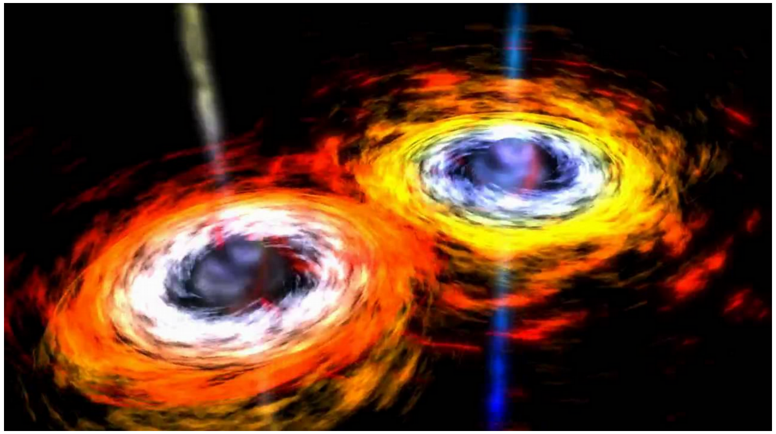
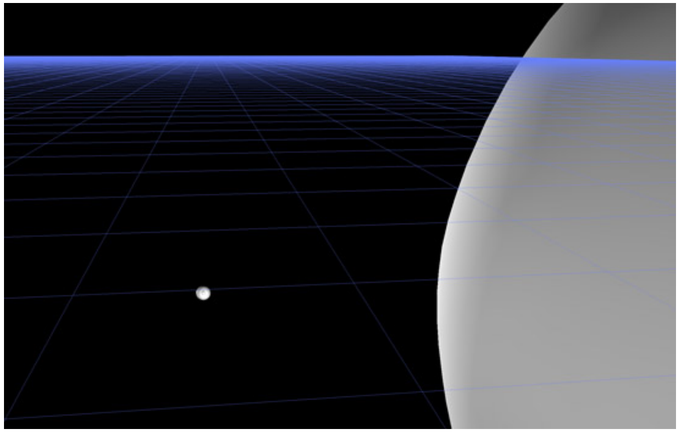
# Gravitational Waves – Sources and Detectors



Orbiting Massive Bodies

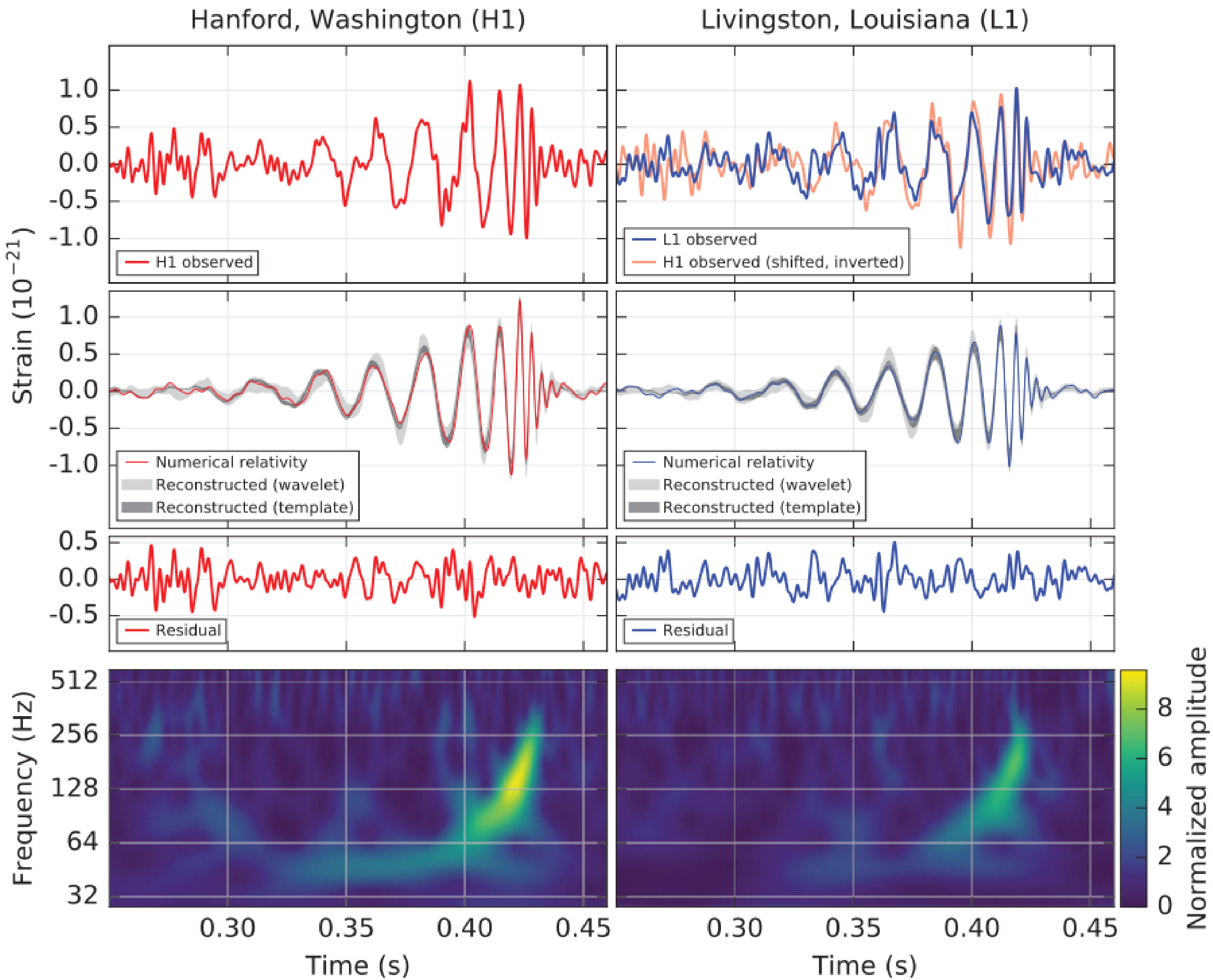
Supernova, NGC7293-Helix Nebula



Extreme Mass Ratio Blackhole Mergers

https://www.rit.edu/news/story.php?id=47936

# Gravitational Wave GW150914




**Direct Observation by LIGO** of a gravitational wave. Announced on 11 February 2016 for a wave on 14 September 2015---event labeled GW150914.

SNR = 24, so a greatre than **5 sigma** detection!

