we've discussed several properties of electro magnetic radiation.

⇒ Intensity / travelling wave
⇒ vector Properties (polarization)
⇒ speed Properties (index of refraction / lenses)

Today we'll add another

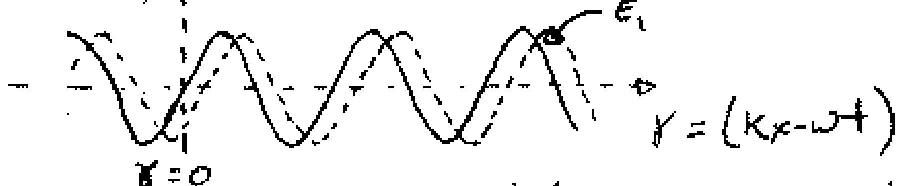
consider 2 travelling waves

both have E_0 the same
have same polarization
have same λ & f

we want to add these together

$$E_0 \sin(kx-\omega t) + E_0 \sin(kx-\omega t) = 2E_0 \sin(kx-\omega t)$$

now, consider what happens if one is shifted a little when we add (for instance)



these 2 waves are separated in space or time if you like or angle

The darker wave, lets call $E_1 = E_0 \sin(\gamma)$
where $\gamma = kx-\omega t$

lighter wave lets call $E_2 = E_0 \sin(\gamma - \phi)$

$$E_{tot} = E_1 + E_2 = E_0 \left(\sin(\gamma) + \sin(\gamma - \phi) \right)$$

$$\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$E_{tot} = 2E_0 \sin\left(\frac{\gamma + (\gamma - \delta)}{2}\right) \cos\left(\frac{\gamma - (\gamma - \phi)}{2}\right)$$

$$= 2E_0 \sin\left(\gamma - \frac{\delta}{2}\right) \cos\left(\frac{\delta}{2}\right) = 2E_0 \sin\left(kx-\omega t - \frac{\delta}{2}\right) \cos\left(\frac{\phi}{2}\right)$$

notice, this is 0 when $\frac{\phi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

and reaches its |maximum| when

$$\frac{\phi}{2} = 0, \pi, 2\pi, 3\pi, 4\pi$$

and if you express $\frac{\phi}{2}$ in terms of a difference in path length δ

For $\left|\cos\frac{\phi}{2}\right|$ maximum $\quad \frac{\phi}{2} = 0, \pi, 2\pi, 3\pi$

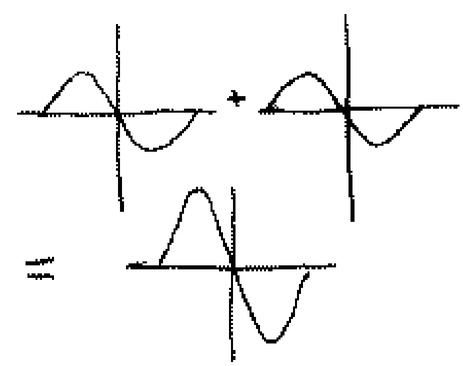
$$\phi = 0, 2\pi, 4\pi, 6\pi = k\delta = \frac{2\pi}{\lambda}\delta$$

$$\text{or } \delta = 0, \lambda, 2\lambda, 3\lambda$$

which is what you expect, if the waves are shifted by a whole λ, they add constructively

δ is a path difference

ϕ is a phase difference



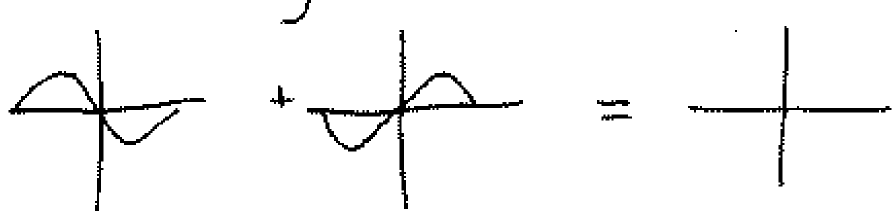
For $\cos\frac{\phi}{2}$ minimal $= 0$

$$\frac{\phi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$\phi = \pi, 3\pi, 5\pi, 7\pi = \frac{2\pi}{\lambda}\delta$$

