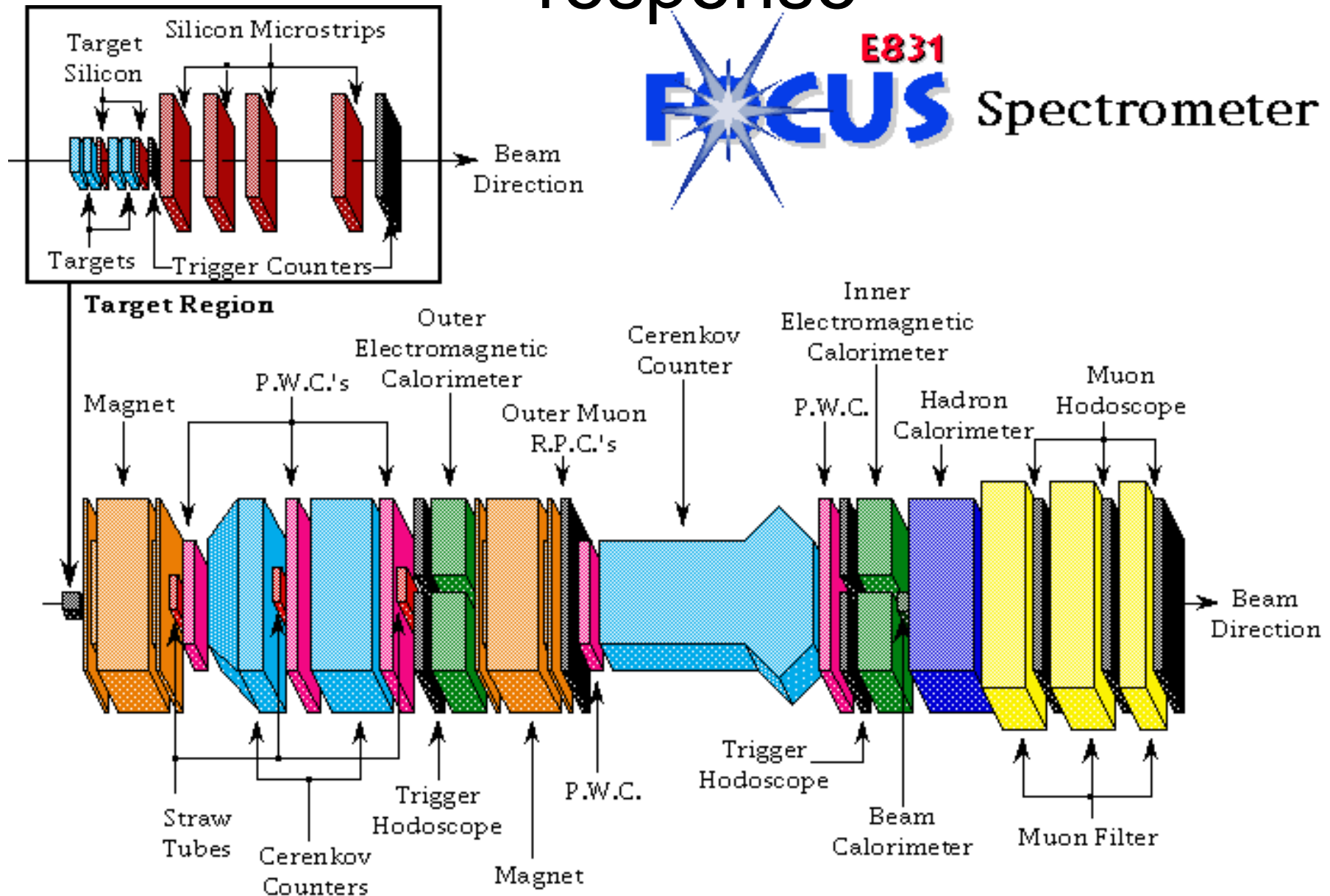


# And did I mention Complex?

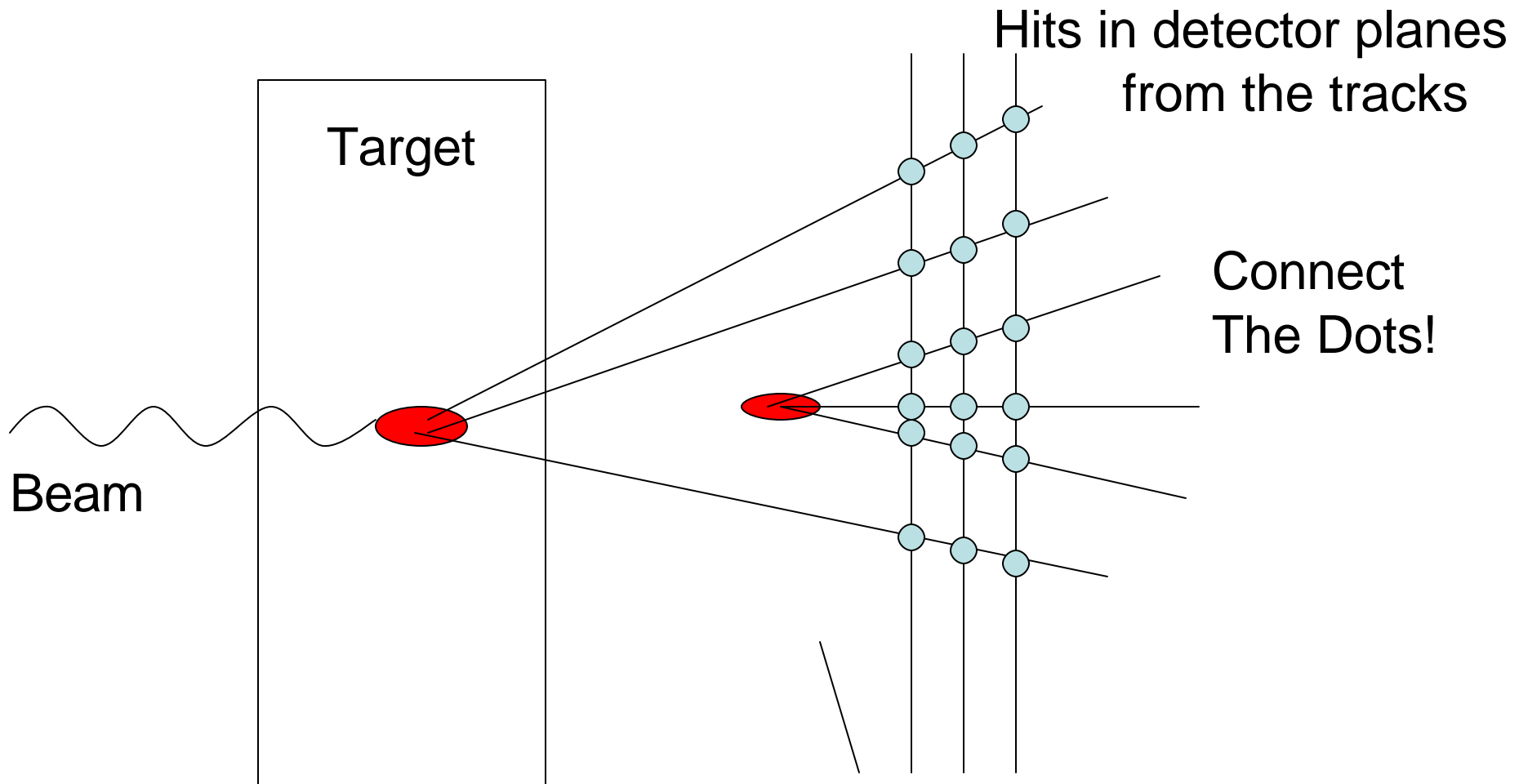




# So you need to simulate the detector response



# Signature of Charm Mesons

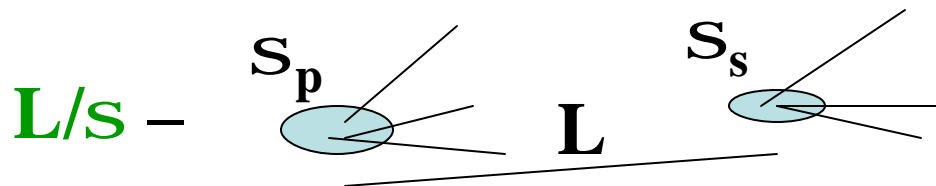
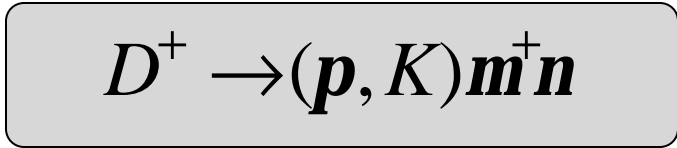


Primary Interaction in Target make tracks

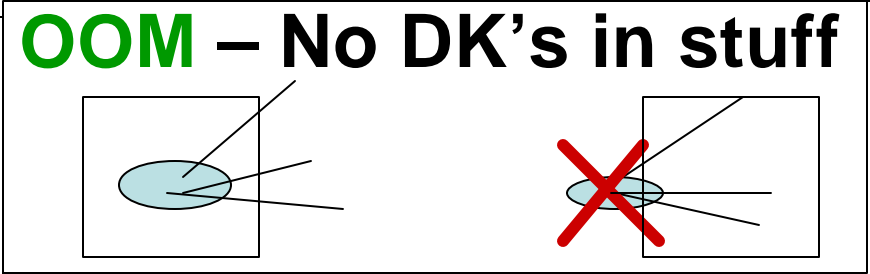
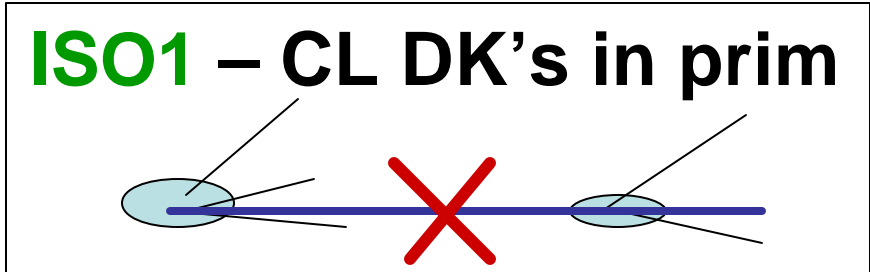
Since the D meson is moving very nearly the speed of light, a 1 ps lifetime becomes  $\sim 1$  cm of travel

