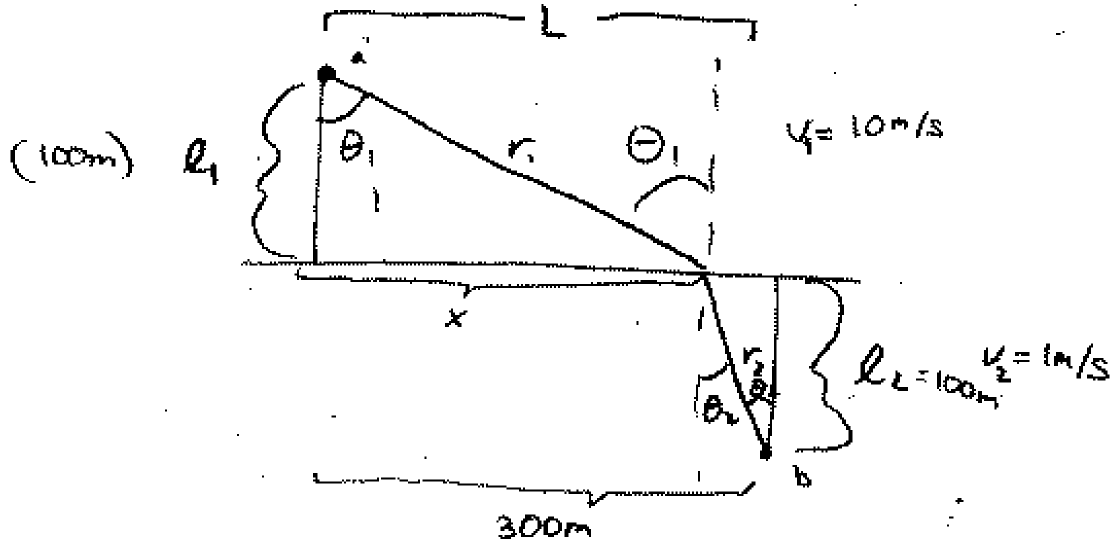


what's the shortest time to get from a to b?
won't solve, but we'll find a general principle

$t_1 = \frac{r_1}{v_1}$ $\quad t_2 = \frac{r_2}{v_2}$

$r_1 = \sqrt{\ell_1^2 + x^2}$ $\quad r_2 = \sqrt{\ell_2^2 + (L-x)^2}$

$$t_{tot} = \frac{\sqrt{\ell_1^2 + x^2}}{v_1} + \frac{\sqrt{\ell_2^2 + (L-x)^2}}{v_2} = \frac{1}{\ }$$

at extreme $\frac{dt}{dx} = 0$

have to be a little careful!

$$\frac{x}{v_1\sqrt{\ell_1^2 + x^2}} - \frac{(L-x)}{v_2\sqrt{\ell_2^2 + (L-x)^2}} = 0$$

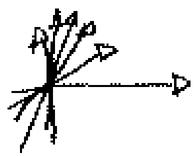
dirty trick $\sin\theta_1 = \frac{x}{\sqrt{\ell_1^2 + x^2}}$ $\quad \sin\theta_2 = \frac{(L-x)}{\sqrt{\ell_2^2 + (L-x)^2}}$

$$\frac{1}{v_1}\sin\theta_1 - \frac{1}{v_2}\sin\theta_2 = 0$$

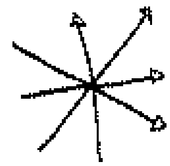
$(c)\ \frac{1}{v_1}\sin\theta_1 = \frac{1}{v_2}\sin\theta_2\ (c)$ $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$

If we don't have polarized e-m waves we can make them with a polarizer

Initial em waves about all directions

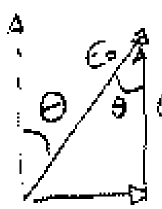


head on view



Pass them through a polarizer with

transmission axis

before	after	
 E_0	 E_0	along Trans axis
	$E_0 \cos\Theta$	waves at an angle with Trans axis
	0	waves ⊥ to Trans axis

