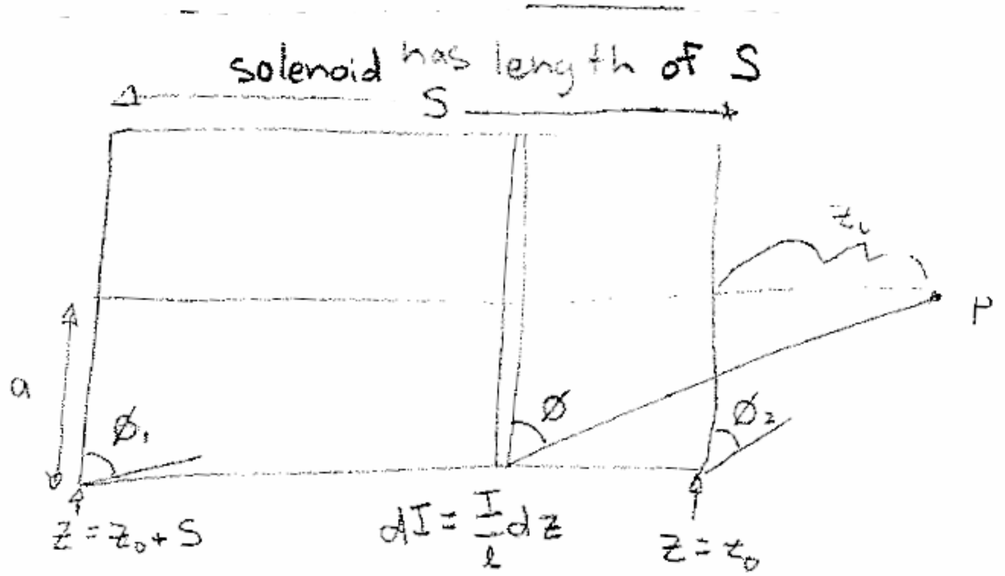
Finite Solenoid Demonstration



Test Solenoid

(1)

$S = 23.43\,cm$

There are 480 turns of wire (coils)

so $\frac{I}{\ell} = I\,\frac{480}{0.2343\,m}$

$= I\,(2048.6/m)$

$z_0 = 5\,cm$

$a = 2.5\,cm$

at Point P $\quad dB = \frac{\mu_0 dI}{2}\,\frac{a^2}{(a^2+z^2)^{3/2}}$

$$B = \int dB = \left(\frac{\mu_0 \frac{I}{\ell}}{2}\right) \int_{z_0}^{z_0+S} \frac{a^2}{(a^2+z^2)^{3/2}}\,dz$$

$a^2 + z^2 = \frac{a^2}{\cos^2\phi}$, $\frac{a}{(a^2+z^2)^{1/2}} = \cos\phi$, $\frac{z}{a} = \tan\phi$, $dz = \frac{a}{\cos^2\phi}\,d\phi$

test solenoid

$\tan^{-1}\left(\frac{z_0+S}{a}\right) = \tan^{-1}\left(\frac{0.05+0.2343}{0.025}\right) = 85°$

$\tan^{-1}\left(\frac{z_0}{S}\right) = \tan^{-1}\left(\frac{0.05}{0.025}\right) = 63.4°$

$\phi = \tan^{-1}\left(\frac{z_0+S}{a}\right)$

$$B = \int_{\phi = \tan^{-1}\frac{z_0}{a}} \frac{\mu_0(I/\ell)}{2}\, a\, \frac{\cos\phi}{(a^2/\cos^2\phi)}\, \frac{a}{\cos^2\phi}\,d\phi = \frac{\mu_0(I/\ell)}{2}\left[\sin\phi\right]_{\tan^{-1}\left(\frac{z_0}{a}\right)}^{\tan^{-1}\left(\frac{z_0+S}{a}\right)}$$

So $B = \frac{\mu_0\left(\frac{I}{\ell}\right)}{2}\sin\theta\Big|_{63.4°}^{85°} = \frac{\mu_0\left(\frac{I}{\ell}\right)}{2}(0.1)$ to get $B_{earth} = \frac{1}{20{,}000}\,T = \frac{\left(4\pi\times10^{-7}\frac{T_m}{A}\right)(2048.6/m)\,I\,(0.1)}{2}$

$I = 0.39\,A$

more than if we were at the center

Infinte Solenoid

$$B = \frac{\mu_0 (I/\ell)}{2}\left[\sin\phi\Big|_{-\pi/2}^{\pi/2}\right] = \mu_0 \frac{I}{\ell} = \mu_0 \underbrace{\frac{n}{\ell}}_{\frac{\#\text{turns}}{\text{length}}} I$$

at center of Finite solenoid

$$B_{center} = \frac{\mu_0 (I/\ell)}{2}\left[\sin\phi\Big|_{\tan^{-1}\left(\frac{\text{length}/2}{\text{radius}}\right)}^{\tan^{-1}\left(\frac{\text{length}/2}{\text{radius}}\right)}\right]$$

example solenoid

$$B_{center} = \frac{\mu_0 I/\ell}{2}\left[\sin\phi\Big|_{-\tan^{-1}\left(\frac{.2343/2}{0.0025}\right)=-78^\circ}^{\tan^{-1}\left(\frac{.2343/2}{0.025}\right)=78^\circ}\right]$$

$$= \frac{\mu_0 (I/\ell)}{2}(196) \quad \text{pretty good approximation!}$$

ξ to get B_{earth} @ center need $\sim 0.02A$ or so,

Our test we saw $I \sim 0.08A$, any guesses as to why?
(why not 0.4A?)