# And you can do lots with tracks

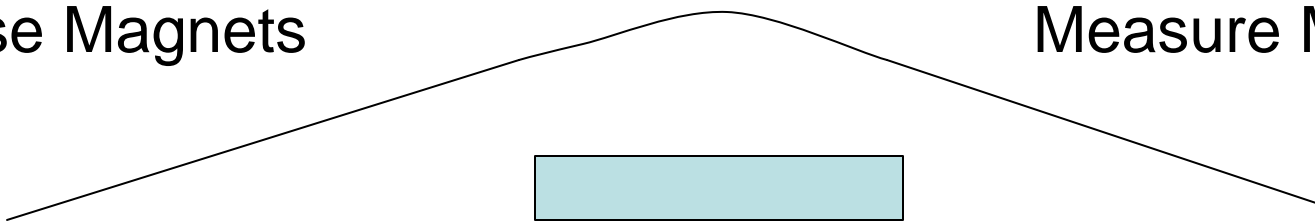
Vertexing "cuts":



**DCL** – CL of DK vertex



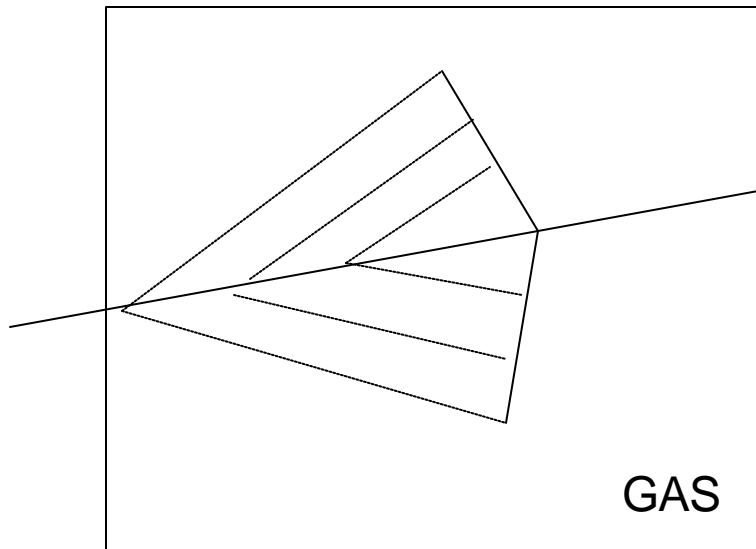
Use Magnets



Measure Momentum

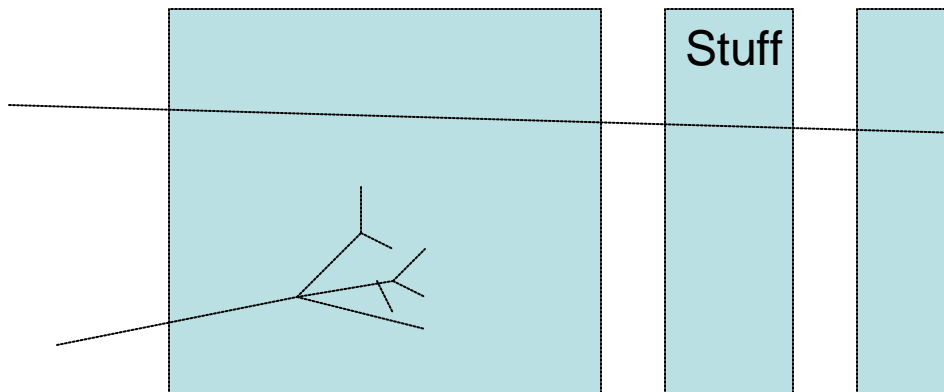


# Signature of other particles



## Cherenkov Detectors

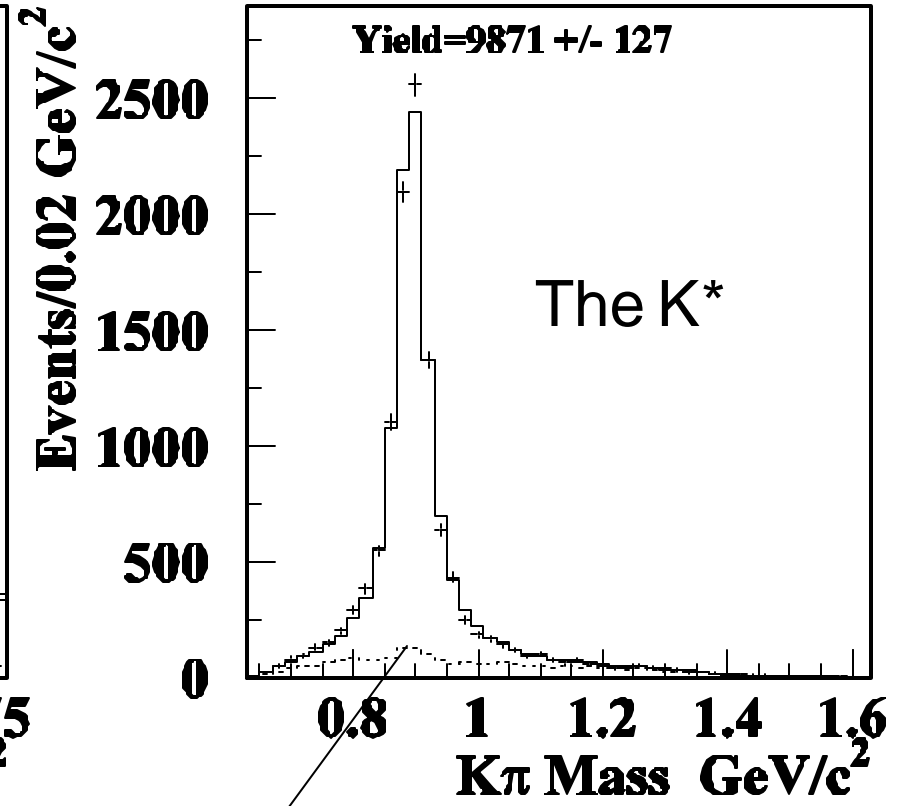
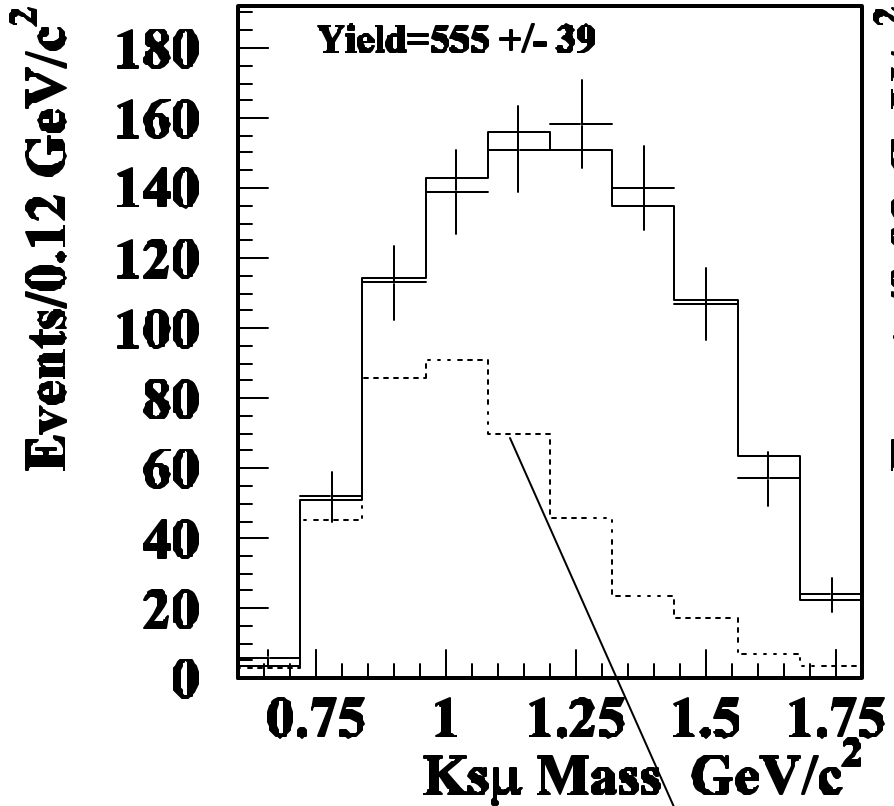
Particles moving faster than  $c/n$  in a gas, make a “shock Wave” of light (Mass sensitive)  
(assuming  $n(\lambda)$  is  $\sim$ constant)



## Muon Detectors

Muons have a much higher chance of penetrating material than pions, electrons...