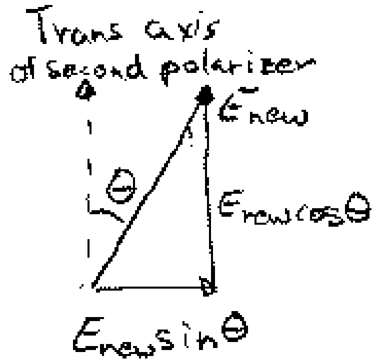
on average, unpolarized light has $\frac{1}{2}$ E along & $\frac{1}{2}$ ⊥ to Trans axis

$$I \Rightarrow \frac{I}{2}$$

and the remaing light has an E along the transmission axis

– what happens if we put in another polarizer?

If polarizer makes an angle Θ



$$E_{transmitted} = E_{new}\cos\Theta$$

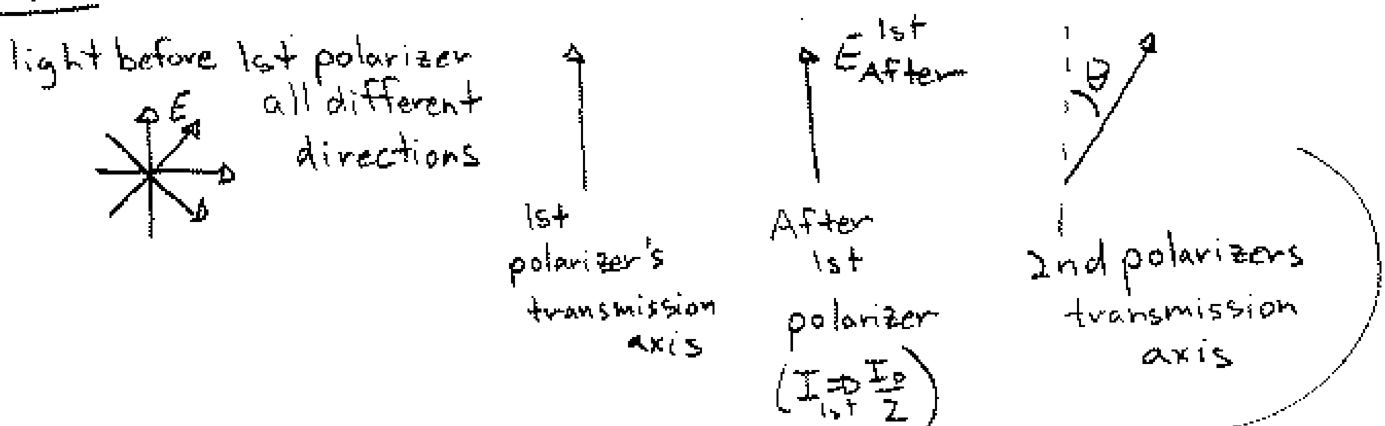
$$I_{trans} = \left(\frac{I_0}{2}\right)\cos^2\Theta$$

if $\Theta = 90°$ though $E_{trans} = 0$!

Now, if we put in a third when $\Theta = 90°$ in between

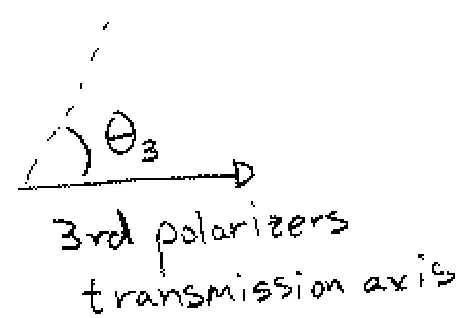
$E_{trans} \neq 0$ what's happening?

