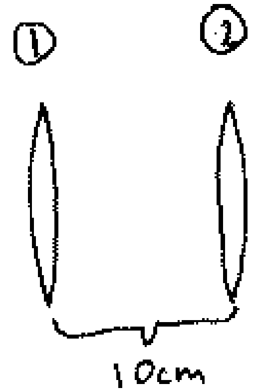
not all lenses are perfect

- spherical abberations — focal point not the same for all points of the lens
- chromatic abberations — light of different wave length focuses different places

We can actually fix a lot of these problems at the cost of some light

Last time we saw how an overhead projector worked & did some calculations. In order to do a better job with things like this, we should try to model the behavior of the overhead with 2 lenses



lens 1 has focal length 0.4m

lens 2 has focal length 0.3 m

If I put an object 0.5 m in front of 1 where is the image?

lens 1 forms an image $\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$

$$q_1 = \frac{p_1 f_1}{p_1 - f_1}$$

$$= \frac{(0.4m)(0.5m)}{0.5m - 0.4m}$$

$$= 2m!$$

This image will be way behind lens 2 in fact it has a negative object length for lens 2 $\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$

consider the case at lens 2

the object is 1.9m behind ($p_2 = -1.9m$) lens (thats the impression the light rays give)

so
$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-1.9m)(0.3m)}{-1.9m - 0.3m}$$

$$= 0.259m$$

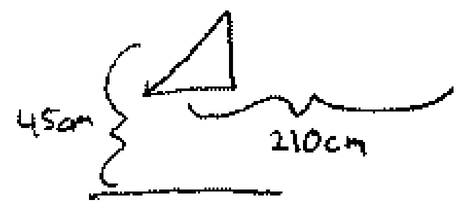
Notice, we can choose lens 2 to tune in where we want the image to be a little

if $f_2 = 1m$

$$q_2 = \frac{(-1.9m)(1.0m)}{-1.9m - 1m} = .67m$$

but lets recall about what we saw with the over head

$$\frac{1}{45\text{cm}} + \frac{1}{210\text{cm}} = \frac{1}{f}$$



$$f \sim 37\text{ cm}$$

Each lens does about $\frac{1}{2}$ of the work

$$f_1 = f_2 \sim \text{guess } 74\text{cm}$$

$$g_1 = \frac{0.45\text{m}\,(.74\text{m})}{(.45\text{m} - 0.74\text{m})} = -1.15\text{m}$$

$$P_2 = 1.25\text{m}$$

$$g_2 = \frac{1.25\text{m}\ 0.74\text{m}}{1.25\text{m} - 0.74\text{m}} = 1.81\text{m}$$

not bad

(use carefully!) have to know how to apply this

(in your lab) $$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

in practice this is not too useful.
so, procedure is to take things step by step.

1) Take object
2) Find Image
3) use image as object
   (IF the image is beyond the next lens, the new object could be negative)

now, consider the case when

$$f_1 = f_2 = x/2$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} = \frac{2}{f_1} - \frac{2\cancel{f_1}}{f_1 \cancel{f_1}} = \frac{1}{f_{eff}}$$

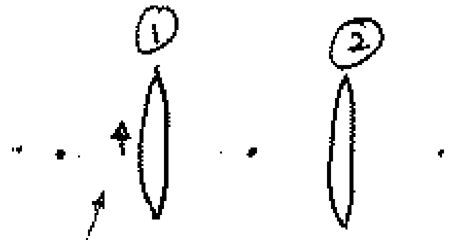
$$= 0$$

$f_{eff} = \infty$? work a problem

$f_1 = 10\text{cm}$

$f_2 = 10\text{cm}$

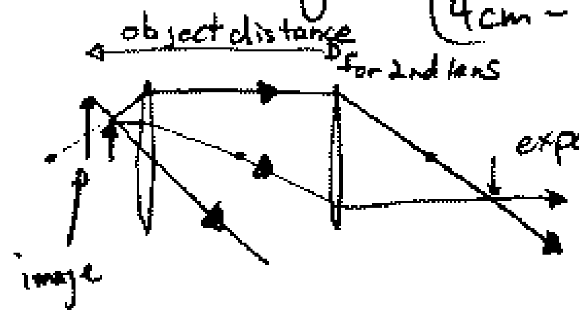
$x = 20\text{cm}$



Object@ 4 cm

1

$$q_1 = \frac{(4\text{cm})(10\text{cm})}{(4\text{cm} - 10\text{cm})} = -6.66\text{cm}$$