# Form invariant Quantities (Mass)



Since Ks decays into  
2 pions, this is the mass  
of the 2 pions and a muon

Estimate Backgrounds

(Interesting since K\* can decay into a Kshort and a neutral pion as well as a charged kaon and a charged pion)

# And be especially careful!

- The  $K^*$  has a lifetime  $< 10^{-20}$ s
- But the  $K_{\text{short}}$  has a lifetime of 90 ps

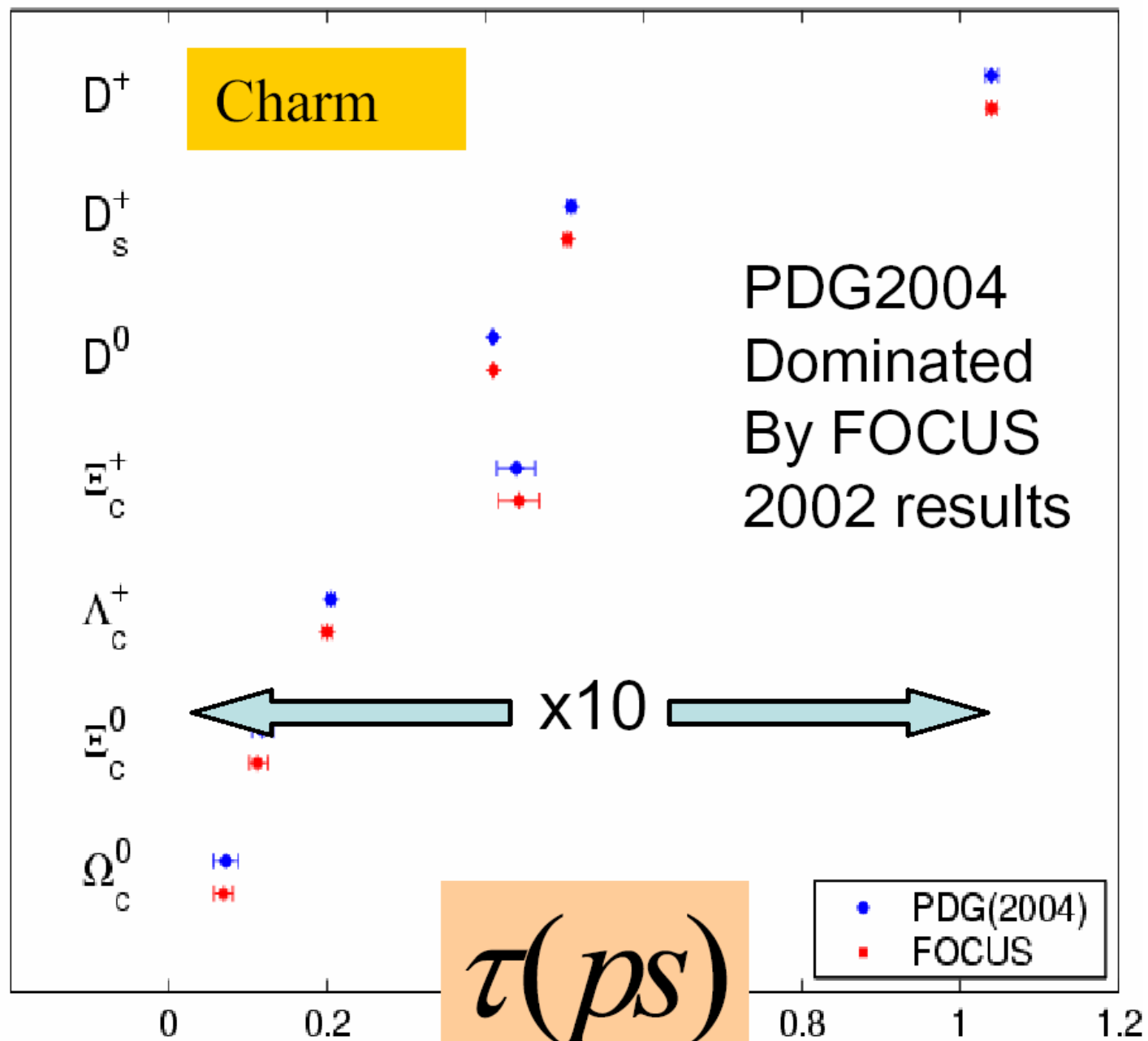
Only a fraction of the  $K_{\text{short}}$ s decay  
in the same area as the D meson

But that fraction is in a VERY well  
understood part of the detector....

Because the super strength of FOCUS  
is the measurement of short lifetimes

# FOCUS Lifetimes Comparison

From ICHEP04 talk  
by Ian Shipsey



But to be sure, you vary lots of things,  
and see the effect on the ratio

Systematic Contribution	Value
Normalized $K_S^0$ Mass Cut	0.008
Secondary Vertex Location	0.017
$K_S^0$ Vertex Location	0.013
Muon Magnet Consistency	0.012
Muon Momentum Cut	0.008
$M_{\text{pole}}$ and $f_-/f_+$ variation	0.015
Contribution from fit variations	0.013
S-wave Fraction ( $K^*$ ratio only)	0.003

# So you can compare with confidence

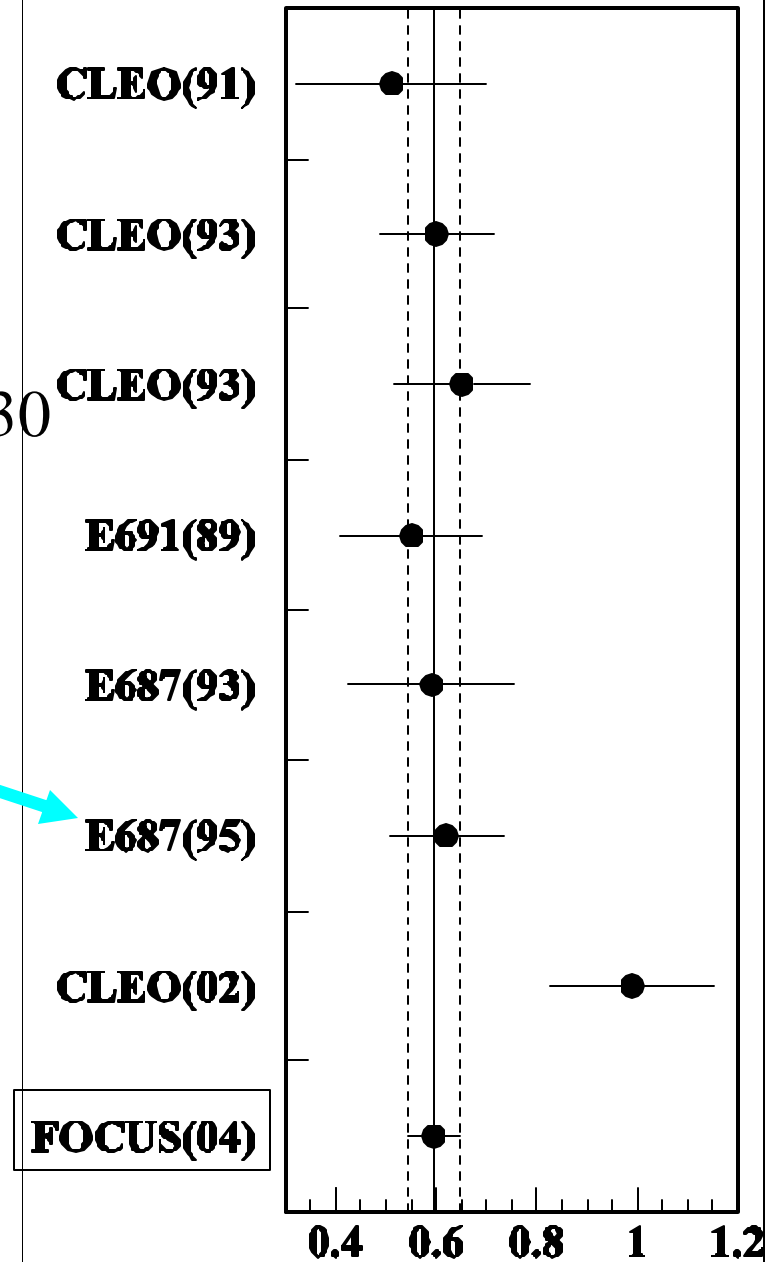
Final **FOCUS(04)** Result:

$$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} m^+ n)}{\Gamma(D^+ \rightarrow \bar{K}^0 m^+ n)} = 0.594 \pm 0.043 \pm 0.030$$

Old Thesis Paper:

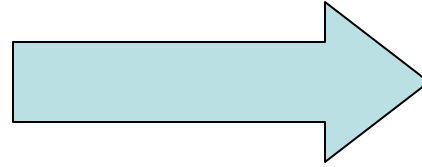
$$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} m^+ n)}{\Gamma(D^0 \rightarrow K^- m^+ n)} = 0.62 \pm 0.11$$