<u>head on</u>



$$E^{2nd}_{After} = E^{1st}_{After}\cos\Theta$$

$$I_{2nd} = I_{1st}\cos^2\theta$$



$$\vec{E}^{3rd}_{After} = E^{2nd}_{After}\cos\Theta_3 \neq 0 \ !$$

example

Unpolarized light is incident on 2 polarizers, the angle between the transmission axis of the 2 polarizers is 30°. what is the intensity of the transmitted light relative to the incident.

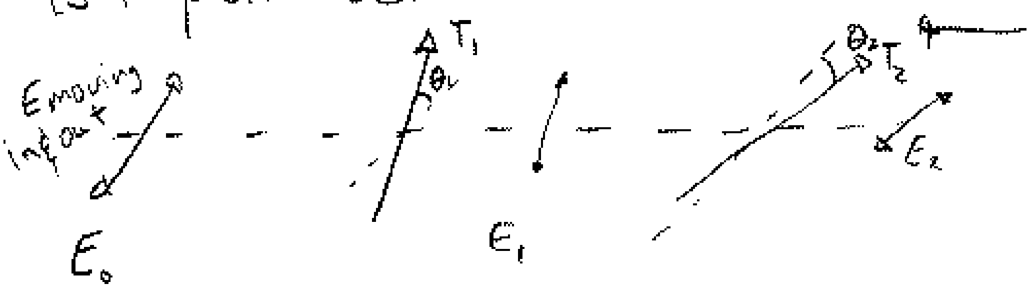
$$I^{1st}_{After} = \frac{I_0}{2}\cos^2\theta = \frac{I_0}{2}\cos^2 30° = 0.38\, I_0$$

$\propto E^2$

example

E moving in a particular direction

polarized light is incident upon 2 polarizers the axis of the 2 polarizers is 30°. what is the intensity of the transmitted light relative to the intensity of the incident? ⇒ need to know the angle of the 1st polarizer relative to the incident light



angle relative to in & out

θ_2 is negative

$$E_1 = E_0 \cos\theta_1$$

$$E_2 = E_1 \cos\left(|\theta_2| + \theta_1\right)$$

angle between $\vec{E}$ & $\vec{T}$ (transmission axis)

$$I_2 = I_1 \cos^2(|\theta_2| + \theta_1) = I_0 \cos^2(\theta_1)\cos^2(|\theta_2| + \theta_1)$$