$$p_2 = 26.6\text{cm}$$

$$q_2 = \frac{(26.6\text{cm})(10\text{cm})}{(26.6\text{cm} - 10\text{ cm})} = 15.95\text{cm}$$

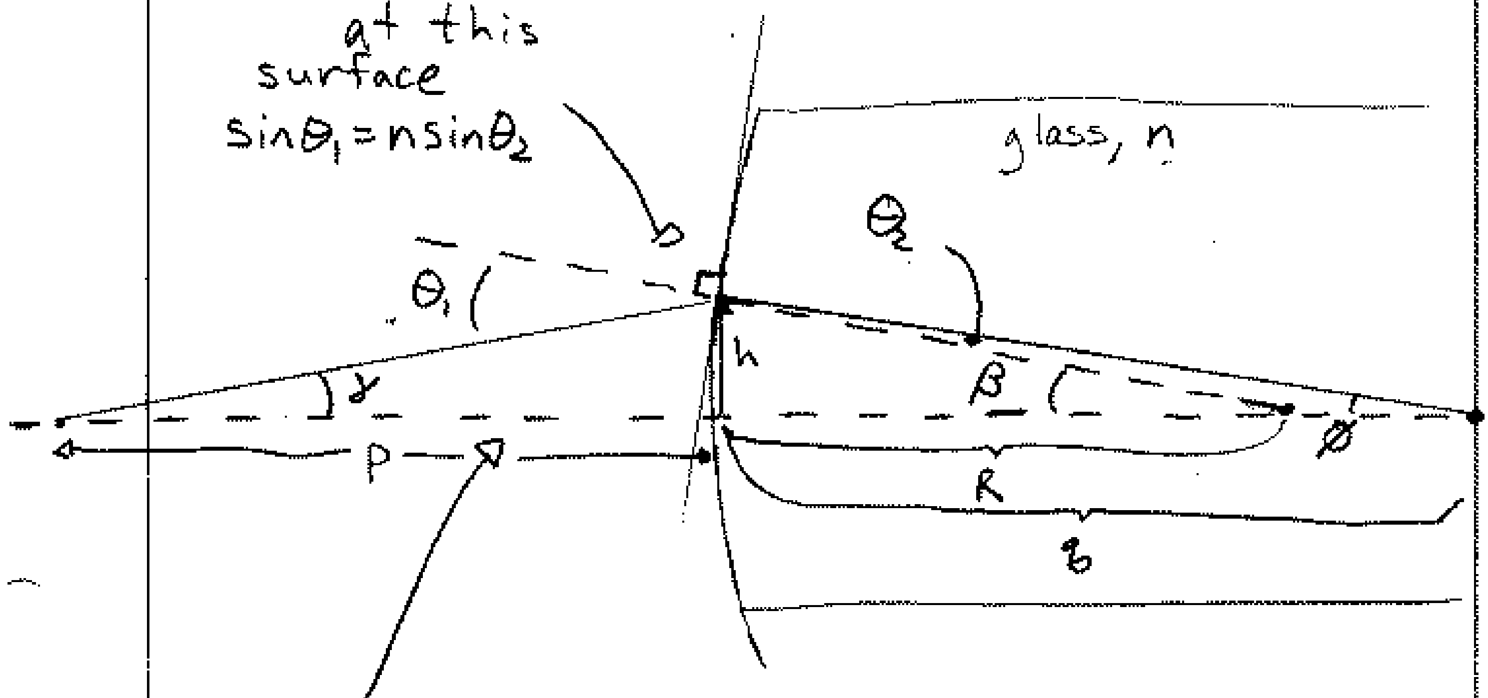
$$m = -\frac{q_2}{p_2}$$ (inverted from original & smaller)

$$= -\frac{15.95}{26.6} = 0.6$$

so, the f_{eff} formula fails miserably!

now, how did that image form in the first place?