Looks like ratio is  
closer to 0.5 again



# And Compare to Models etc.

Models cluster around the data results

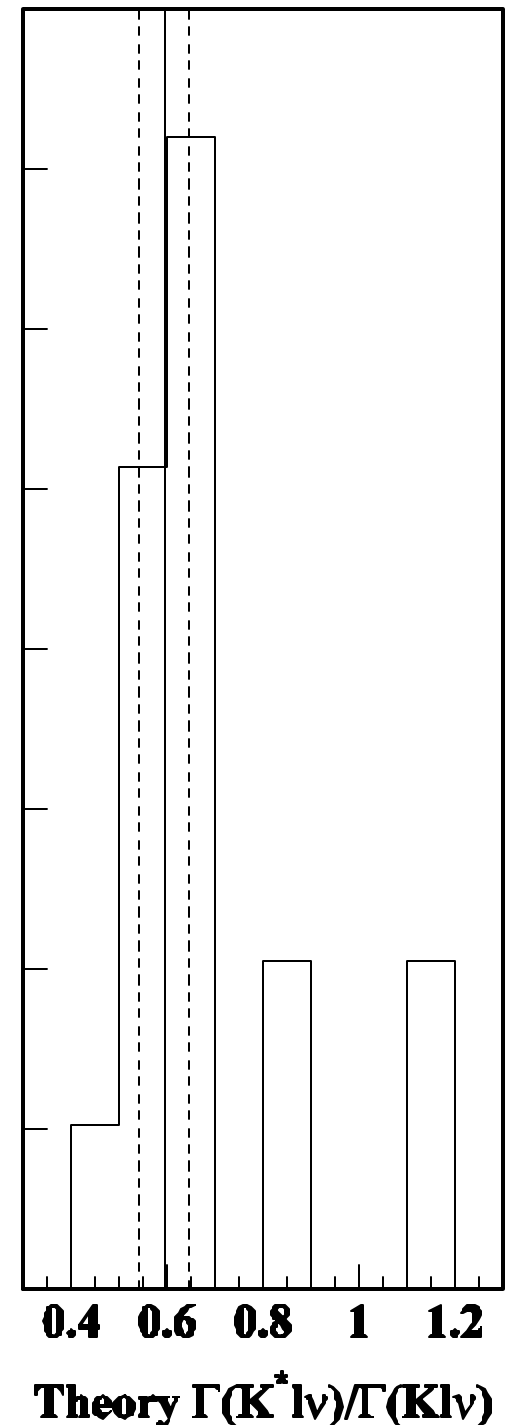


$$\Gamma D^+(\mathbf{m}) / \Gamma D^+(e) = 1.42 \pm 0.22$$

$$\Gamma D^+(\mathbf{m}) / \Gamma D^0(\mathbf{m}) = 1.15 \pm 0.15$$

$$\Gamma D^+(\mathbf{m}) / \Gamma D^0(e) = 1.02 \pm 0.13$$

New result agrees with neutral D meson



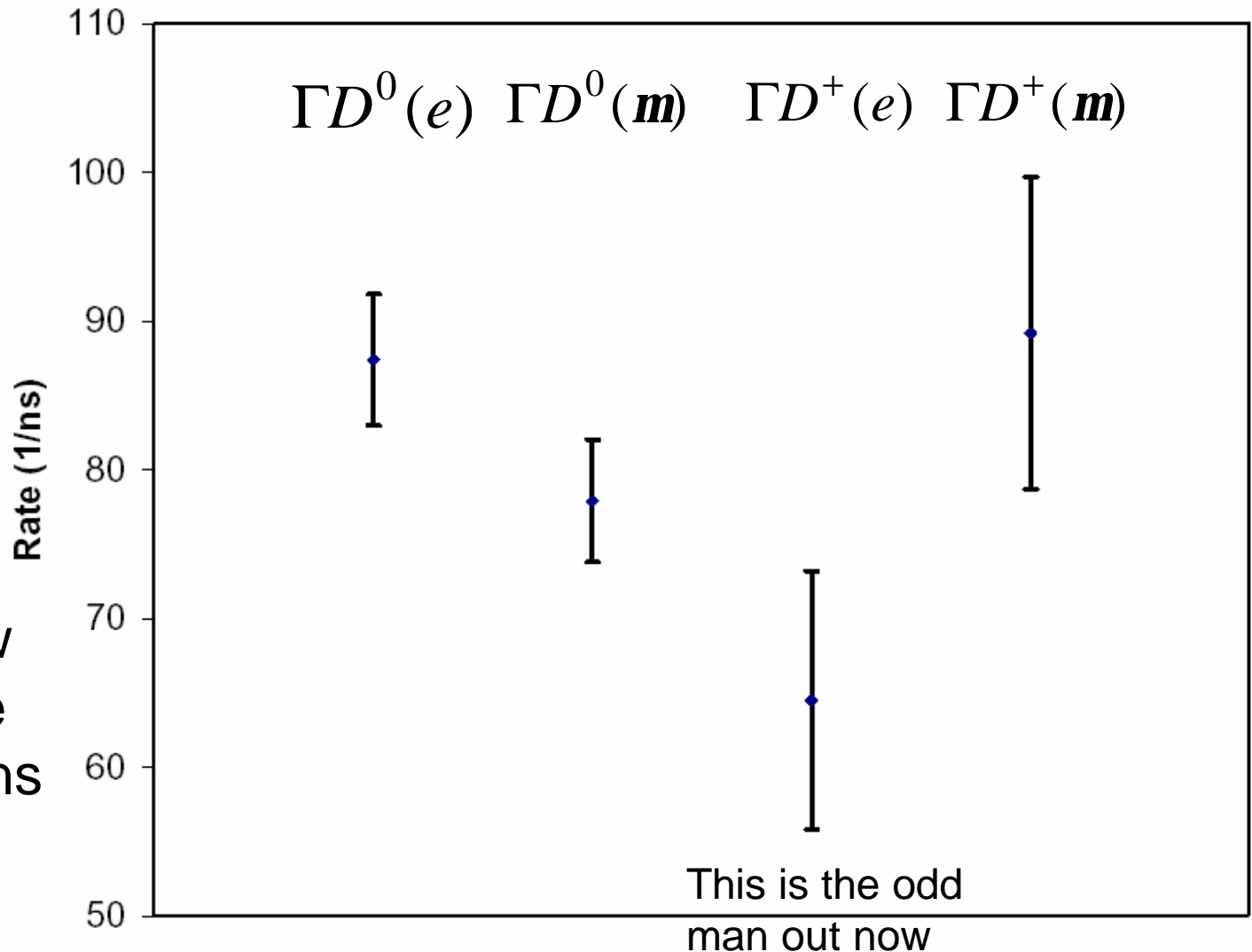
Experiment	Quantity	Result
CLEO(91)[5]	$\frac{\Gamma(D^0 \rightarrow K^{*-} e^+ \nu)}{\Gamma(D^0 \rightarrow K^- e^+ \nu)}$	$0.51 \pm 0.18 \pm 0.06$
CLEO(93)[6]	$\frac{\Gamma(D^0 \rightarrow K^{*-} e^+ \nu)}{\Gamma(D^0 \rightarrow K^- e^+ \nu)}$	$0.60 \pm 0.09 \pm 0.07$
CLEO(93)[6]	$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} e^+ \nu)}{\Gamma(D^+ \rightarrow \bar{K}^0 e^+ \nu)}$	$0.65 \pm 0.09 \pm 0.10$
E691(89)[13]	$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} e^+ \nu)}{\Gamma(D^0 \rightarrow K^- e^+ \nu)}$	$0.55 \pm 0.14$
E687(93)[14]	$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu)}{\Gamma(D^0 \rightarrow K^- \mu^+ \nu)}$	$0.59 \pm 0.10 \pm 0.13$
E687(95)[15]	$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu)}{\Gamma(D^0 \rightarrow K^- \mu^+ \nu)}$	$0.62 \pm 0.07 \pm 0.09$
CLEO(02)[4]	$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} e^+ \nu)}{\Gamma(D^+ \rightarrow \bar{K}^0 e^+ \nu)}$	$0.99 \pm 0.06 \pm 0.07 \pm 0.06 (\pm 0.12)^a$
FOCUS(04)	$\frac{\Gamma(D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu)}{\Gamma(D^+ \rightarrow \bar{K}^0 \mu^+ \nu)}$	$0.594 \pm 0.043 \pm 0.030$

<sup>a</sup> The PDG00 [16] error for  $\Gamma(D^+ \rightarrow \bar{K}^0 \ell^+ \nu) / \Gamma_{Total}$ , omitted [18] in the CLEO [4] result, is shown in parentheses.

That measurement that got everything started, wasn't really a measurement!



# In terms of our original interest



Errors grow since these comparisons tend to be indirect

# And the conclusion of the analysis

- The ratio of the  $K^*/K$  ratio is indeed closer to  $\frac{1}{2}$  than 1 (evidence is overwhelming now)
- Charged D meson rate into a kaon and an electron is probably underestimated
- Some other conclusions ancillary to this talk  
(It's in PLB 598, pg 33-41)

And then the Results started to come in from the summer conferences (and preprints)

# I'm happiest about this one

- At ICHEP04 in Beijing (Jiangchuan Chen)