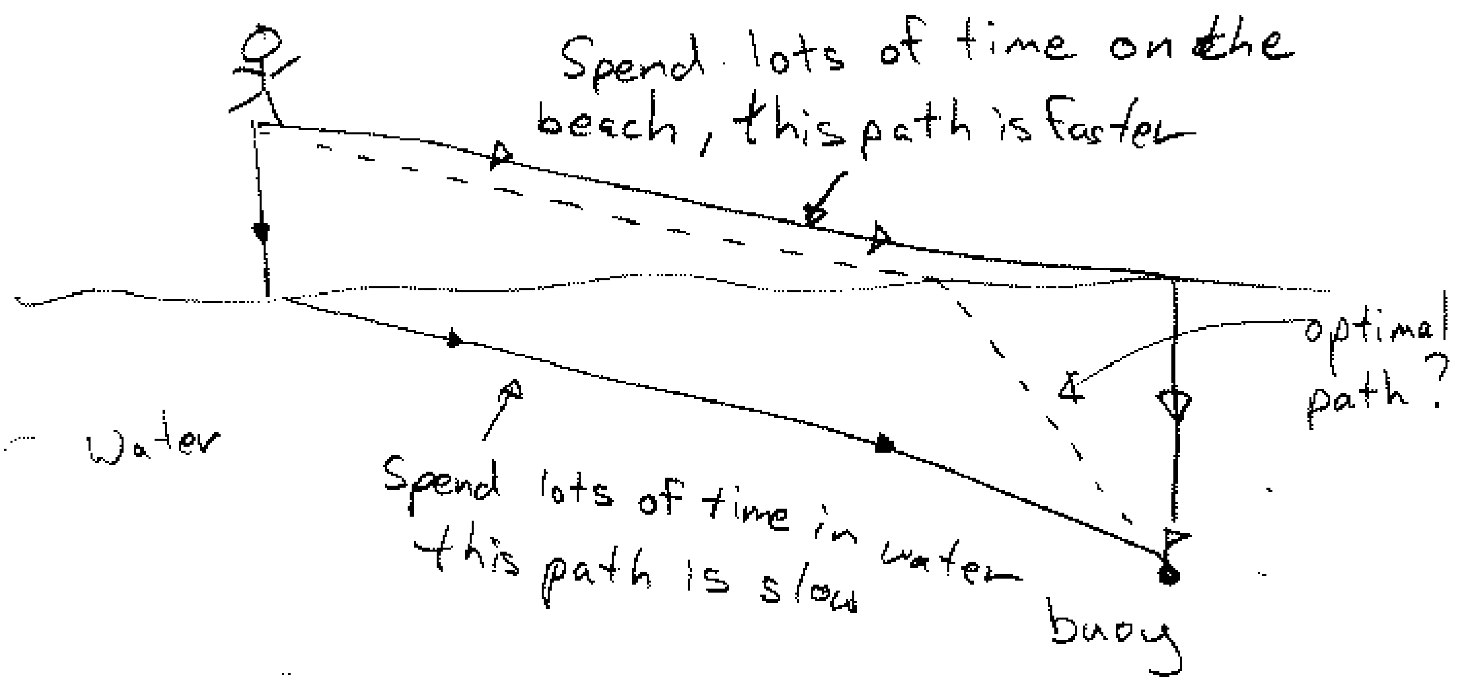
One other property of EM Radiation that is very interesting is speed. The fastest EM radiation goes is c, in most materials though, the speed of the radiation is less than c. This has interesting consequences. Light traversing 2 mediums follow Fermat's Principle: Light choses the path of minimum time between 2 points. here's an every day example:

Suppose you are in a race and the 1st person to reach a buoy in the water wins

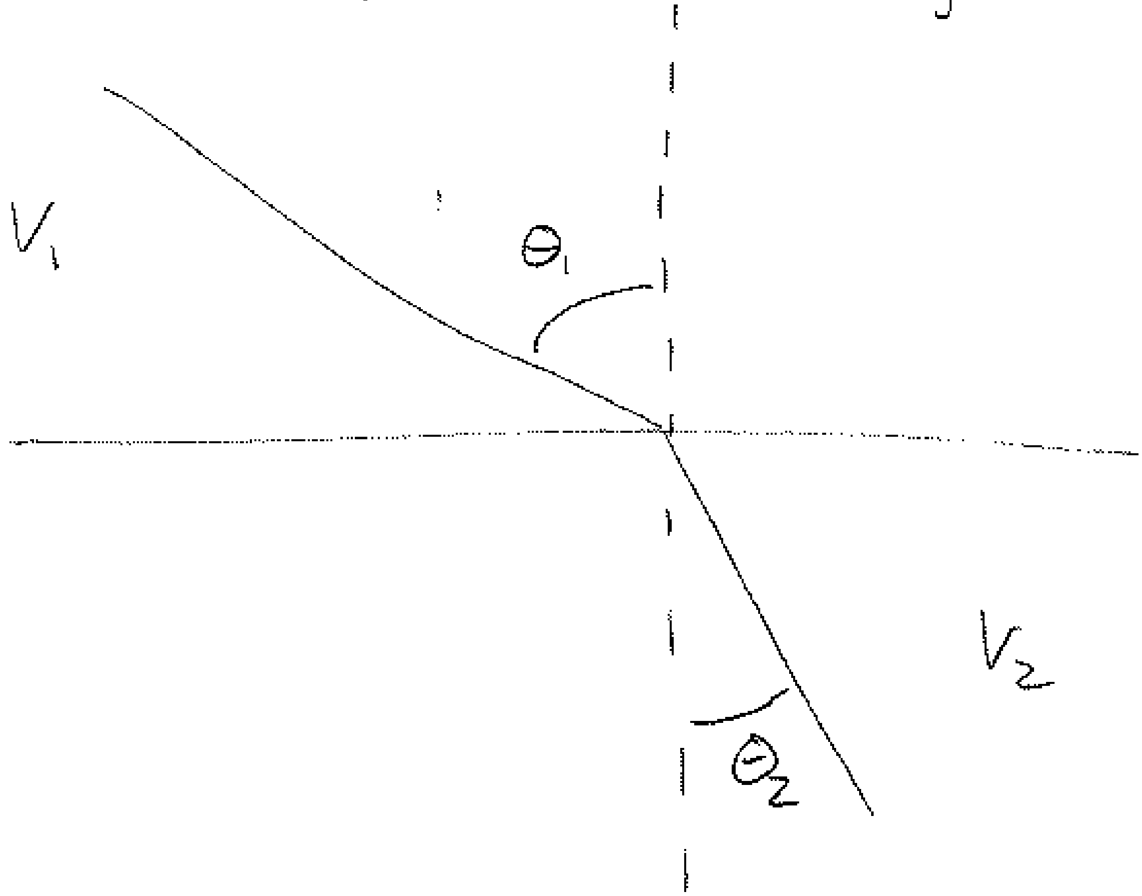
Along the beach you can run 8 m/s

In the water you can swim 1 m/s

What's the best path to take?



Turns out the optimal path is always



$$\frac{1}{V_1}\sin\theta_1 = \frac{1}{V_2}\sin\theta_2$$