Consider a round piece of glass



In this triangle, the big angle $= 180° - \gamma - \beta$

$$= 180° - \theta_1$$

① $\theta_1 = \gamma + \beta$

same trick for the tiny triangle

② $\beta = \theta_2 + \phi$

now $p\tan\gamma = h$, $R\tan\beta = h$, $q\tan\phi = h$

for small angles, $\sin\theta \approx \theta$, $\tan\theta \sim \theta$

③ $p\gamma = R\beta = q\phi$ ④ $\theta_1 \sim n\theta_2$

put ① into ② using ④

$$\gamma + \beta = n(\beta - \phi)$$
$$\theta_1' = n\,\theta_2$$

get everything in terms of β using ③

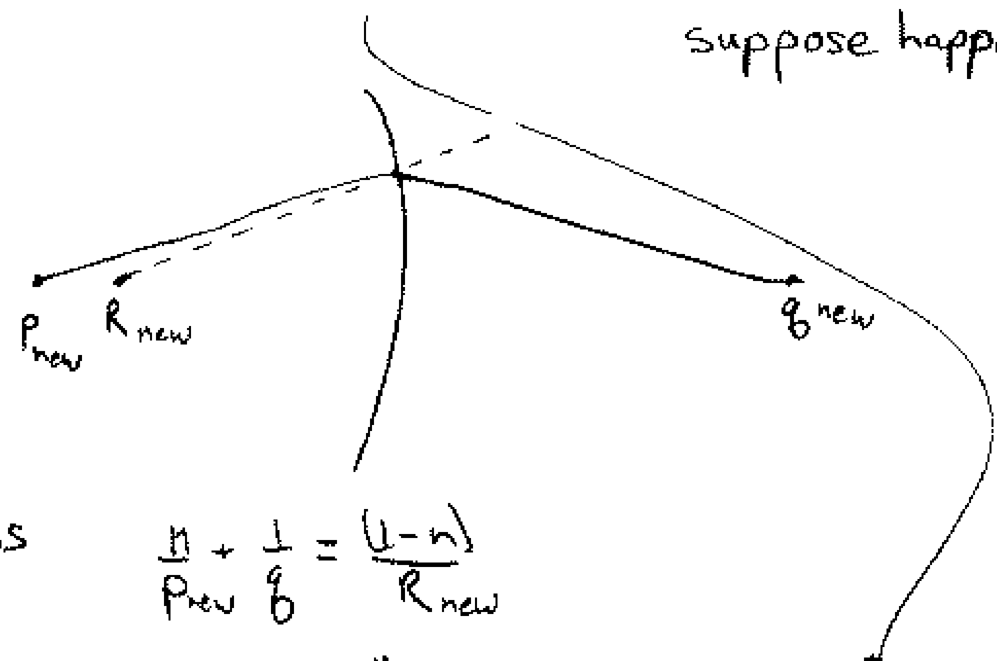
$$\gamma = \frac{R\beta}{p} \qquad \phi = \frac{R\beta}{q}$$

$$\frac{R\beta}{p} + \beta = n\beta - \frac{R\beta n}{q}$$

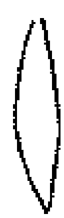
$$\frac{1}{R}\left(\frac{R\cancel{\beta}}{p} + \frac{R\cancel{\beta}}{q}\right) = \left((n-1)\cancel{\beta}\right)\frac{1}{R}$$