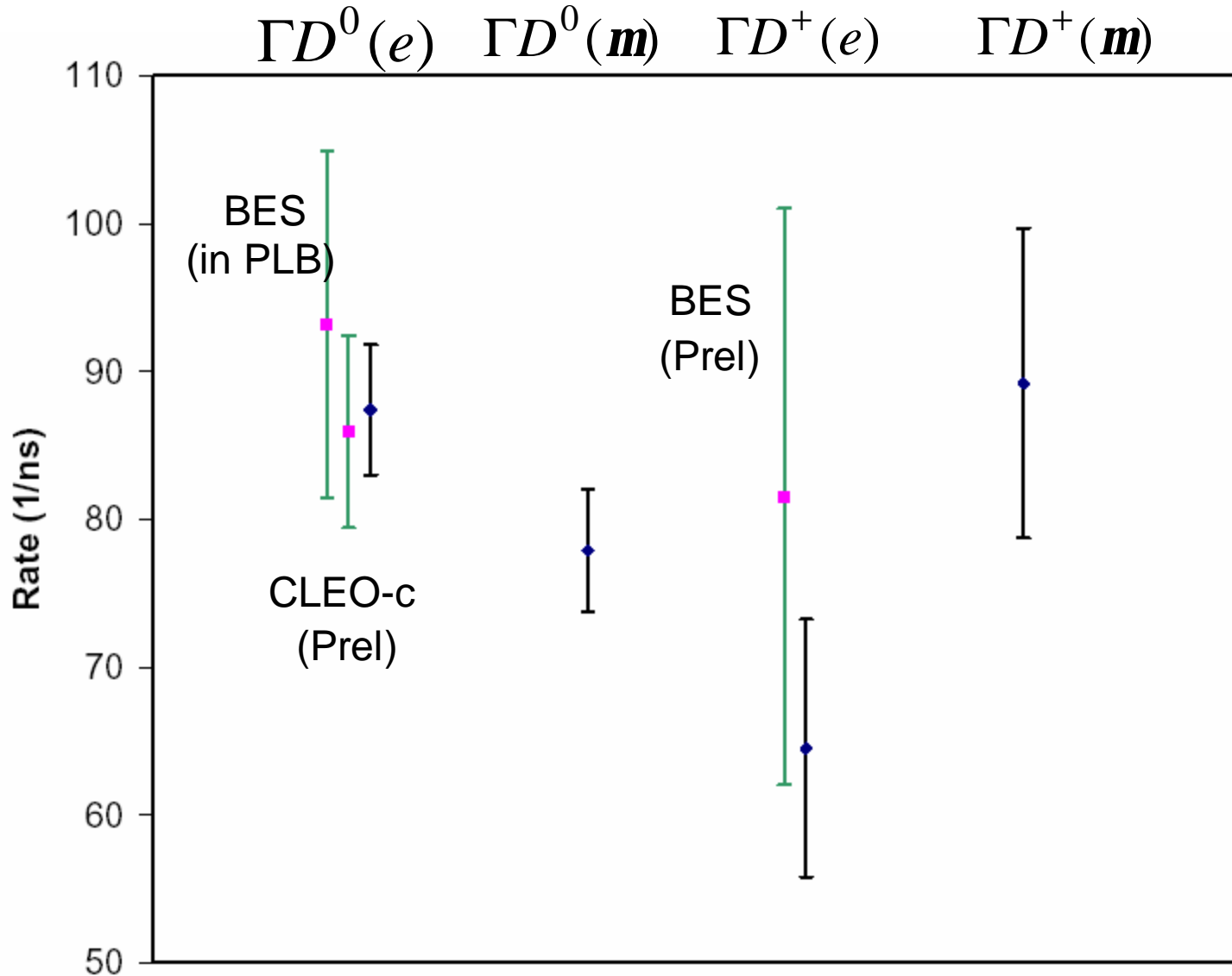
## ◆ Ratio of partial width

	BES	MARK III	PDG04
$\frac{\Gamma(D^0 \rightarrow K^- e^+ \nu_e)}{\Gamma(D^+ \rightarrow \bar{K}^0 e^+ \nu_e)}$	$1.15 \pm 0.29 \pm 0.09$ preliminary	$1.44 \pm 0.62$	$1.4 \pm 0.2$

Obtained based on the branching fractions quoted from PDG04

BES preliminary results consistent to the isospin conservation held in D meson semi-leptonic decays

# And here's where they all fall.



# And you might be interested to know

KTeV has done a measurement

$$\frac{\% K_L^0(\mathbf{m})}{\% K_L^0(e)} = 0.6640 \pm 0.0026 \quad (\text{new pred} = 0.666 \pm 0.0029)$$

Which gives:

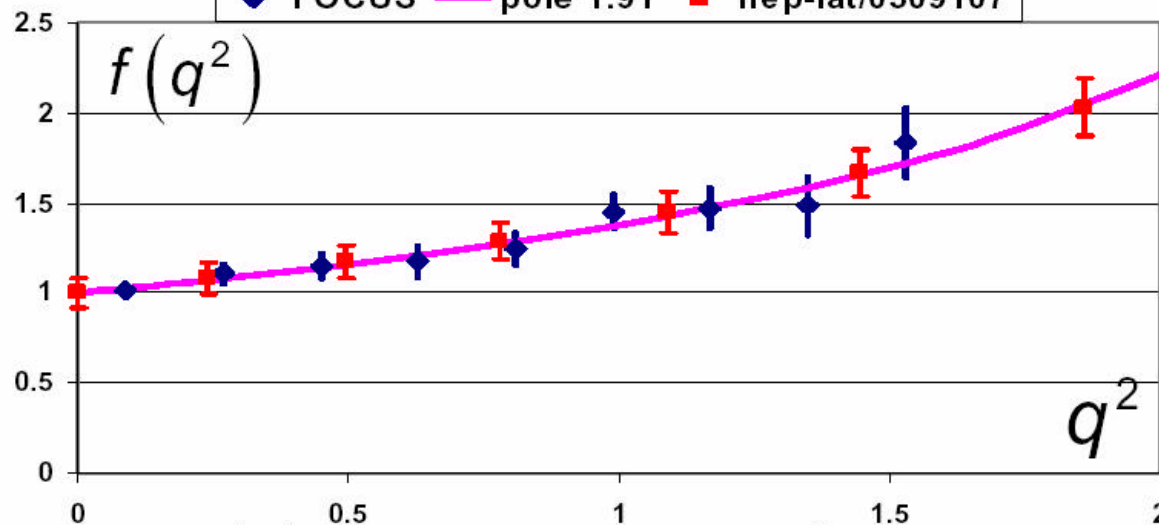
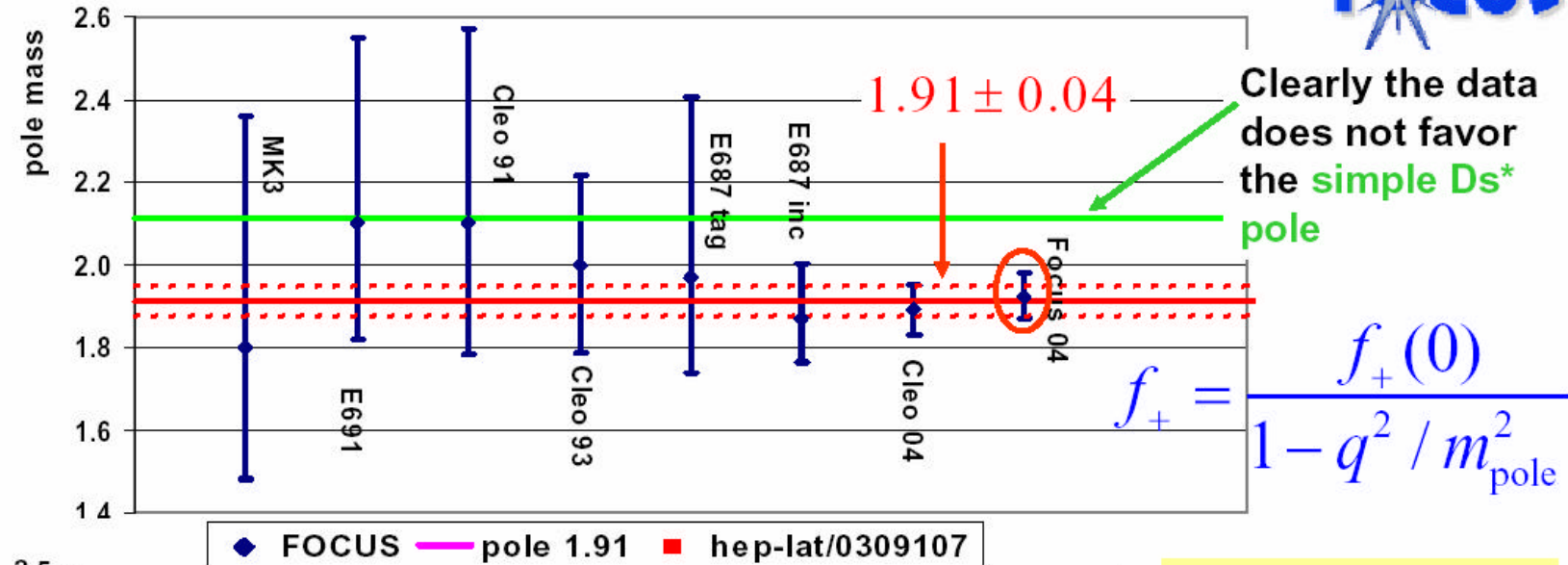
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9982 \pm 0.0019$$

And the new CLEO-c measurements give:

$$\frac{D^0 \rightarrow K^{*-} e^+ \mathbf{n}}{D^0 \rightarrow K^- e^+ \mathbf{n}} = 0.59 \pm 0.09$$

And now we are beginning to really measure the Matrix Element

## Pole mass fits and LQCD calculations



Agreement with LQCD unquenched calculation is also very impressive!

This calculation predicts  $f_{+\pi}(0)/f_{+K}(0)=0.85$  in great agreement with the FOCUS result.

# Round up and the future

>For now this charm decay crisis looks solved  
(electron mode for the charged D appears low)

- The ratio of  $K^*/K$  is grounded around 0.6
- Interest in these decays is accelerating!  
(Big  $e^+/e^-$  samples are finally appearing)

Hmmmm... Couplings measured in a few years!

- There's actually more FOCUS data sitting around, and with a lot of work, the muon sample could double, and we could do  $e$ 's and maybe measure that swell matrix element...

# Lot of references used in this talk

The Particle Data Group tables and summaries (2000-2004)

The ICHEP04 talks of Ian Shipsey and Jiangchuan Chen

The DPF04 talk of Lorenzo Agostino

All the references in PLB 598, pg 33-41

The Klong papers hep-ex 0406001 v1 & 0406003 v1

I'm sure to have missed a couple...