Works either way

(Physicists usually multiply both sides by c and call $\frac{c}{V}$ the "index of refraction" labelled n

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad n \geq 1$$

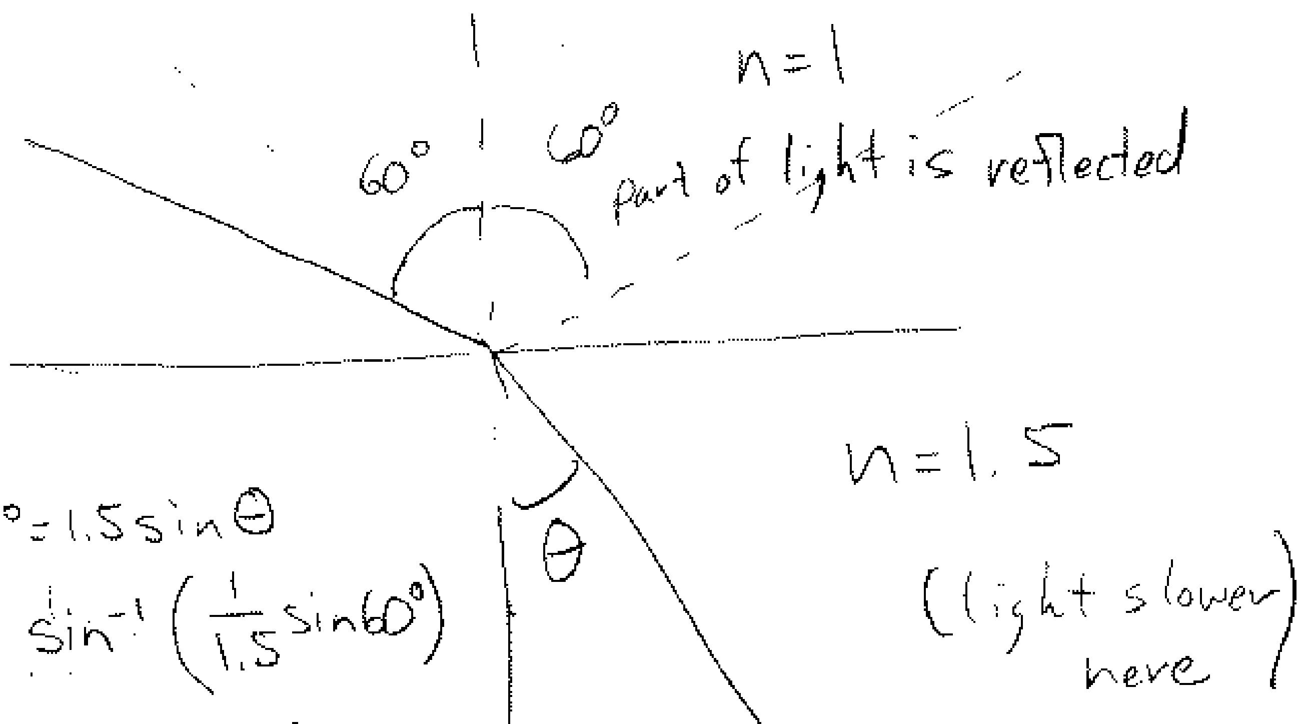
going from v_{fast} to $v_{smaller}$ the angle gets smaller