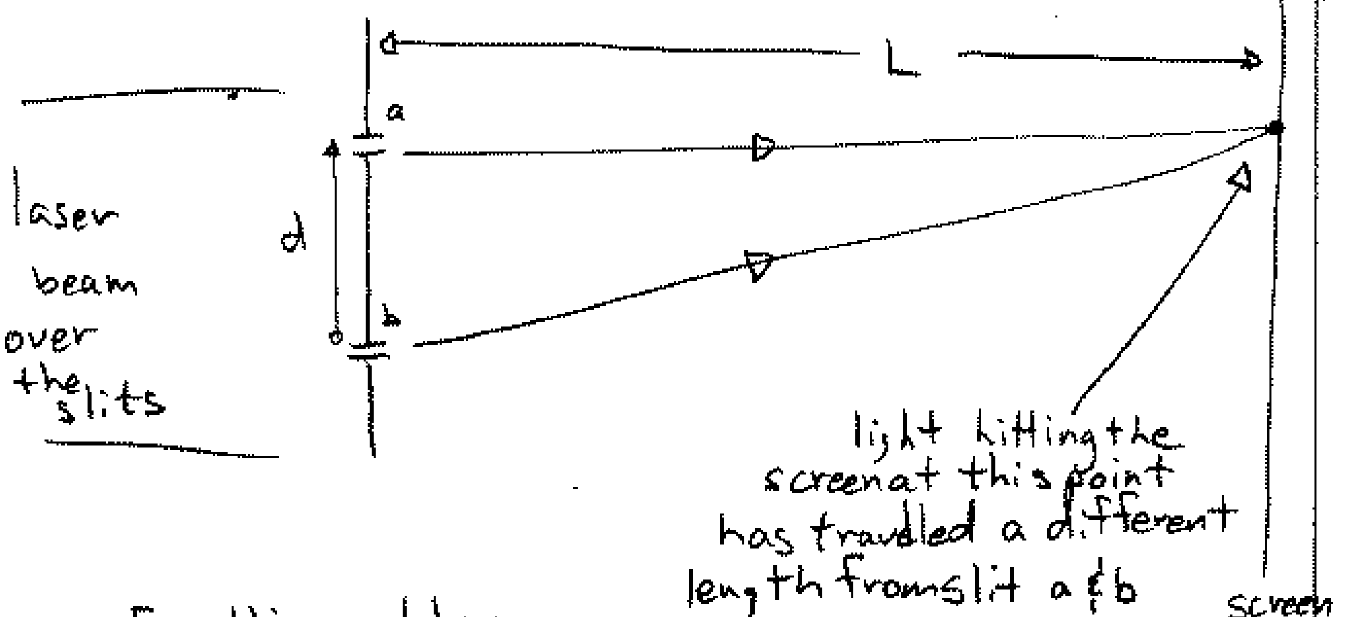
$$\delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$$

if the waves differ by ½ wavelength, they add destructively

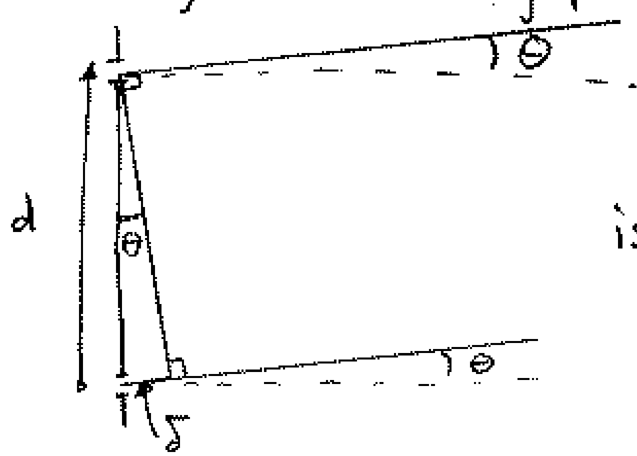


To demonstrate this, we will need a source of EM waves that are pretty similar (good source is a laser OK source is a the sun too)