## And special thanks to:

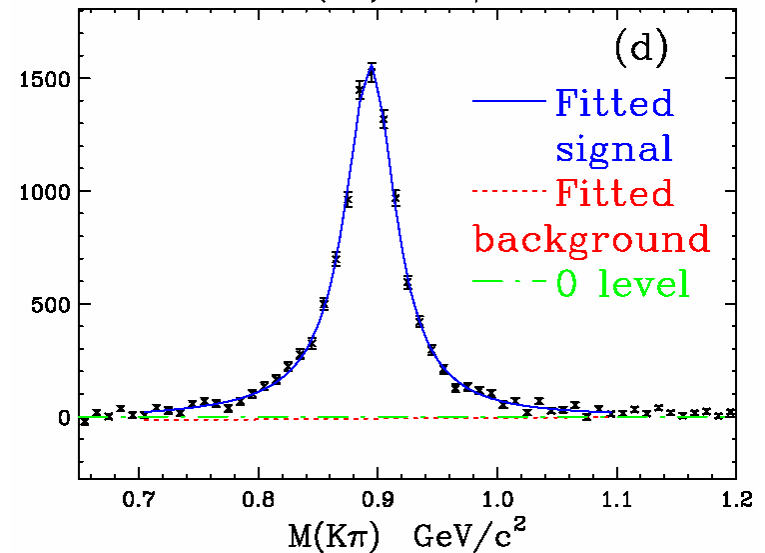
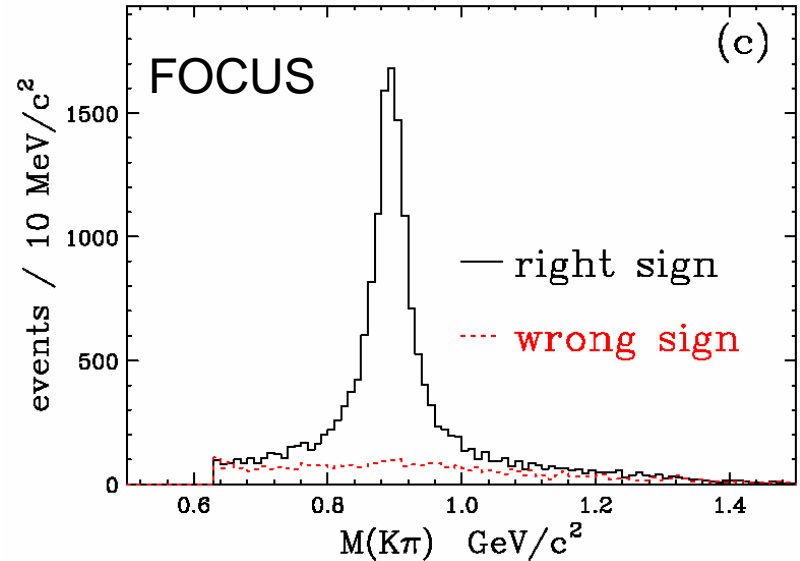
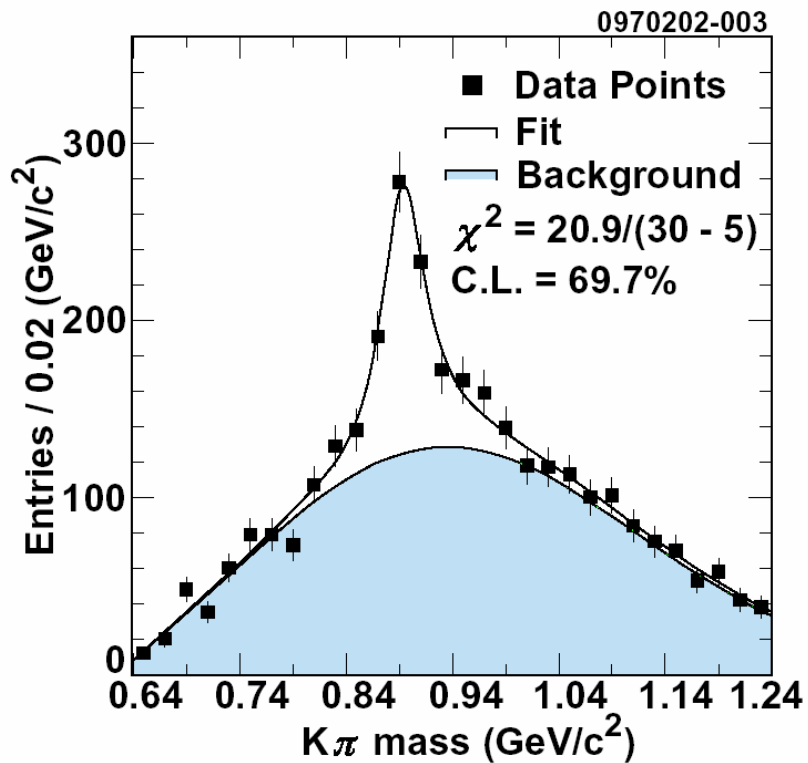
University of Colorado and Vanderbilt for supporting the sabbatical during which this work was performed

(And Cynthia for projector help)



# Differences in the $K^*$ analysis

Other Experiment



# Semileptonic Charm Decays

More than just CKM measurement tools...

## "Simple" Equation:

(D decay, No form factors,  
V decays to spin 0 particles)

$$\frac{d^2\Gamma}{d\cos\mathbf{q}_V d\cos\mathbf{q}_\ell} \propto$$

Neutrino is left handed

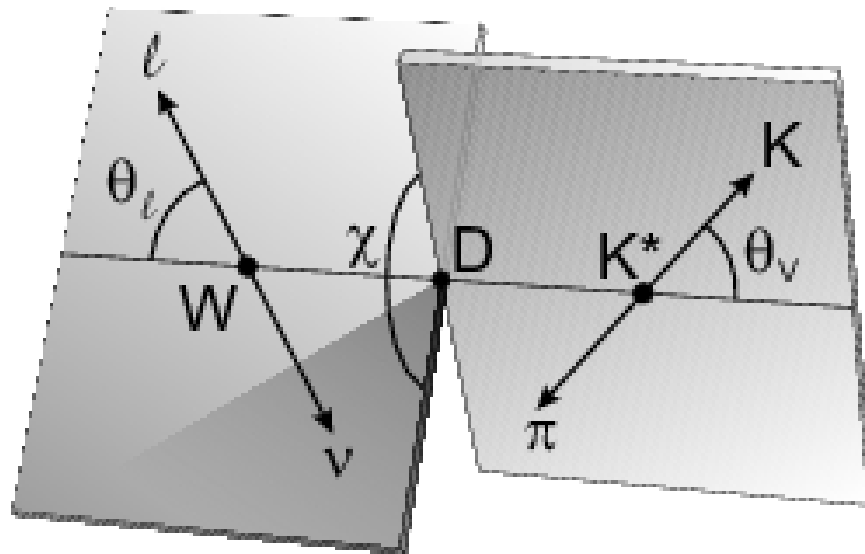
$$\{(1 + \cos\mathbf{q}_\ell)^2 \Gamma_+ + (1 - \cos\mathbf{q}_\ell)^2 \Gamma_-\} \sin^2\mathbf{q}_V$$

Prefers W spin along muon, e

$$+ 4 \sin^2\mathbf{q}_\ell \cos^2\mathbf{q}_V \Gamma_0$$

Prefer  $L_z=0$

V products spinless



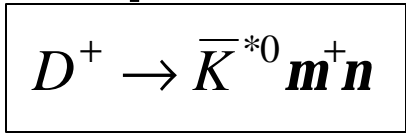
## Gets Complicated...

Scalar Resonance?

CP?

Form Factors

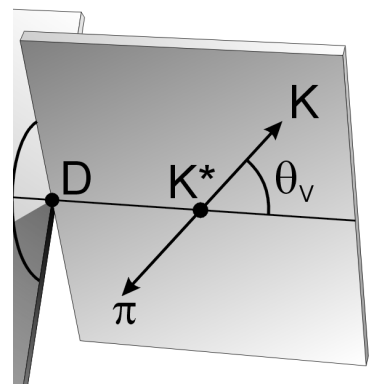
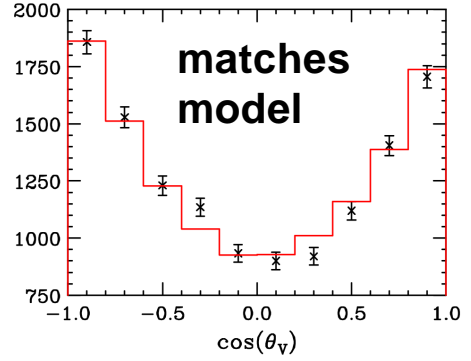
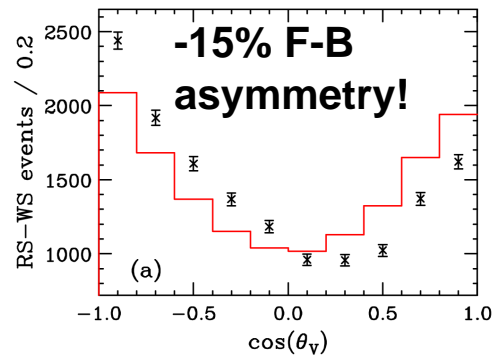
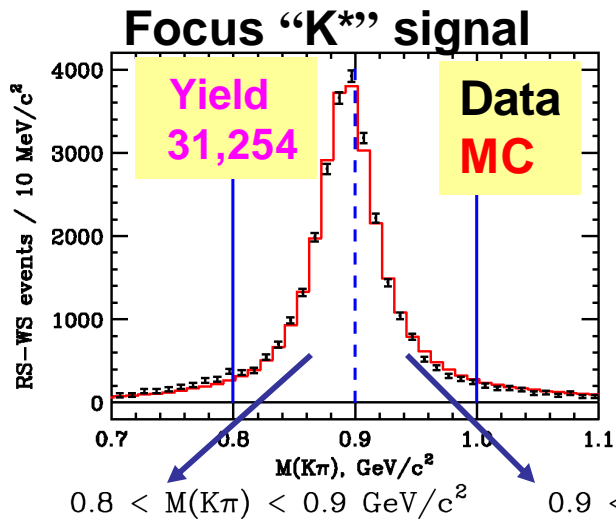
# FOCUS saw discrepancies in the data