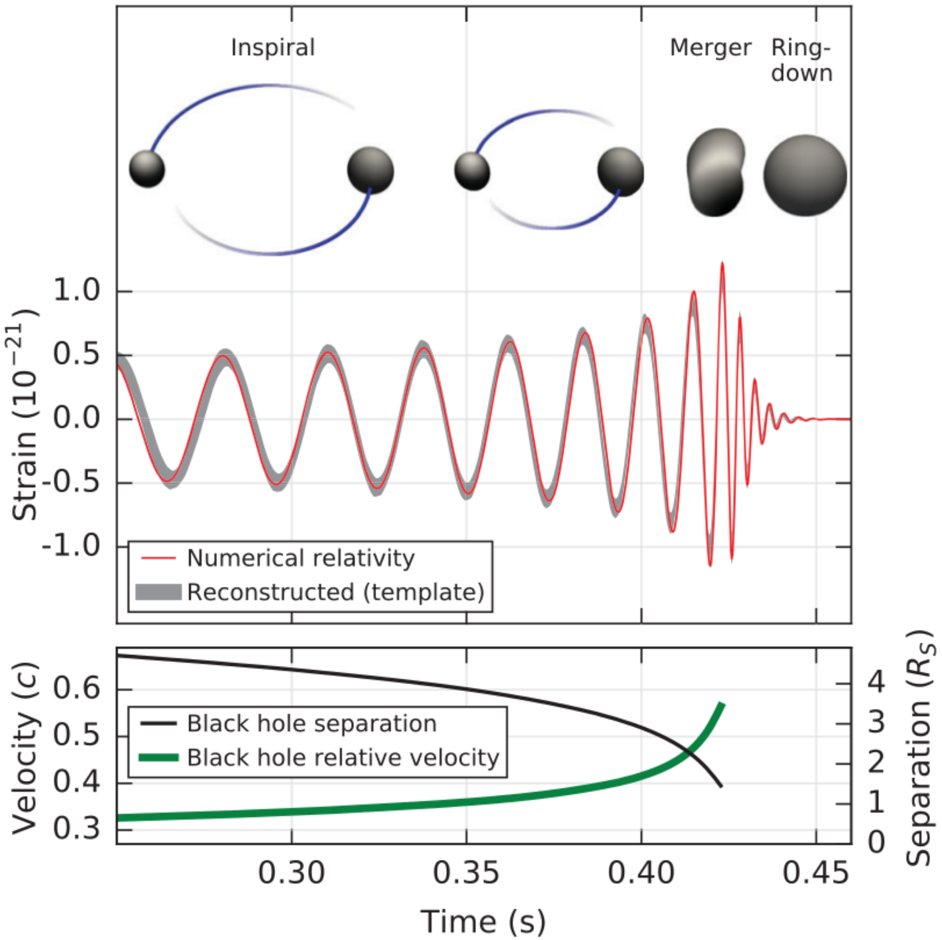
Pretty after filtering out the known seismic noise frequencies.

First at L1 and 6.9ms later at H1.

# Merging Black Holes

ref: http://www.techinsider.io/binary-black-holes-confirmed-gravitional-waves-2016-2

# Interpretation as a Binary Black Hole Merger




Ref: https://losc.ligo.org/events/GW150914/

**GW150914** signal is best fit with the model of a 36 Msun and 29 Msun black hole coalescing into a 62 Msun BH. Radiated 3 Msuns of energy into the GW. About 410 Mpc away, 1.3 billion ly.

Also the discovery of Black Holes!

Oddly heavy BHs compared to best ideas on how BHs are created/formed.

# Second Direct Detection GW151226 and "trigger" LVT151012

**Second Direct Detection by LIGO** called GW151226 which occurred on Chirstmas Day, 2015, in the USA.

SNR = 11 (LHO) and 8 (LLO), so not as pretty, greater than **5 sigma** significance (it could happen by chance).

14 Msun BH orbiting 7.5 Msun BH
21 Msun BH final, radiated about 1 Msun in GW
440 Mpc away (+180 Mpc -190 Mpc)

**Third event called LVT151012.** Called a "trigger" or "candidate."

SNR = 9.7, and *significance* of 1.7 sigma.

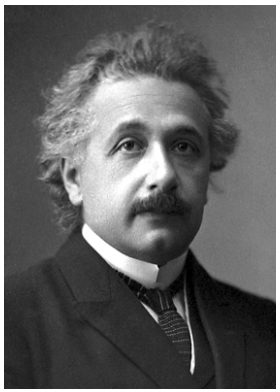
23 Msun BH orbiting 13 Msun BH
35 Msun BH final, radiated about 1 Msun in GW

Ref: 15 June 2016 press conference.
https://losc.ligo.org/events/LVT151012/
https://losc.ligo.org/events/GW151226/

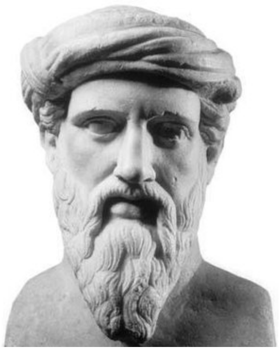
# More about LIGO Tomorrow morning.

# General Relativity is used to describe gravity...spacetime.

Thank you, again, **Albert Einstein**!



GR has a function that lets you calculate distances in spacetime...the **metric**. Think Pythagoras!



$$c^2\, d\tau^2 = g_{00} c^2\, dt^2 - g_{11} dx^2 - g_{22} dy^2 - g_{33} dz^2$$

Find the metric by solving Einstein's Field Equations...just a set of nonlinear, coupled, second order partial differential equations.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

**Matter and Energy, it is what curves space.**

**Geometry, derivatives of the metric.**

**Accelerating Expansion**

# Gravitational Radiation predicted by Einstein in 1916

Einstein predicted the wave-like behavior of small perturbations to the metric in his 1916 paper. The formula for masses was corrected by another physicist later (factor of 2?). Oddly Einstein and his assistants decided it could not exist around 1936 and wrote many papers saying so.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h_{\mu\nu} \ll 1$$

**Linearized** General Relativity

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Like that in the Special Theory of Relativity, Minkowski metric/matrix. Gives invariant:

$$d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2$$

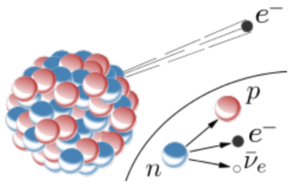
$$\boxed{\frac{1}{c^2}\frac{\partial^2}{\partial t^2} h_{\mu\nu} - \boldsymbol{\nabla}^2 h_{\mu\nu} = 0}$$

**Linearized** General Relativity yields the wave equation in the small perturbation.

Ref: Kennefick, *Traveling at the Speed of Thought*, 2007.