& a way to make a path difference
Here is the classic way (2 slit interference)



for this problem,
it is useful to consider the following approximation
if the screen is very far away (d/L is tiny, try it!) the light beams coming out are nearly parallel



the path difference is very nearly

$\delta = d\sin\theta$

for constructive interference

$$d\sin\theta = 0,\ \lambda,\ 2\lambda,\ 3\lambda$$

destructive

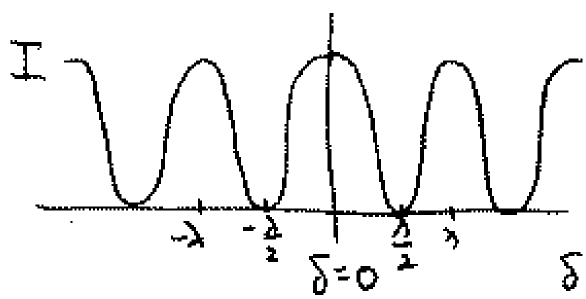
$$d\sin\theta = \frac{\lambda}{2},\ \frac{3\lambda}{2},\ \frac{5\lambda}{2}$$

the net effect is to get fringes since the intensity

$$I \propto E^2 = (2E_0)^2 \sin^2\left(kx - \omega t - \frac{\phi}{2}\right)\cos^2\frac{\phi}{2}$$

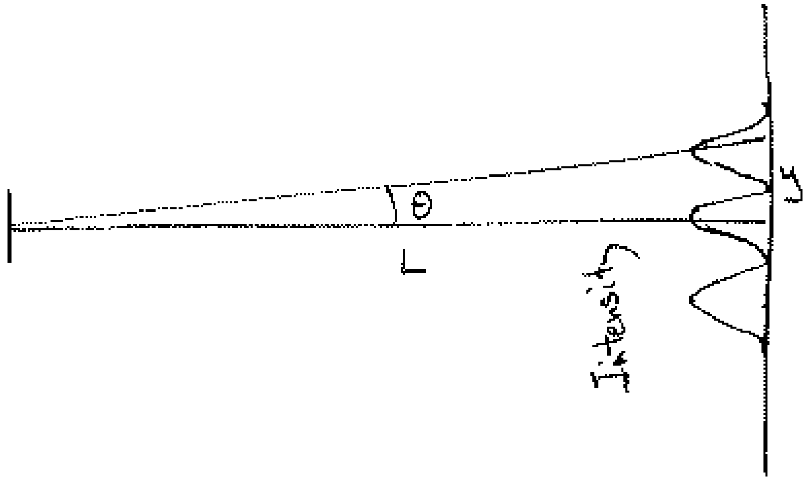
$$\phi = k\delta = \frac{2\pi}{\lambda}\delta$$

$$= (2E_0)^2 \sin^2\left(kx - \omega t - \frac{\phi}{2}\right)\cos^2\left(\pi\frac{\delta}{\lambda}\right)$$



this translates directly to the screen

for small θ $\quad \sin\theta \sim \theta \sim \frac{y}{L}$



$d\left(\frac{y}{L}\right) \approx \delta$

$= 0, \lambda, 2\lambda$

for bright spots

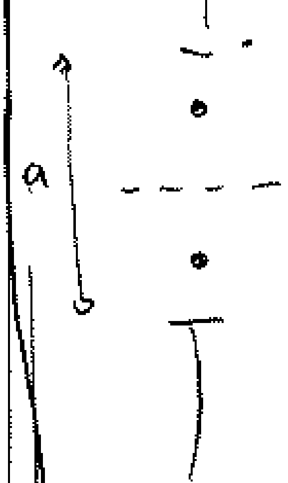
$y = 0,\ \lambda\left(\frac{L}{d}\right),\ 2\lambda\left(\frac{L}{d}\right)$

Lets do a practical example & try to determine the wavelength of a laser

Pay attention! you'll do this in lab

laser demo

You'll also notice another pattern, this is due to the slits themselves. You treat a single slit almost the same as a double



single slit

cut the slit in half each half acts like a "slit" in a double slit $d \Rightarrow \frac{a}{2}$

$\frac{a}{2} \sin\theta = \frac{\lambda}{2}$ for the first minima

single slit, hold your fingers to the light!

now, there's another way to get interference

Ever wonder why most camera lenses are blue? We want to capture as much visible light as possible so we try to minimize reflections of visible light