$$d^5\Gamma$$

$$\frac{d^5\Gamma}{dm_{kp} dq^2 d \cos q_V d \cos q_l dc} \propto$$

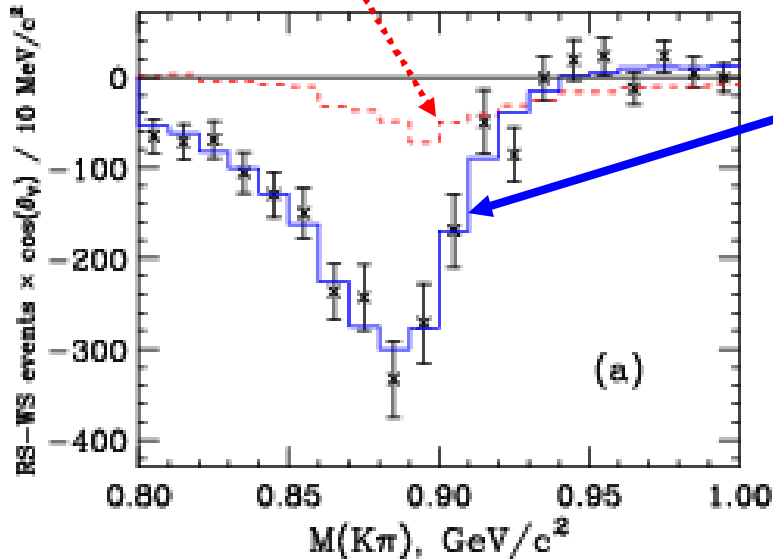
$$\left[ \begin{aligned} & \sin q_V (1 + \cos q_l) e^{ic} B_{K^{*0}} H_+(q^2) \\ & - \sin q_V (1 - \cos q_l) e^{-ic} B_{K^{*0}} H_-(q^2) \\ & - 2 \cos q_V \sin q_l B_{K^{*0}} H_0(q^2) \end{aligned} \right]^2$$



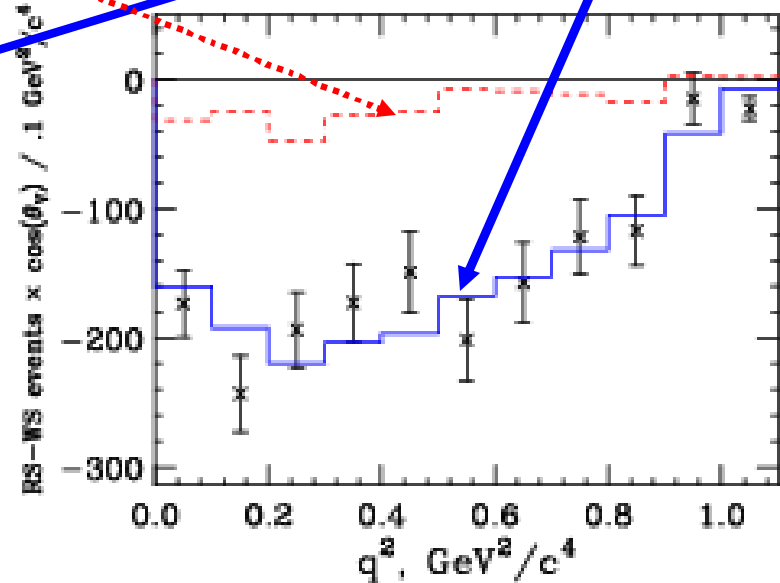
# FOCUS added a term, things got better

Signal Events weighted  
by  $\text{avg}(\cos \mathbf{q}_V)$ :

No added term

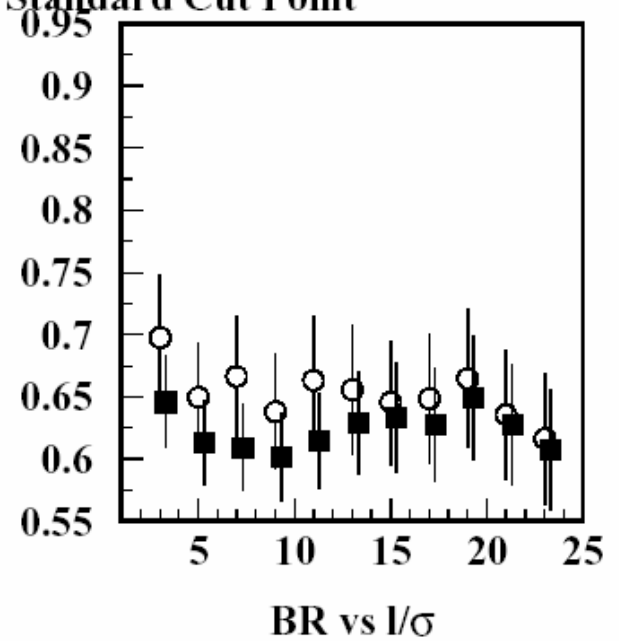
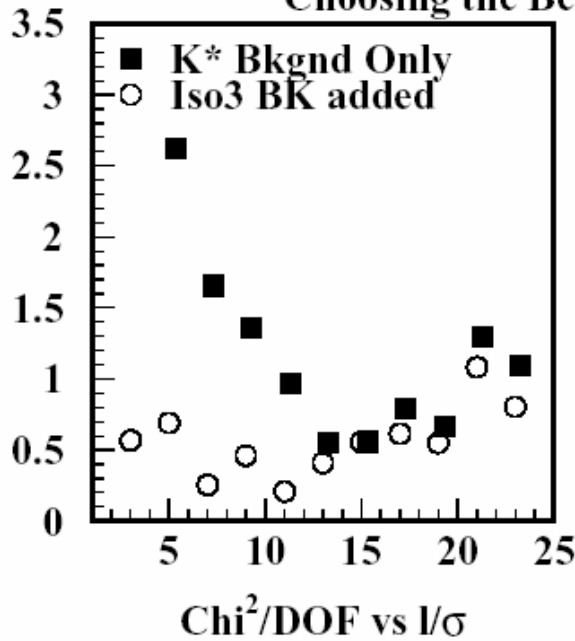


$$\frac{d^5 \Gamma}{dm_{kp} dq^2 d \cos \mathbf{q}_V d \cos \mathbf{q}_\ell dc} \propto \left\{ \begin{array}{l} (1 + \cos \mathbf{q}_\ell) \sin \mathbf{q}_V e^{ic} B_{K^*0} H_+(q^2) \\ - (1 - \cos \mathbf{q}_\ell) \sin \mathbf{q}_V e^{-ic} B_{K^*0} H_-(q^2) \\ - 2 \sin \mathbf{q}_\ell (\cos \mathbf{q}_V B_{K^*0} + \underbrace{Ae^{id}}_{\text{L=0 ansatz}}) H_0(q^2) \end{array} \right\}^2$$

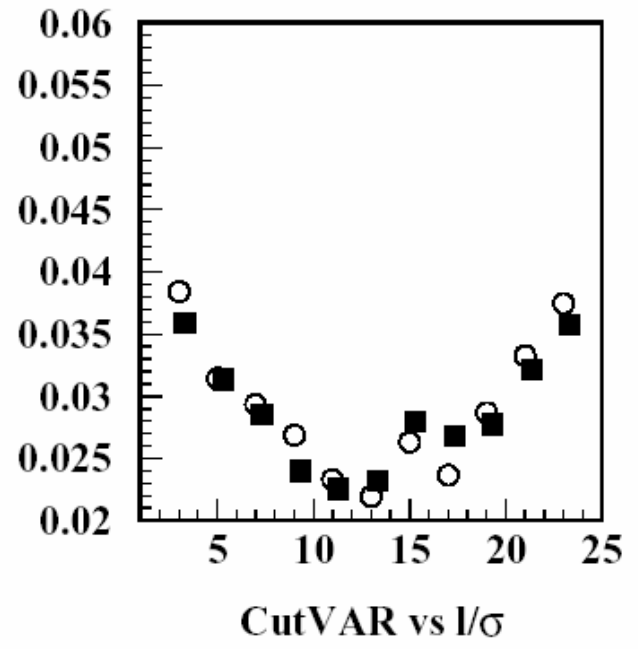
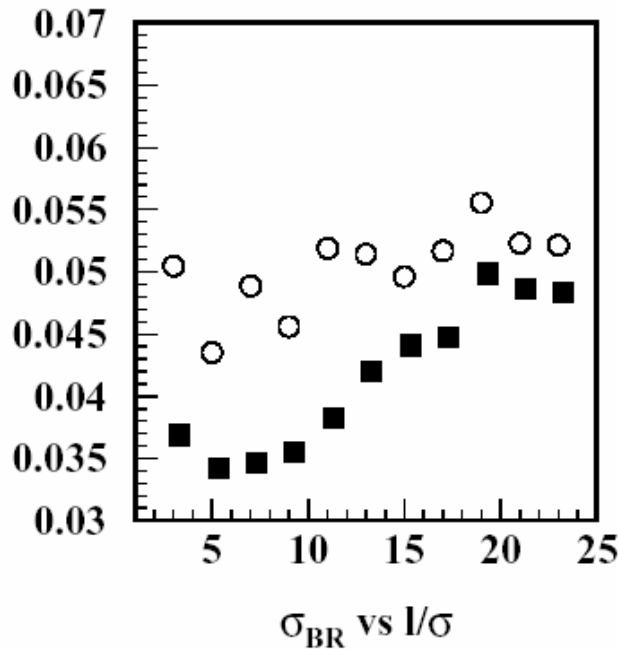