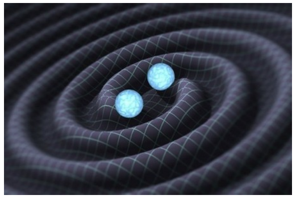
# Gravitational Waves and Neutrinos

History of Discovery in physics not always quick nor easy.  Compare the prediction of Neutrinos and Gravitational Waves.




**Neutrinos:**

- Postulated by W. Pauli in 1930.
- To explain energy, momentum, and spin conservation in beta decay.
- Direct observation not unitl 1956 by Cowan and Reines.  Nobel Prize in 1995.



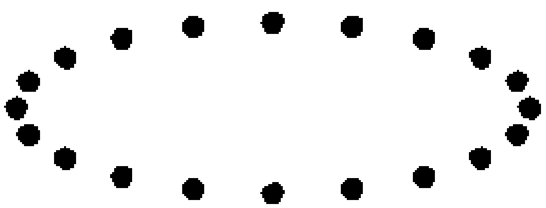
**Gravitational Waves:**

- Consequence of GR, A. Einstein 1916.
- Long, sordid debate on its existence and nature---even in 1970 when most agreed it existed many believed orbiting bodies were **protected** from Grav. Waves.
- Discovery of Pulsar PSR1913+16 by Hulse and Taylor in 1974, eventually showed orbit energy loss and perihelion precession consistent with GWs.  Nobel 1993.
- Direct detection by LIGO in 2016.

# Nature of Gravitational Waves, and $h_{\mu\nu}$

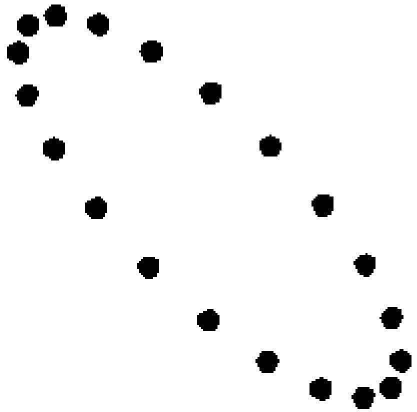
Of all 10 possible components to h, it simplifies to two polarizations. One with a motion that shrinks (grows) in one direction while growing (shrinking) in the other direction.  The other polarization is just rotated by 45 degrees.

$h_{+}$



hplus polarization

$h_{\times}$



hcross polarization

Wave headed into the page, Y up, X right.

# Multipoles - Electrostatic Potential Energy

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \, d^3x'$$

Multipole expansion when far from a complicated source, $|\boldsymbol{x}'| \ll |\boldsymbol{x}|$ .

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left[ \frac{q}{r} + \frac{\mathbf{p} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} + \cdots \right]$$

Monopole Moment $\quad q = \int_{V'} \rho(\boldsymbol{x}') \, d^3x'$

Dipole Moment $\quad \mathbf{p} = \int \mathbf{x}' \rho(\mathbf{x}') \; d^3x'$

Quadrupole Moment $\quad Q_{ij} = \int (3x_i' x_j' - r'^2 \delta_{ij}) \rho(\mathbf{x}') \; d^3x'$

Ref: Jackson, *Classical Electrodynamics*

# Multipoles – EM Radiation

Multipole expansion for electromagnetic radiation, when the source has charge changing in time. $\rho(\boldsymbol{x}')$ is the charge distribution (later for GW it will be the mass distribution!).

Charge is conserved, dq/dt = 0.

Monopole Moment $$\mathcal{P}^{EM} = k\,\mu_0 c\,\dot{q}^2 = 0 \qquad q = \int_{V'} \rho(\boldsymbol{x}')\,d^3x'$$

First non-zero term.

Electric Dipole Moment $$\mathcal{P}^{\mathrm{ED}} = \frac{\mu_0}{6\pi c}\,|\ddot{\mathbf{p}}|^2 \qquad \mathbf{p} = \int_{V'} \mathbf{x}'\,\rho(\mathbf{x}')\,d^3x'$$

Magnetic Dipole Moment $$\mathcal{P}^{\mathrm{MD}} = \frac{\mu_0}{6\pi c^3}\,|\ddot{\mathbf{m}}|^2 \qquad \mathbf{m} = \frac{1}{2}\int_{V'} \mathbf{x}' \times \mathbf{J}(\mathbf{x}')\,d^3x'$$

Electric Quadrupole Moment $$\mathcal{P}^{\mathrm{EQ}} = \frac{\mu_0}{720\pi c^3}\sum \dddot{Q}_{ij}\dddot{Q}_{ij}$$

$$Q_{ij} = \int_{V'} (3x_i'x_j' - \delta_{ij}r^2)\,\rho(\mathbf{x}')\,d^3x'$$

Ref: Brau, *Modern Problems in Classical Electrodynamics*

# Quadrupole Moment

Now $\rho(\boldsymbol{x}')$ is the mass distribution of the source.

Mass is conserved, dM/dt = 0.

Monopole Moment $\mathcal{P}^{EM} = k\,\frac{G}{c}\,\dot{M}^2 = 0$

$$M = \int_{V'} \rho(\boldsymbol{x}')\, d^3x'$$

Center of Mass
does not change.

*Electric* Dipole Moment $\mathcal{P}^{\rm ED} = k\,\frac{G}{c^3}\,|\ddot{\mathbf{p}}|^2 = 0$

$$\mathbf{p} = \int_{V'} \mathbf{x}'\, \rho(\mathbf{x}')\, d^3x'$$

Angular Momentum
does not change.