$$\frac{1}{p} + \frac{n}{q} = \frac{n-1}{R}$$

looks pretty close, now, what do you suppose happens

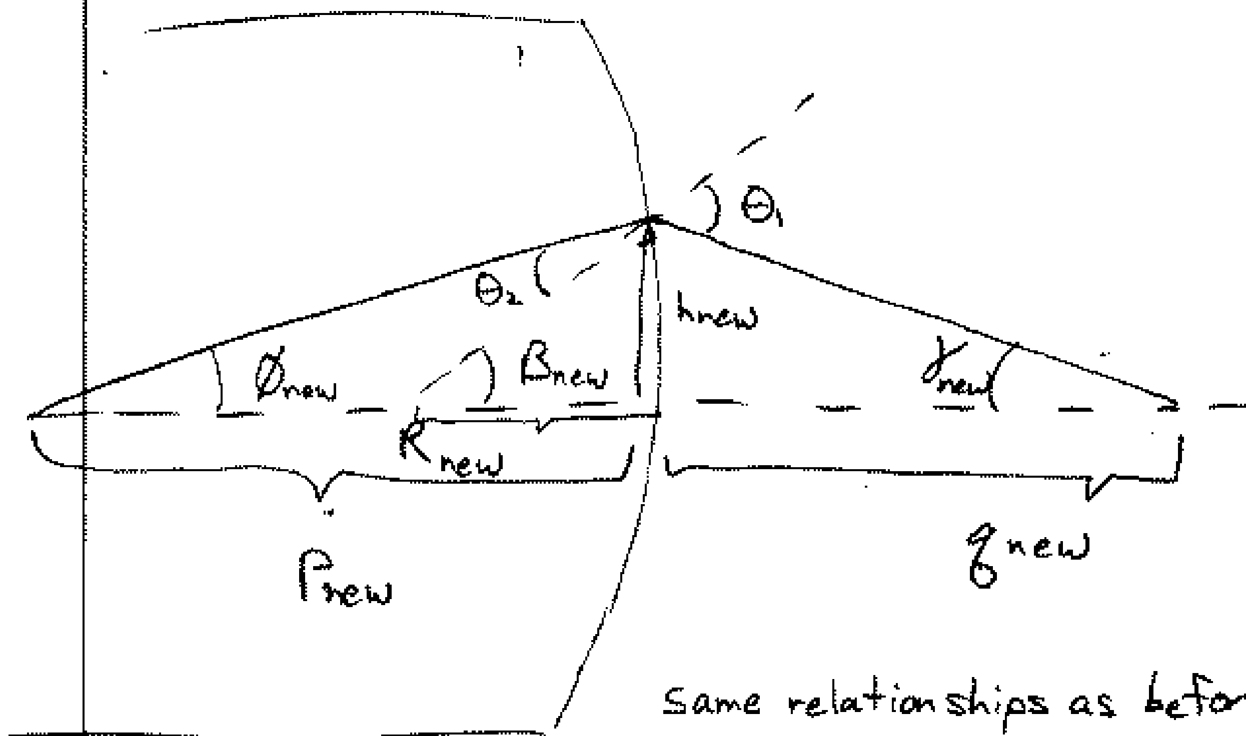


guess $\frac{n}{p_{new}} + \frac{1}{q} = \frac{(1-n)}{R_{new}}$

& if we have  get then $p_{new} = q$

$$\frac{1}{p_{old}} + \frac{1}{q_{new}} = (n-1)\left(\frac{1}{R} + \frac{1}{R_{new}}\right)$$