Here we are taking a background from the data where events likely had an extra track and comparing it to a background dominated by  $K^*\mu\nu$ . (Which happens for sure when the signal is very clean)

### Choosing the Best Standard Cut Point

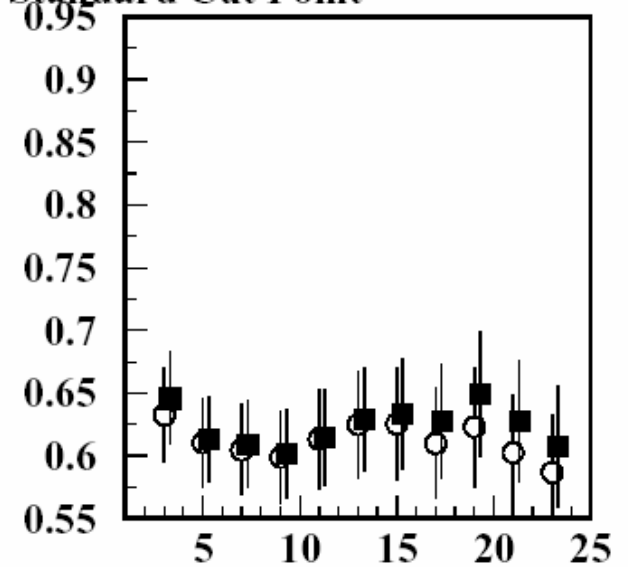
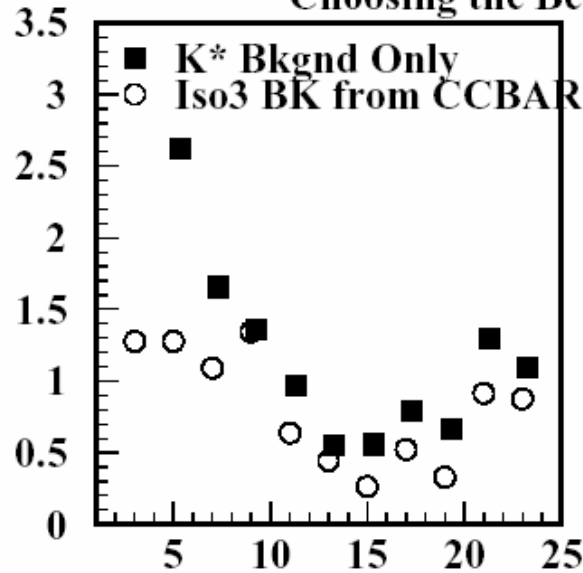


The tighter you cut, the less statistics you have. But it's worse here since the data based background has a small component of signal in it: we've correlated the signal and background in a bad way!



But you can get the same sort of background from the simulation. Since the majority of unmodelled junk occurs at low separation ( $l/\sigma$ ) we expect agreement to be better at higher separation.

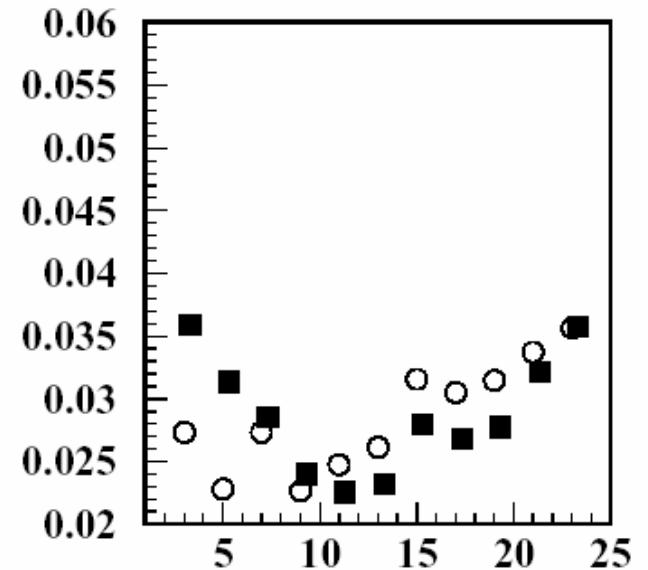
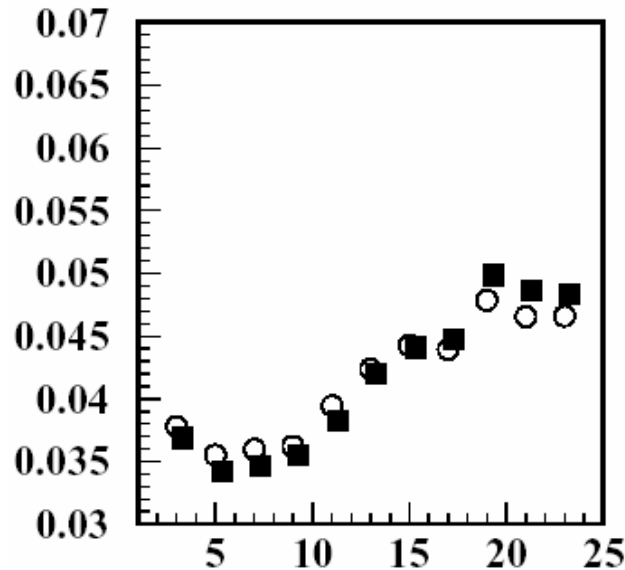
### Choosing the Best Standard Cut Point



$\chi^2/\text{DOF}$  vs  $l/\sigma$

BR vs  $l/\sigma$

Except in the simulation, we know when the signal is leaking into the background, so we can remove it *a-priori*, and the error “bonus” goes away!



$\sigma_{\text{BR}}$  vs  $l/\sigma$

CutVAR vs  $l/\sigma$

# Matrix Element Parameterization

$$M = G_F V_{cs} \left[ f_+(q^2)(p_D + p_K)^\sigma + f_-(q^2)(p_D - p_K)^\sigma \right] (\bar{u}_\nu \gamma_\sigma (1 - \gamma^5) u_\mu) \quad (1.5.6)$$

which leads to a decay rate in the  $D^0$  center of mass:<sup>[15]</sup>

$$\begin{aligned} \frac{d^2\Gamma}{dE_K dE_\mu} = & \frac{G_F^2}{4\pi^3} |V_{cs}|^2 \\ & \left( |f_+(q^2)|^2 \left[ M_D(2E_\mu(M_D - E_\mu - E_K) - M_D(E_K^{max} - E_k)) + \right. \right. \\ & \left. \left. \frac{1}{4}M_\mu^2(E_K^{max} - E_k) - M_\mu^2(M_D - E_\mu - E_K) \right] + \right. \\ & \left. Re\{f_-(q^2)/f_+(q^2)\} \left[ M_\mu^2((M_D - E_\mu - E_K) - \frac{1}{2}(E_K^{max} - E_k)) \right] + \right. \\ & \left. |f_-(q^2)|^2 \left[ \frac{1}{4}M_\mu^2(E_K^{max} - E_k) \right] \right) \end{aligned} \quad (1.5.7)$$

where we parameterize the form factors:

$$f_\pm(q^2) = \frac{f_\pm(0)}{1 - q^2/M_{D^*}^2}, \quad E_K^{max} = \frac{M_D^2 + M_K^2 - M_\mu^2}{2M_D} \quad (1.5.8)$$

and using the  $K^+$  decay as an example we predict:

$$f_-(0)/f_+(0) = -\frac{M_D^2 - M_K^2}{M_{D^*}^2} = -0.7 \quad (1.5.9)$$

# Evolution of the strong current

$$M = G_F f_K (p_K + p_\pi)^\sigma \left[ \frac{g^{\sigma\delta} - p^\sigma p^\delta / M_{K^*}^2}{q^2 - M_{K^*}^2} \right] (\bar{u}_\nu \gamma_\delta (1 - \gamma^5) \gamma u_e) \quad (1.5.1)$$

Assuming a massive propagator

which can be written:

$$M = G_F \frac{f_K}{q^2 - M_{K^*}^2} \left[ (p_K + p_\pi)^\sigma - \frac{M_K^2 - M_\pi^2}{M_{K^*}^2} (p_K - p_\pi)^\sigma \right] (\bar{u}_\nu \gamma_\sigma (1 - \gamma^5) u_e) \quad (1.5.2)$$

A more general approach is usually taken where one defines the hadronic current in terms of the form factors  $f_+$  and  $f_-$ :

$$M = G_F \left[ f_+(q^2) (p_K + p_\pi)^\sigma + f_-(q^2) (p_K - p_\pi)^\sigma \right] (\bar{u}_\nu \gamma_\sigma (1 - \gamma^5) u_e) \quad (1.5.3)$$

and comparing to the parameterization (1.5.2):

$$\frac{f_-(0)}{f_+(0)} = -\frac{M_K^2 - M_\pi^2}{M_{K^*}^2} = -0.3 \quad (1.5.4)$$