*Magnetic* Dipole Moment $\mathcal{P}^{\rm MD} = k\,\frac{G}{c^5}\,|\ddot{\mathbf{m}}|^2 = 0$

$$\mathbf{m} = \frac{1}{2}\int_{V'} \mathbf{x}' \times \boldsymbol{J}_{mass}(\mathbf{x}')\, d^3x'$$

*Electric* Quadrupole Moment $\mathcal{P}^{\rm EQ} = \frac{G}{5c^5} \sum \dddot{Q}_{ij}\dddot{Q}_{ij}$ **First non-zero moment! Quadrupole**

$$Q_{ij} = \int_{V'} (3x'_i x'_j - \delta_{ij} r^2)\, \rho(\mathbf{x}')\, d^3x'$$

Ref: Misner, Thorne, and Wheeler, *Gravitation*

# Quadrupole Moments

- m1 fixed and m2 in circular orbit
- quadrupole moment is:

$$\begin{pmatrix} -a^2 \operatorname{Sin}[t\,\omega]^2\, m_2 & a^2 \operatorname{Cos}[t\,\omega] \operatorname{Sin}[t\,\omega]\, m_2 & 0 \\ a^2 \operatorname{Cos}[t\,\omega] \operatorname{Sin}[t\,\omega]\, m_2 & -a^2 \operatorname{Cos}[t\,\omega]^2\, m_2 & 0 \\ 0 & 0 & -a^2\, m_2 \end{pmatrix}$$

- monopole moment is: m1+m2
- dipole moment is:

$$\begin{pmatrix} 0 \\ 0 \\ a^2\, \omega\, m_2 \end{pmatrix}$$


$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} a \operatorname{Cos}[t\,\omega] \\ a \operatorname{Sin}[t\,\omega] \\ 0 \end{pmatrix}$$

# GW strain for Circular Orbit

$$h_0 = \frac{r_{s1} \cdot r_{s2}}{r \cdot R}$$

$$\frac{\omega_s^2}{c^2} = \frac{(r_{s1} + r_{s2})}{2\,R^3}$$

$$h_+(t) \quad = \quad h_o\left(\frac{1 + \cos^2 i}{2}\right) \cos 2\omega_s t$$

$$h_\times(t) \quad = \quad h_o \, \cos i \, \sin 2\omega_s t$$

- rs1 is 2GM_1/c^2, Schwarzschild radius for mass M_1, etc.
- r is distance from Earth to system.
- R is the separation of the two bodies.



ref: Maggiore around Eqn. 3.332.

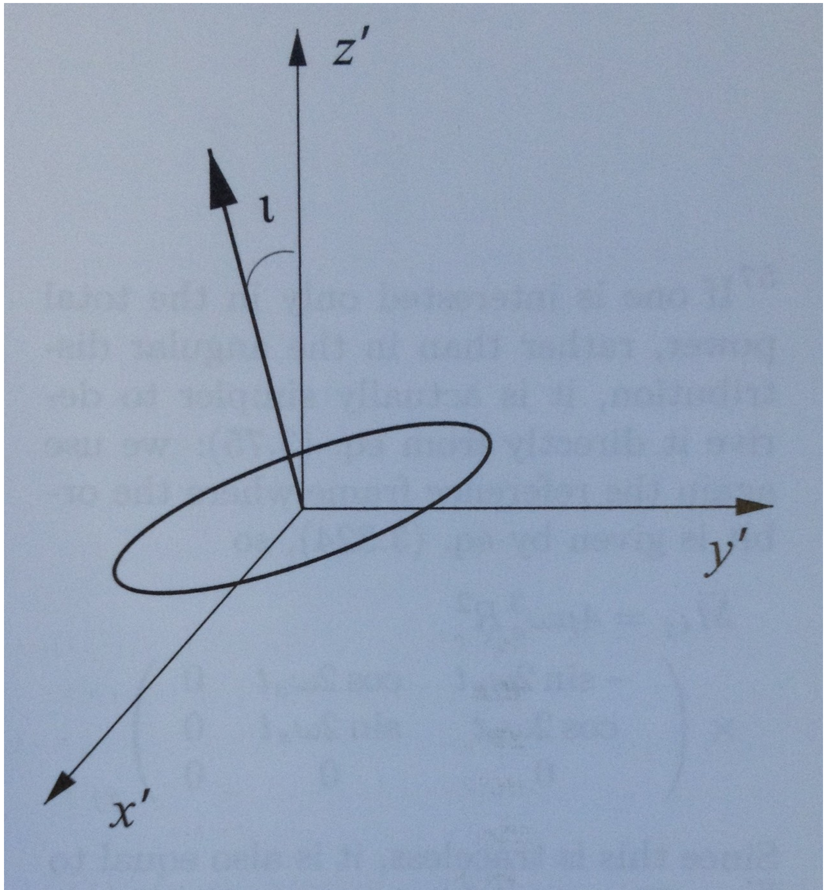
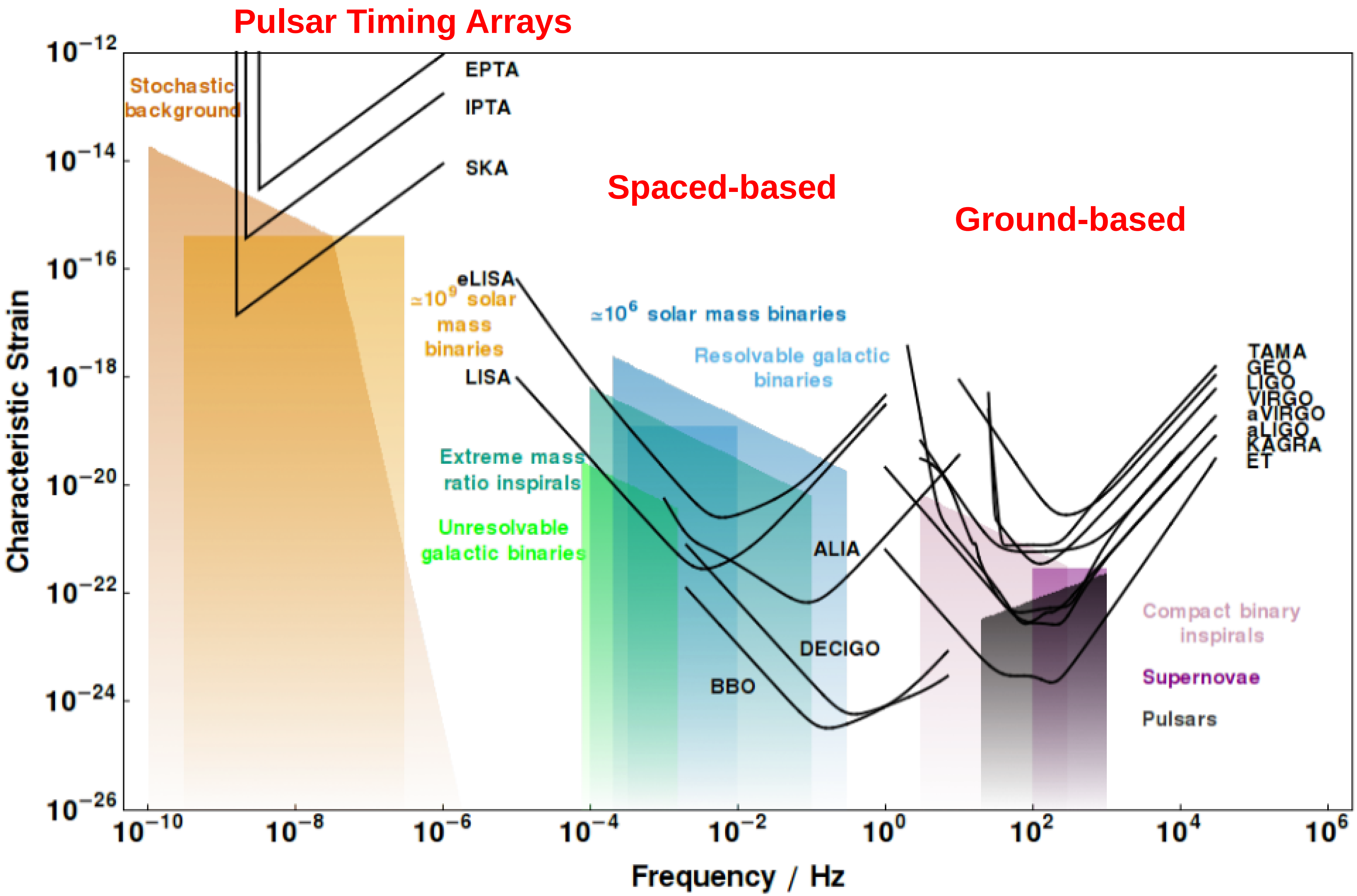



