Gravitational Waves – Sources and Detectors

Orbiting Massive Bodies

Supernova, NGC7293-Helix Nebula

Extreme Mass Ratio Blackhole Mergers

Gravitational Wave GW150914

Direct Observation by LIGO of a gravitational wave. Announced on 11 February 2016 for a wave on 14 September 2015---event labeled GW150914.

SNR = 24, so a greater than 5 sigma detection!

Pretty after filtering out the known seismic noise frequencies.

First at L1 and 6.9ms later at H1.
Merging Black Holes

GW150914 signal is best fit with the model of a 36 Msun and 29 Msun black hole coalescing into a 62 Msun BH. Radiated 3 Msuns of energy into the GW. About 410 Mpc away, 1.3 billion ly.

Also the discovery of Black Holes!

Oddly heavy BHs compared to best ideas on how BHs are created/formed.

Ref: https://losc.ligo.org/events/GW150914/
Second Direct Detection by LIGO called GW151226 which occurred on Christmas Day, 2015, in the USA.

SNR = 11 (LHO) and 8 (LLO), so not as pretty, greater than 5 sigma significance (it could happen by chance).

14 Msun BH orbiting 7.5 Msun BH
21 Msun BH final, radiated about 1 Msun in GW
440 Mpc away (+180 Mpc -190 Mpc)

Third event called LVT151012.
Called a “trigger” or “candidate.”

SNR = 9.7, and significance of 1.7 sigma.

23 Msun BH orbiting 13 Msun BH
35 Msun BH final, radiated about 1 Msun in GW

https://losc.ligo.org/events/LVT151012/
https://losc.ligo.org/events/GW151226/
More about LIGO Tomorrow morning.
General Relativity is used to describe gravity...spacetime.

Thank you, again, Albert Einstein!

GR has a function that lets you calculate distances in spacetime...the metric. Think Pythagoras!

\[ c^2 \, d\tau^2 = g_{00}c^2 \, dt^2 - g_{11} \, dx^2 - g_{22} \, dy^2 - g_{33} \, dz^2 \]

Find the metric by solving Einstein’s Field Equations...just a set of nonlinear, coupled, second order partial differential equations.

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu} \]

Einstein predicted the wave-like behavior of small perturbations to the metric in his 1916 paper. The formula for masses was corrected by another physicist later (factor of 2?). Oddly Einstein and his assistants decided it could not exist around 1936 and wrote many papers saying so.

\[ g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu} \quad h_{\mu \nu} \ll 1 \]

\[ \eta_{\mu \nu} = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \]

\[ \frac{1}{c^2} \frac{\partial^2}{\partial t^2} h_{\mu \nu} - \nabla^2 h_{\mu \nu} = 0 \]

**Linearized** General Relativity

Like that in the Special Theory of Relativity, Minkowski metric/matrix. Gives invariant:

\[ d\tau^2 = dt^2 - dx^2 - dy^2 - dz^2 \]

**Linearized** General Relativity yields the wave equation in the small perturbation.

Gravitational Waves and Neutrinos

History of Discovery in physics not always quick nor easy. Compare the prediction of Neutrinos and Gravitational Waves.

**Neutrinos:**
- Postulated by W. Pauli in 1930.
- To explain energy, momentum, and spin conservation in beta decay.

**Gravitational Waves:**
- Consequence of GR, A. Einstein 1916.
- Long, sordid debate on its existence and nature---even in 1970 when most agreed it existed many believed orbiting bodies were *protected* from Grav. Waves.
- Discovery of Pulsar PSR1913+16 by Hulse and Taylor in 1974, eventually showed orbit energy loss and perihelion precession consistent with GWs. Nobel 1993.
- Direct detection by LIGO in 2016.
Nature of Gravitational Waves, and $h_{\mu\nu}$

Of all 10 possible components to $h$, it simplifies to two polarizations. One with a motion that shrinks (grows) in one direction while growing (shrinking) in the other direction. The other polarization is just rotated by 45 degrees.

$h_+$

$h_\times$

hplus polarization

hcross polarization

Wave headed into the page, Y up, X right.
**Multipoles - Electrostatic Potential Energy**

\[
\Phi(x) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(x')}{|x - x'|} \, d^3x'
\]

Multipole expansion when far from a complicated source, \( |x'| \ll |x| \).

\[
\Phi(x) = \frac{1}{4\pi\varepsilon_0} \left[ \frac{q}{r} + \frac{p \cdot x}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} + \cdots \right]
\]

**Monopole Moment** \( q = \int_{V'} \rho(x') \, d^3x' \)

**Dipole Moment** \( p = \int x' \rho(x') \, d^3x' \)

**Quadrupole Moment** \( Q_{ij} = \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(x') \, d^3x' \)

Ref: Jackson, *Classical Electrodynamics*
Multipoles – EM Radiation

Multipole expansion for electromagnetic radiation, when the source has charge changing in time. $\rho(\mathbf{x}')$ is the charge distribution (later for GW it will be the mass distribution!).