Ever wonder why oil on the water has pretty colors

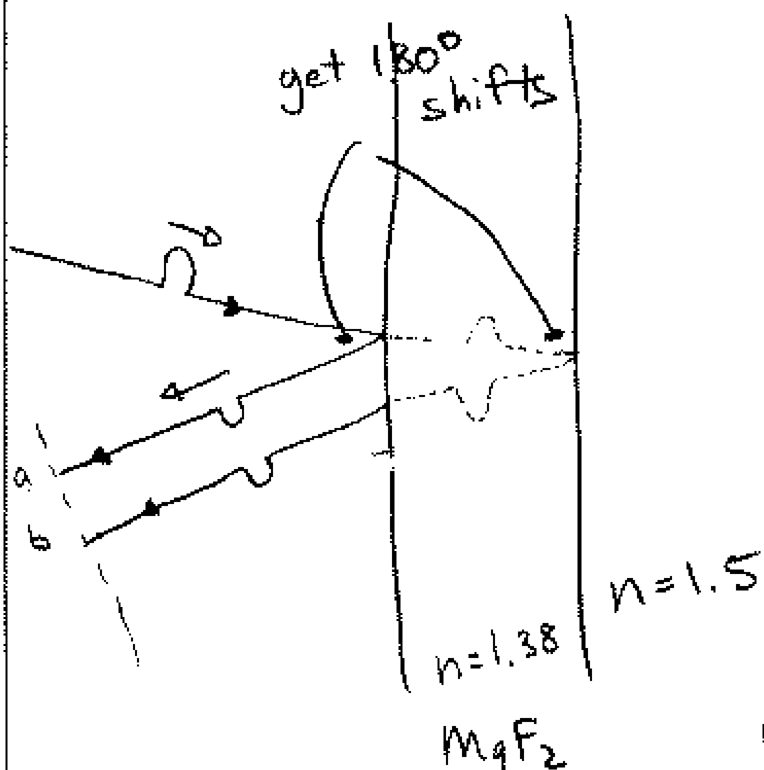
it's interference

To do calculations keep in mind

① ⇒ light reflected from a material with higher index of refraction has a 180° phase change (like a rope tied to a wall)

② λ changes if n changes, this will effect our path length calculation

so for our lens



ϕ_a at line $= \pi$

ϕ_b at line $= \pi + \phi$ due to path length

$\phi = \frac{2\pi}{\lambda}\delta \qquad \delta = 2t$

$\lambda = \lambda_0/n$

$\lambda_0 \sim 550\text{nm}$

now, we want to make sure most of the light passes through so we want to minimize reflections

⇒ destructive interference

$$\underbrace{\phi_a - \phi_b}_{\text{phase difference between 2 waves}} = \pi = \pi - \left(\pi + \frac{2\pi}{\lambda_0/n} 2t\right)$$

or

$$\cancel{\pi} = 4\frac{\cancel{\pi}}{\lambda_0} n t$$

$$\frac{\lambda}{4n} = t = \frac{550\,nm}{4(1.38)}$$

$$= 99.6\,nm$$

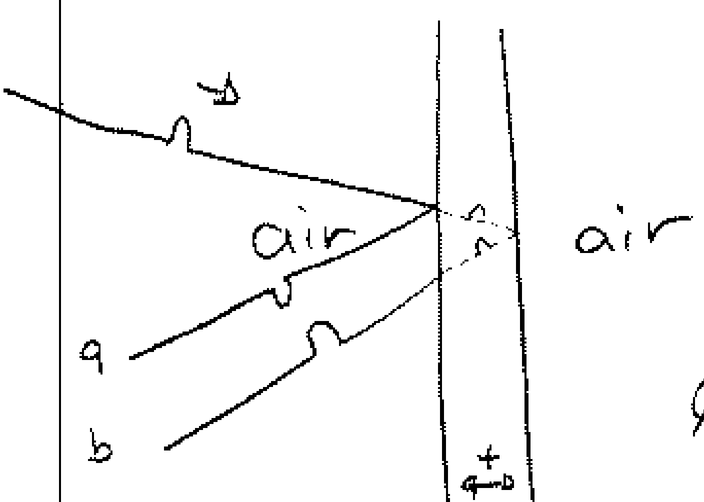
& for the low end of the visible spectrum $\lambda_0 = 350nm$

$$\phi_a - \phi_b = \pi \left(\frac{4(1.38)\,99.6nm}{350nm} \right)$$

$$= 1.6\pi$$

constructive & getting more so as λ_0 gets smaller

2nd example soap bubble



$n = 1.33$
(water)

$$\phi_a = \pi$$

$$\phi_b = 0 + \frac{2\pi}{(\lambda/n)} 2t$$

$$= \frac{4\pi n t}{\lambda_0}$$

$$\phi_a - \phi_b = \pi - \frac{4\pi n t}{\lambda_0} = 0, 2\pi, 4\pi \text{ const}$$

$\pi, 3\pi, 5\pi$ des

or $\frac{4nt}{\lambda} = 1, 3, 5$ cons

$\frac{4nt}{\lambda} = 0, 2, 4 \ldots$ des

Soap bubble demo (different colors!)