**Fig. 3.6** The geometry of the problem in a frame $(x', y', z')$ where a fixed observer is at large distance along the positive $z'$ axis. The normal to the orbit makes an angle $\iota$ with the $z'$ axis.

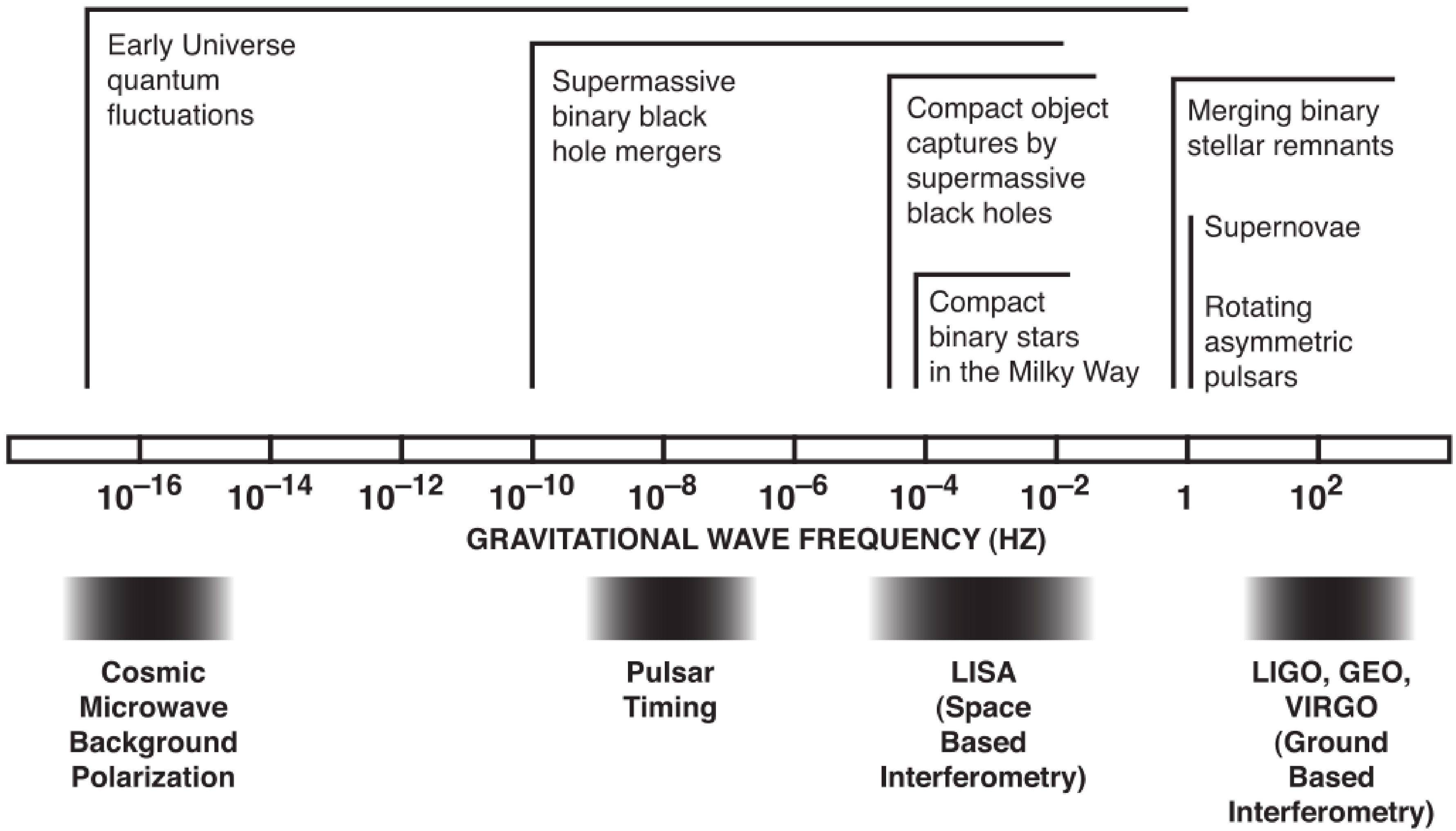
# Strain for binary system GW150914

- Now work it for our **GW150914**, 36 Msun, 29 Msun, at distance of 410 Mpc (1.3 billion ly),
- And separation of how much?
- Check the strain on slide 2.