let's check



same relationships as before

except $p_{new} \hookrightarrow q_{old}$

$q_{new} \hookrightarrow p_{old}$

$R_{new} \hookrightarrow R_{old}$

should get

$$\frac{1}{q_{new}} + \frac{n}{p_{new}} = \frac{n-1}{R_{new}}$$

$$p_{new} = -q_{old}$$

$$\frac{1}{p_{old}} + \frac{n}{q_{old}} = \frac{n-1}{R_{old}}$$

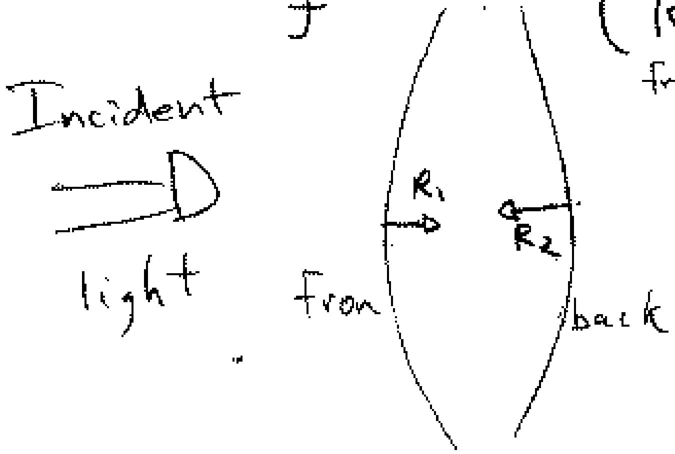
add

$$\frac{1}{p_{old}} + \frac{1}{q_{new}} = (n-1)\left(\frac{1}{R_{new}} + \frac{1}{R_{old}}\right)$$

The convention is that R_{new} is negative if it points into the glass

Lens makers equation

$$\frac{1}{f} = (n-1)\left(\underset{\text{front}}{\frac{1}{R_1}} - \underset{\text{back}}{\frac{1}{R_2}}\right)$$



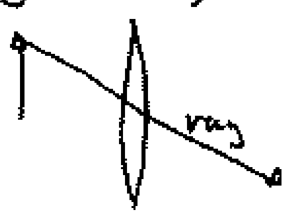
R_1 is along the light $R_1 > 0$

R_2 is opposite the light $R_2 < 0$

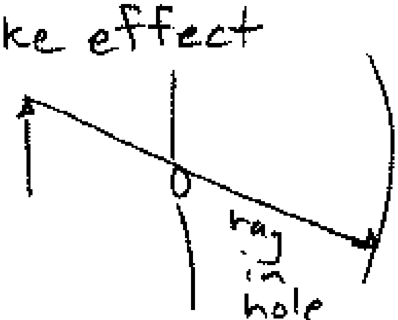
end up with

$$\frac{1}{p} + \frac{1}{6} = \frac{1}{f}$$

In all our ray tracing, we've had a ray that goes right through the center of the lens.



we can get a lens like effect with a pinhole



<u>Demo</u>

1) Pinhole glasses

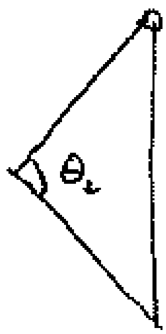
angular magnification ⇒ 2) Pinhole magnifier (near point)

3) Demonstration of your "inverted" eye

we've talked about how to get magnification from lenses using $m = \frac{h'}{h} = -\frac{q}{p}$

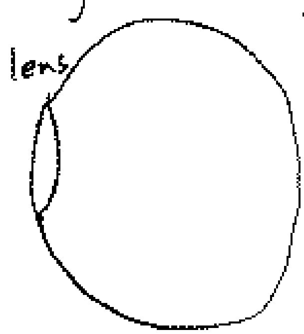
But there's another kind of magnification

$\Rightarrow$ angular magnification

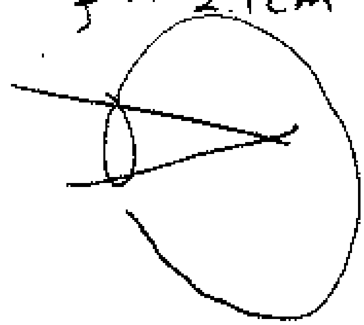


usually see this with optical instruments
discuss this a little

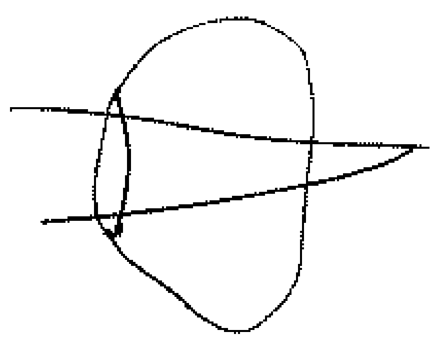
Really, our only optical instument is our eye



$f \sim 2.1\,cm$



near sighted



far sighted