- **Monopole Moment**
  \[ P^{EM} = k \mu_0 c \dot{q}^2 = 0 \]

- **Electric Dipole Moment**
  \[ P^{ED} = \frac{\mu_0}{6\pi c} |\mathbf{p}|^2 \]

- **Magnetic Dipole Moment**
  \[ P^{MD} = \frac{\mu_0}{6\pi c^3} |\mathbf{m}|^2 \]

- **Electric Quadrupole Moment**
  \[ P^{EQ} = \frac{\mu_0}{720\pi c^3} \sum Q_{ij} \bar{Q}_{ij} \]

  \[ Q_{ij} = \int_{V'} (3x_i'x_j' - \delta_{ij}r^2) \rho(\mathbf{x}') d^3 x' \]

Charge is conserved, $dq/dt = 0$.

First non-zero term.

Ref: Brau, *Modern Problems in Classical Electrodynamics*

25 July 2016

Gabella GravWaves
Quadrupole Moment

Now $\rho(x')$ is the mass distribution of the source.

Monopole Moment
$$p^{EM} = k \frac{G}{c} \dot{M}^2 = 0$$

Electric Dipole Moment
$$p^{ED} = k \frac{G}{c^3} |\ddot{p}|^2 = 0$$

Magnetic Dipole Moment
$$p^{MD} = k \frac{G}{c^5} |\dddot{m}|^2 = 0$$

Electric Quadrupole Moment
$$p^{EQ} = \frac{G}{5c^5} \sum \dddot{Q}_{ij}$$

Mass is conserved, $dM/dt = 0$.

Center of Mass does not change.

Angular Momentum does not change.

First non-zero moment! Quadrupole

$$M = \int_{V'} \rho(x') \, d^3x'$$

$$p = \int_{V'} x' \rho(x') \, d^3x'$$

$$m = \frac{1}{2} \int_{V'} x' \times J_{mass}(x') \, d^3x'$$

$$Q_{ij} = \int_{V'} (3x'_i x'_j - \delta_{ij} r^2) \rho(x') \, d^3x'$$

Ref: Misner, Thorne, and Wheeler, *Gravitation*

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Quadrupole Moments

- m1 fixed and m2 in circular orbit
- quadrupole moment is:

\[
\begin{pmatrix}
-a^2 \sin^2 t \omega & a^2 \cos t \omega \sin t \omega & m2 \\
-a^2 \cos t \omega \sin t \omega & -a^2 \cos^2 t \omega & m2 \\
0 & 0 & -a^2 m2
\end{pmatrix}
\]

- monopole moment is: m1+m2
- dipole moment is:

\[
\begin{pmatrix}
0 \\
0 \\
a^2 \omega m2
\end{pmatrix}
\]
GW strain for Circular Orbit

\[ h_0 = \frac{r s_1 \cdot r s_2}{r \cdot R} \]
\[ \omega_s^2 = \frac{(r s_1 + r s_2)}{2 R^3} \]

\[ h_+ (t) = h_0 \left( \frac{1 + \cos^2 i}{2} \right) \cos 2\omega_s t \]
\[ h_\times (t) = h_0 \cos i \sin 2\omega_s t \]

- \( r s_1 \) is 2GM_1/c^2, Schwarzschild radius for mass M_1, etc.
- \( r \) is distance from Earth to system.
- \( R \) is the separation of the two bodies.

ref: Maggiore around Eqn. 3.332.
Strain for binary system GW150914

- Now work it for our GW150914, 36 Msun, 29 Msun, at distance of 410 Mpc (1.3 billion ly),
- And separation of how much?
- Check the strain on slide 2.
Strain Curves from Moore et al.