# Strain Curves from Moore et al.



# Gravitational Wave Spectrum



# Ground-Based, aka aLIGO

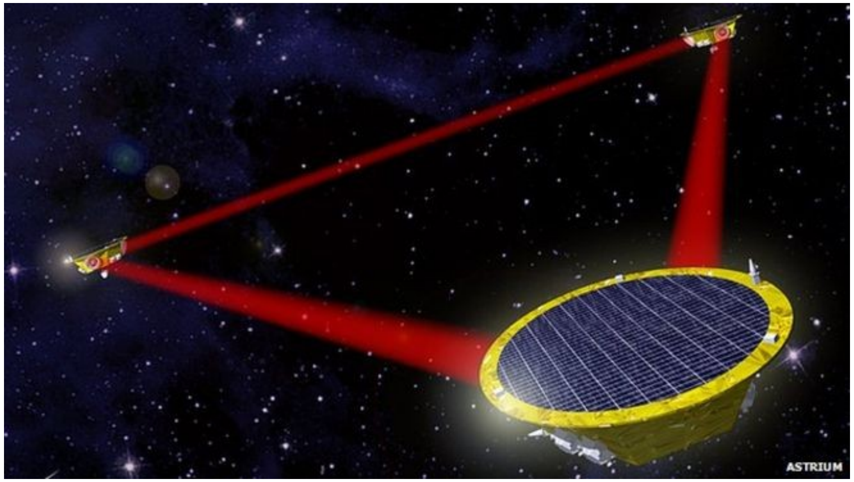
- Both interferometers using light as a clock to measure the change in lengths.
- **More tomorrow:)**
- Besides interferometers there are also still “bars.”

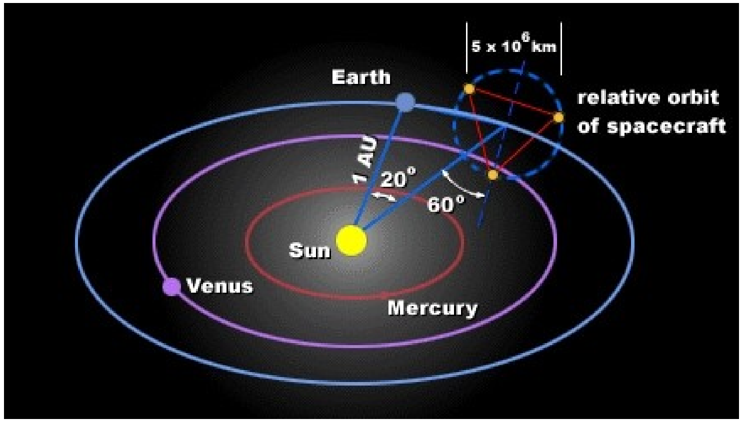
# Space-Based, aka LISA

- Laser Interferometer Space Antenna, LISA, is a set of satellites with lasers bouncing in a triangle arrangement.
  - No seismic effects!
  - Very long baseline, 5e9 m or 1e9 m each leg.
  - Control of craft suble, “drag-free flying” with free test mass inside as reference.
- Currently mostly European eLISA with some NASA/US support. eLISA could launch in 2032.
- LISA Pathfinder satellite experiment tested several components for eLISA, early 2016.

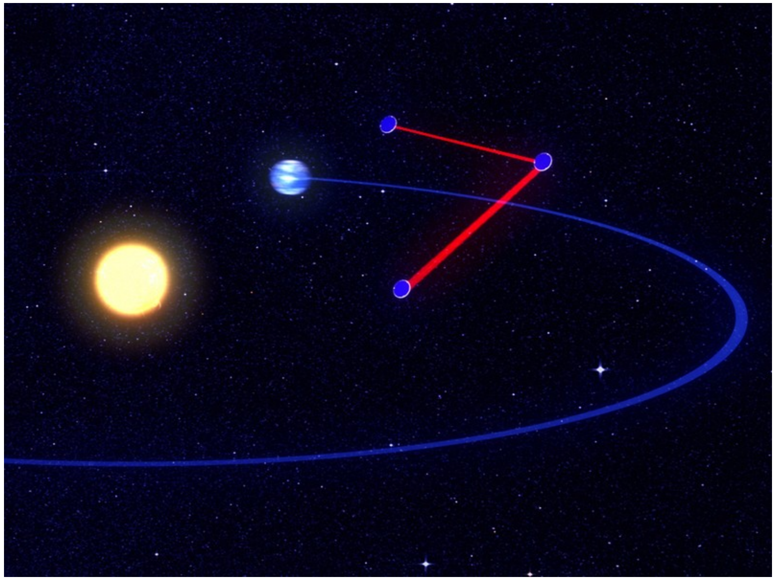
# LISA and eLISA

- LISA has a full equilateral triangle of lasers and baseline of 5e9 m.  Not funded.



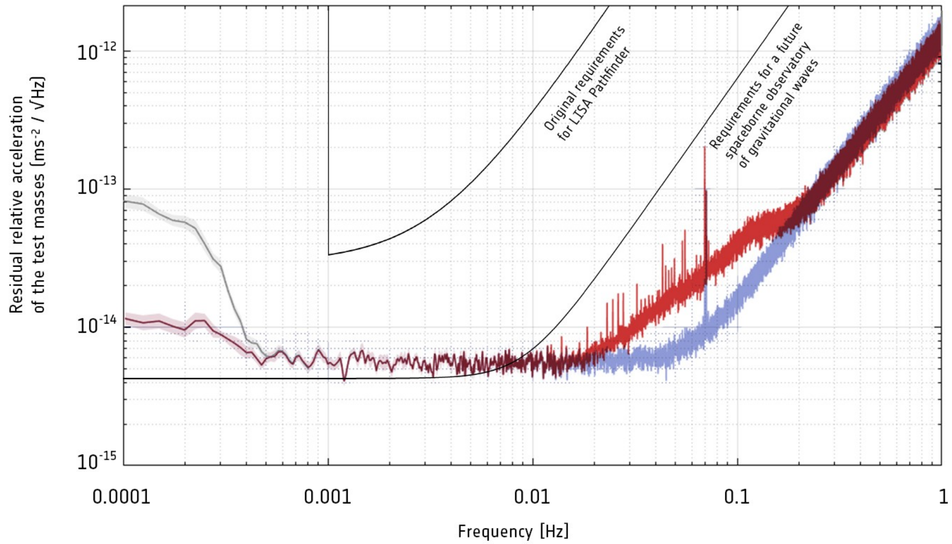

- eLISA uses two legs of the equilateral triangle of lasers and baseline of 1e9 m. Successful LISA Pathfinder mission and LIGO detection means likely to go forward and maybe “fast-tracked.”



# LISA Pathfinder

- LISA Pathfinder successfully demonstrated the technologies for “drag-free” flight of the eLISA space craft.
  - Orbit is *nearly* freely falling around the sun.
  - Microthrusters used to counter solar wind, etc, and allow a free mass inside to react only to gravity!

- Measured relative acceleration as good as 6e-15 m/s^2 per root Hz (tomorrow).