going from $v_{smaller}$ to v_{fast} the angle becomes vast

here's an example, suppose I have a piece of plastic and a laser

$n_{Air} \sim 1$ $\quad n_{plastic} \approx 1.5$

If I hold the laser at 60°

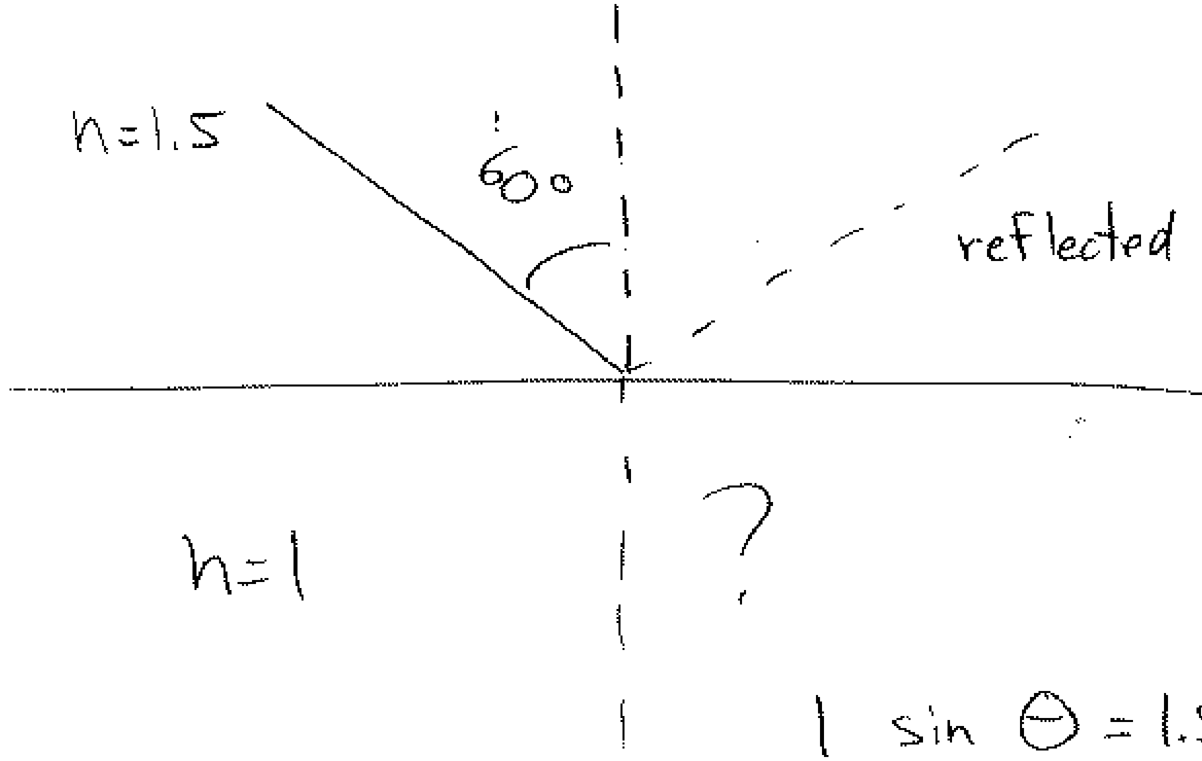


$1 \sin 60^\circ = 1.5 \sin\theta$

$\theta = \sin^{-1}\left(\frac{1}{1.5}\sin 60^\circ\right)$

$\theta = 35.3^\circ$

A strange thing happens though if we have



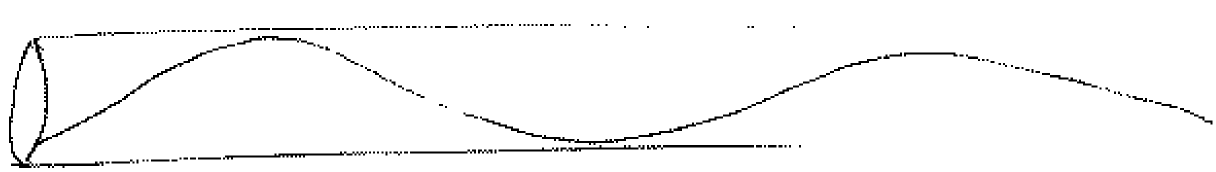
$$1 \sin\theta = 1.5 \sin 60^\circ$$

$$\sin\theta = 1.3 \;!?!$$

can't happen

Indeed, none is transmitted and you get total internal reflection.

This property is exploited in fiber optic cables to keep the light inside



consider what happens at the surface for the incident light.

A crest of the light hits the surface once every $T = \frac{1}{f}$

Since light does not build up at the surface

A crest of light leaves the surface once every $T = \frac{1}{f}$ too

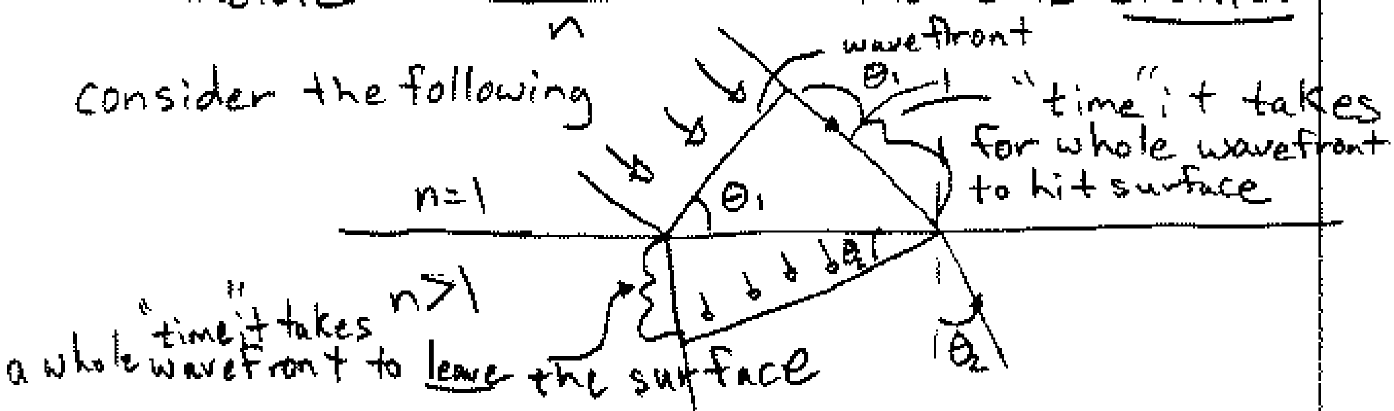
So $\Rightarrow$ Frequency of light is constant inside the material, but we know the speed slows down

$$\left\{\begin{array}{l} v_{light} = \lambda_{inside} f \\ c = \lambda_{vacuum} f \end{array}\right.$$