Pulsar Timing Arrays

Ground-based

Spaced-based
Gravitational Wave Spectrum

- Early Universe quantum fluctuations
- Supermassive binary black hole mergers
- Compact object captures by supermassive black holes
- Merging binary stellar remnants
- Supernovae
- Rotating asymmetric pulsars

GRAVITATIONAL WAVE FREQUENCY (HZ)

- Cosmic Microwave Background Polarization
- Pulsar Timing
- LISA (Space Based Interferometry)
- LIGO, GEO, VIRGO (Ground Based Interferometry)
Ground-Based, aka aLIGO

- Both interferometers using light as a clock to measure the change in lengths.
- More tomorrow:
- Besides interferometers there are also still “bars.”
Space-Based, aka LISA

- Laser Interferometer Space Antenna, LISA, is a set of satellites with lasers bouncing in a triangle arrangement.
  - No seismic effects!
  - Very long baseline, 5e9 m or 1e9 m each leg.
  - Control of craft suble, “drag-free flying” with free test mass inside as reference.
- Currently mostly European eLISA with some NASA/US support. eLISA could launch in 2032.
- LISA Pathfinder satellite experiment tested several components for eLISA, early 2016.
LISA and eLISA

- LISA has a full equilateral triangle of lasers and baseline of 5e9 m. Not funded.

- eLISA uses two legs of the equilateral triangle of lasers and baseline of 1e9 m. Successful LISA Pathfinder mission and LIGO detection means likely to go forward and maybe “fast-tracked.”
LISA Pathfinder

- LISA Pathfinder successfully demonstrated the technologies for “drag-free” flight of the eLISA space craft.
  - Orbit is *nearly* freely falling around the sun.
  - Microthrusters used to counter solar wind, etc, and allow a free mass inside to react only to gravity!

- Measured relative acceleration as good as $6 \times 10^{-15}$ m/s$^2$ per root Hz (tomorrow).
FIG. 2. A schematic of LPF. The figure shows TM1, TM2, and the optical bench beam paths for measuring $\Delta x$ and $x_1$. The measurement of $\Delta x$ drives the electrostatic suspension of TM2, which applies the necessary electrostatic forces by means of the electrodes represented by the four gold plates facing TM2. All other electrodes surrounding the TMs are not shown. The measurement of $x_1$ drives the drag-free control loop that uses the micronewton thrusters to exert forces on the spacecraft. The figure depicts the $x$ and $y$ axes we use in this Letter, while $z$ is normal to the figure.

Ref: Armano et al., PRL 2016
LISA Pathfinder

LISA Pathfinder Exceeds Expectations

Centrifugal force
The rotation of the spacecraft required to keep the solar array pointed at the Sun and the antenna pointed towards Earth, coupled with the noise of the charmed particles, produces a noisy centrifugal force on the test masses. This noise term has been subtracted, and the source of the residual noise after subtraction is still being investigated.

Gas damping
Inside their housings, the test masses collide with some of the few gas molecules still present. This noise term becomes smaller with time, as more gas molecules are vented to space.

Sensing noise
The sensing noise of the optical metrology system used to monitor the position and orientation of the test masses, at a level of 3σ km/s/kg, has already surpassed the level of precision required by a future gravitational-wave observatory by a factor of more than 100.

www.esa.int

Spacetime: ESA/STIS mission; date: ESA/ESA Pathfinder Calibration

European Space Agency
Pulsar Timing Arrays

- Millisecond pulsars are very good clocks scattered around the Universe.
- By measuring them once in a while, you can detect a hiccup in the reception of the pulse at Earth. With enough “clocks” you can decide if there was a gravitational wave and from where it may have come.
- See chart, sensitive to very, very long wavelengths, and thus very, very small frequencies. Size of the Universe sort of waves!

Ref: [NANOGrav collaboration](http://www.nanograv.org)
Pulsar Timing Arrays

- Several collaborations working at radio astronomy facilities to measure millisecond pulsars:
  - The **International Pulsar Timing Array** uses many radio telescopes.
  - The **European Pulsar Timing Array**
  - and others.
Links

• CalTech GW media assets page.
• Kelly Holley-Bockelman’s TEDx Nashville Talk 2016 about GW150914.
• Pulsar timing array, Nanograv
Backup
## Some Constants

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<th>Schwarzschild radius</th>
<th>$2GM/c^2$</th>
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## SI Prefixes

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Feynman’s Take on Gravitational Radiation

• Consider the spin of the “graviton”---the postulated particle that carries the gravitational force.

• Generically a spin 1 particle is just like electromagnetism, that is like particles repel and opposites attract. This is NOT what we see in gravity. Just one “charge” and they always attract.
  – True for all odd spin particle theories.

• So consider spin 0 in some detail, we find that two hot gas clouds are attracted to each other less than those same clouds if they were colder. Yet they have more energy when hot.

• So that leaves spin 2. That gives rise to a quadrupolar source for radiation.