# LISA Pathfinder

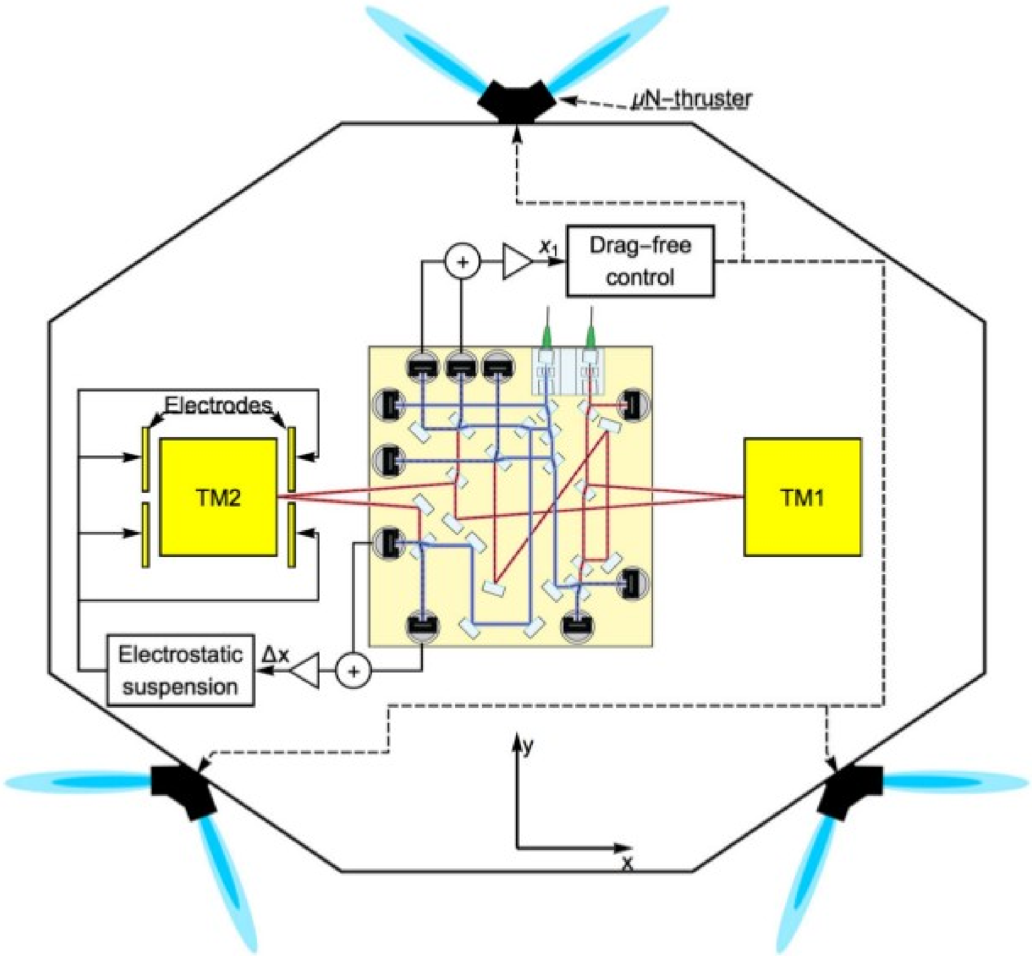
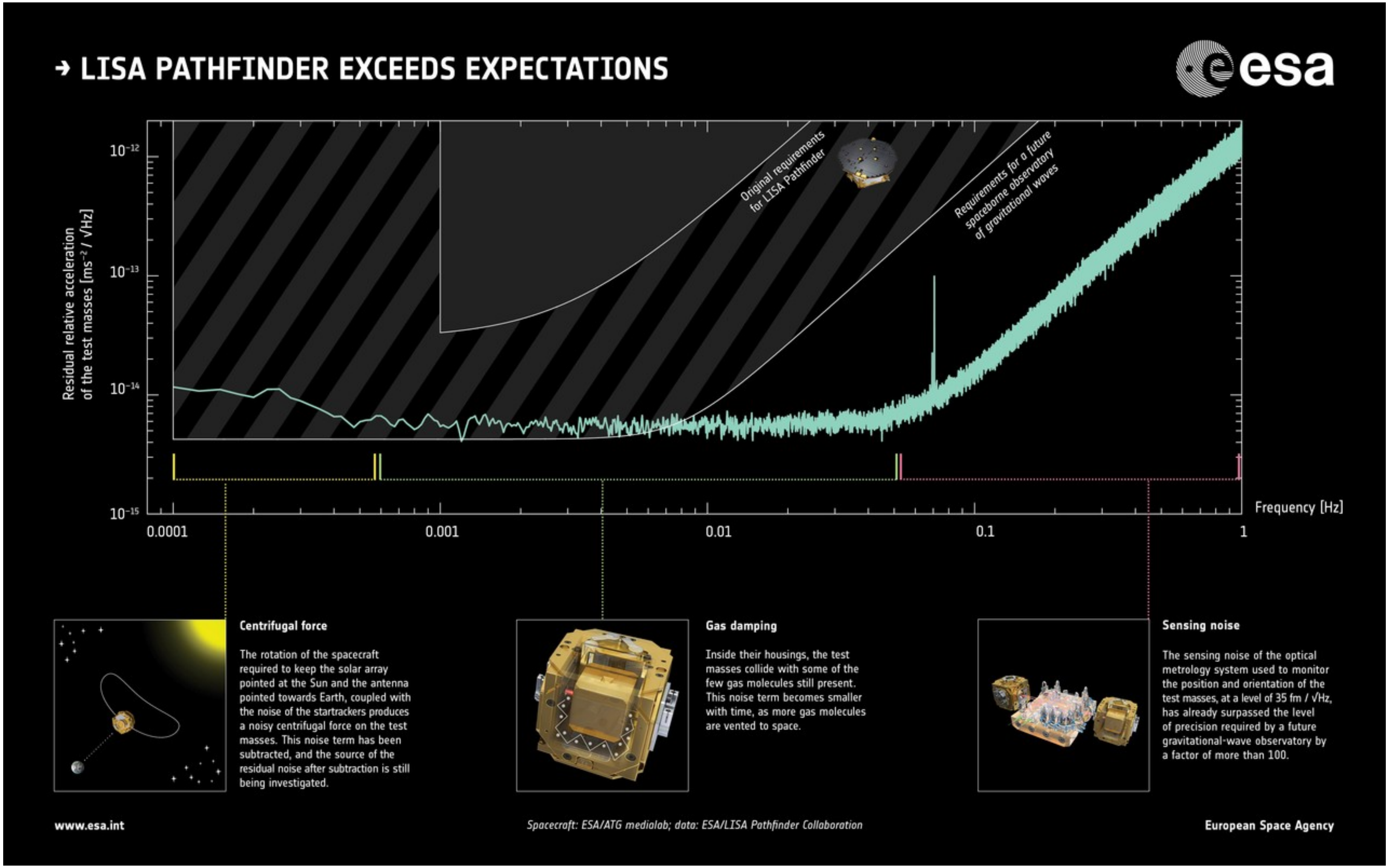