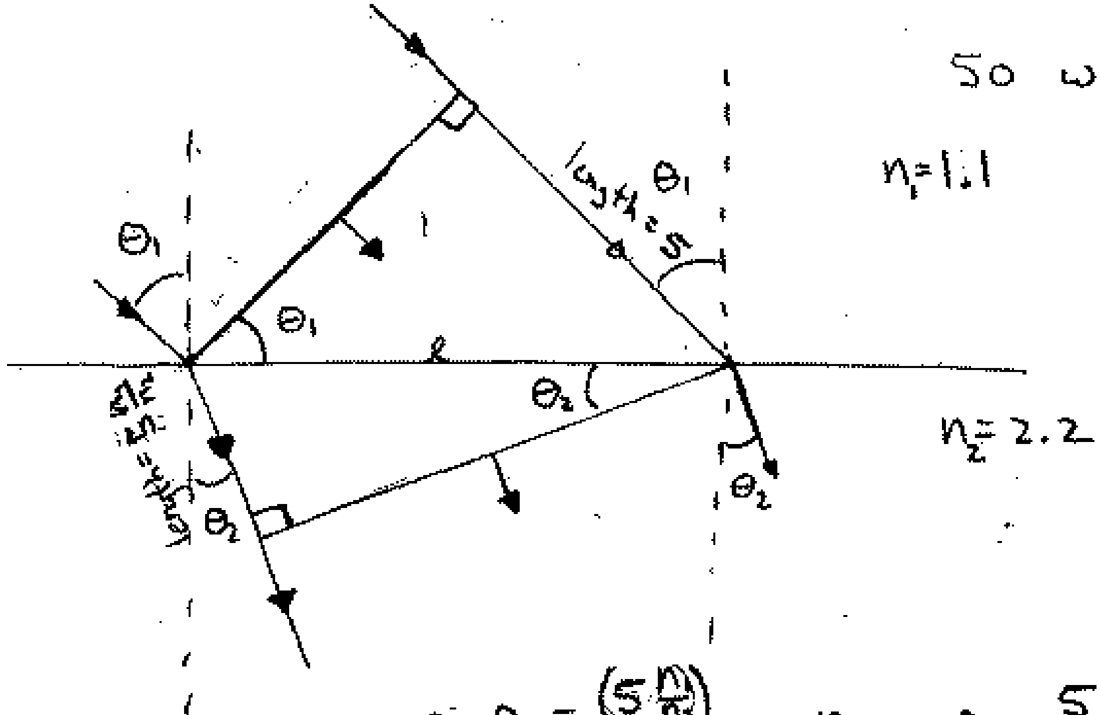
$$\text{or} \quad \frac{v_{light}}{c} = \frac{\lambda_{inside}}{\lambda_{vacuum}} = \frac{1}{n}$$

$$\lambda_{inside} = \frac{\lambda_{vacuum}}{n}$$ λ_{inside} is smaller

consider the following



So what



$$\sin\theta_2 = \frac{\left(5\frac{n_1}{n_2}\right)}{\ell} \qquad \frac{n_2}{n_1}\sin\theta_2 = \frac{5}{\ell}$$

$$\sin\theta_1 = \left(\frac{5}{\ell}\right)$$

$$\sin\theta_1 = \frac{n_2}{n_1}\sin\theta_2$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

(light has no traffic jams, if cars had a length $\propto$ velocity, we wouldn't either .... too bad!)

means wavelength of light $\lambda = \frac{\lambda_0}{n}$

or

$n_1\lambda_1 = n_2\lambda_2$ (in our example above)

$n_1(5) = n_2\left(5\frac{n_1}{n_2}\right) = n_1 5$ ✓