FIG. 2. A schematic of LPF. The figure shows TM1, TM2, and the optical bench beam paths for measuring $\Delta x$ and $x_1$. The measurement of $\Delta x$ drives the electrostatic suspension of TM2, which applies the necessary electrostatic forces by means of the electrodes represented by the four gold plates facing TM2. All other electrodes surrounding the TMs are not shown. The measurement of $x_1$ drives the drag-free control loop that uses the micronewton thrusters to exert forces on the spacecraft. The figure depicts the $x$ and $y$ axes we use in this Letter, while $z$ is normal to the figure.

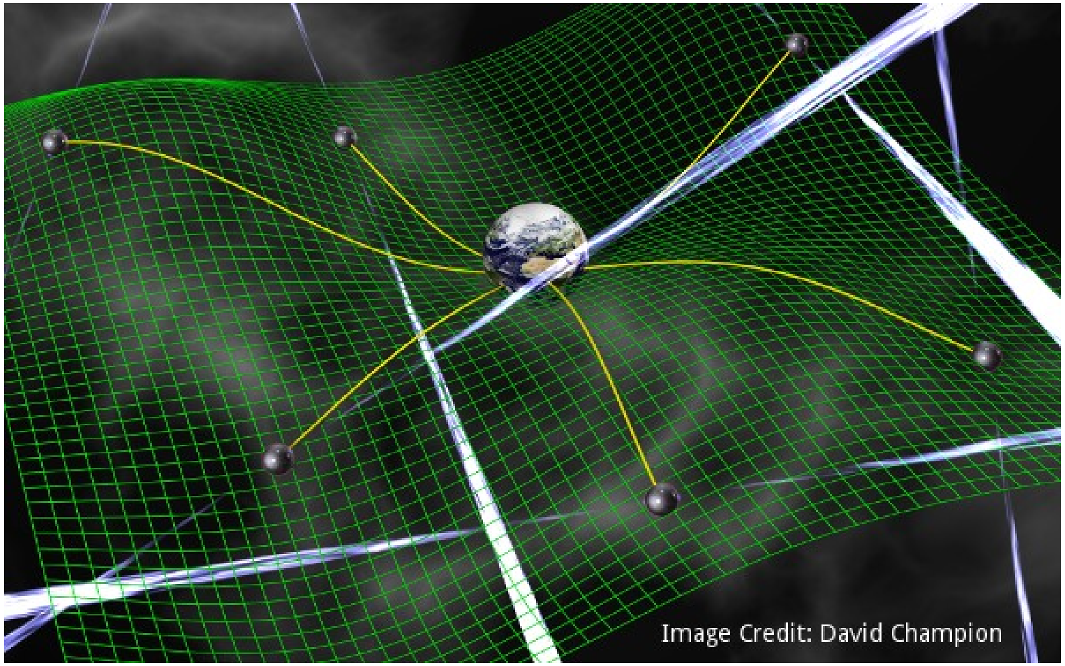
Ref: Armano et al., PRL 2016

# LISA Pathfinder



# Pulsar Timing Arrays

- Millisecond pulsars are very good clocks scattered around the Universe.
- By measuring them once in a while, you can detect a hiccup in the reception of the pulse at Earth.  With enough “clocks” you can decide if there was a gravitational wave and from where it may have come.
- See chart, sensitive to very, very long wavelengths, and thus very, very small frequences.  Size of the Universe sort of waves!




Ref: NANOGrav collaboration

# Pulsar Timing Arrays

- Several collaborations working at radio astronomy facilities to measure millisecond pulsars:
- NANOGrav, US and NASA, using Arecibo and Green Bank.
- The International Pulsar Timing Array uses many radio telescopes.
- The European Pulsar Timing Array
- and others.

# Links

- CalTech GW media assets page.
- Kelly Holley-Bockelman’s TEDx Nashville Talk 2016 about GW150914.
- Pulsar timing array, Nanograv

# Backup

# Some Constants

| | | |
|---|---|---|
| Schwarzschild radius | 2GM/c^2 | 2953 m for Msun |
| GR Units, Mass | GM/c^2 | 1477 m for Msun |
| GR Units, Power | c^5/G | 3.628e52 W |
| GR Units, Energy | c^4/G | 1.210e44 J/m |
| parsec, pc | | 3.09e16 m = 3.262 ly |
| astronomical unit, au | | 149.6e9 m |
| light-year, ly | | 0.946e16 m = 0.307 pc |
| fine structure constant | e^2/(4pi eps0)/hbar c | 0.00730 = 1/137.04 |
| wavelength-energy | h c | 1.24 eV µm |

# SI Prefixes

**Table 5. SI prefixes**

| Factor | Name | Symbol | Factor | Name | Symbol |
|---|---|---|---|---|---|
| $10^{24}$ | yotta | Y | $10^{-1}$ | deci | d |
| $10^{21}$ | zetta | Z | $10^{-2}$ | centi | c |
| $10^{18}$ | exa | E | $10^{-3}$ | milli | m |
| $10^{15}$ | peta | P | $10^{-6}$ | micro | µ |
| $10^{12}$ | tera | T | $10^{-9}$ | nano | n |
| $10^{9}$ | giga | G | $10^{-12}$ | pico | p |
| $10^{6}$ | mega | M | $10^{-15}$ | femto | f |
| $10^{3}$ | kilo | k | $10^{-18}$ | atto | a |
| $10^{2}$ | hecto | h | $10^{-21}$ | zepto | z |
| $10^{1}$ | deka | da | $10^{-24}$ | yocto | y |

# aa

- aa

# Feynman’s Take on Gravitational Radiation

- Consider the spin of the “graviton”---the postulated particle that carries the gravitational force.
- Generically a **spin 1** particle is just like electromagnetism, that is like particles repel and opposites attract. This is NOT what we see in gravity. Just one “charge” and they always attract.
  - True for all odd spin particle theories.
- So consider **spin 0** in some detail, we find that two hot gas clouds are attracted to each other **less than** those same clouds if they were colder. Yet they have more energy when hot.
- So that leaves **spin 2**. That gives rise to a quadrupolar source for radiation.