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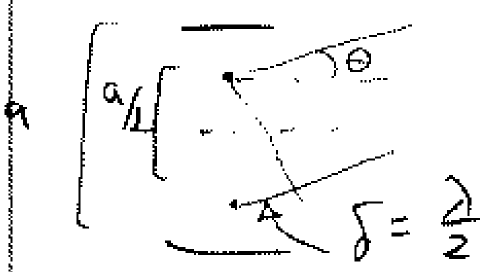

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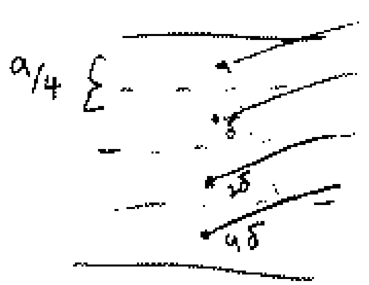

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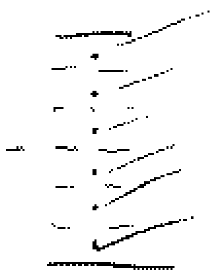

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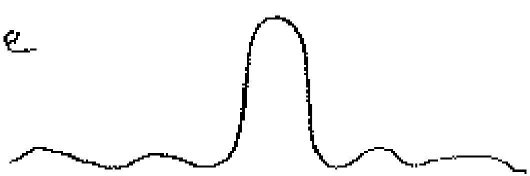

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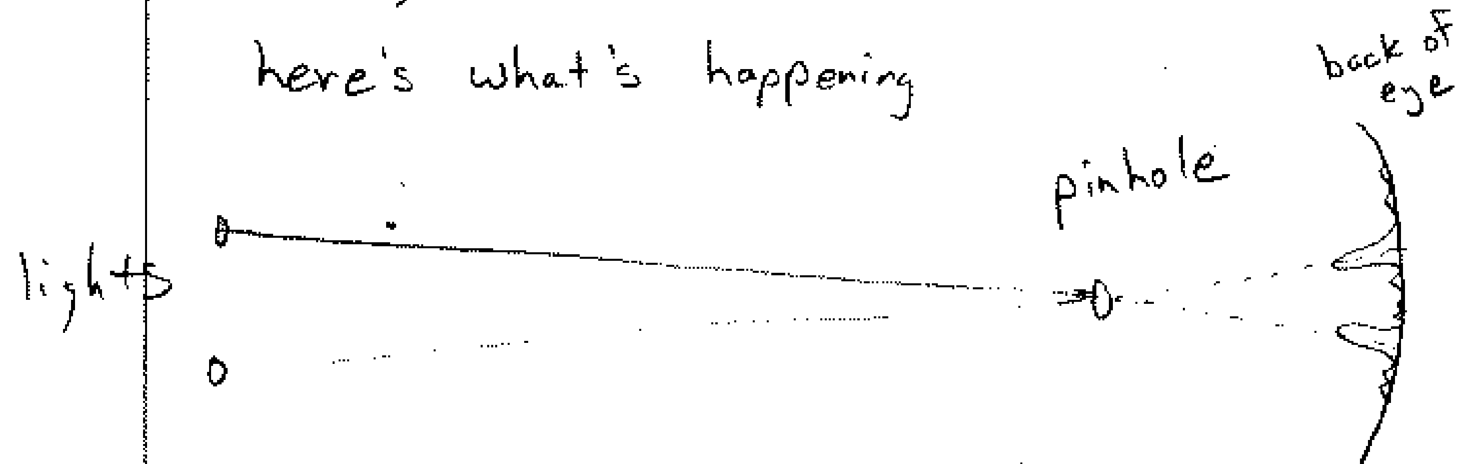

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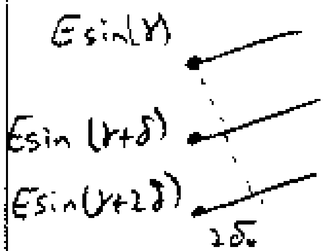

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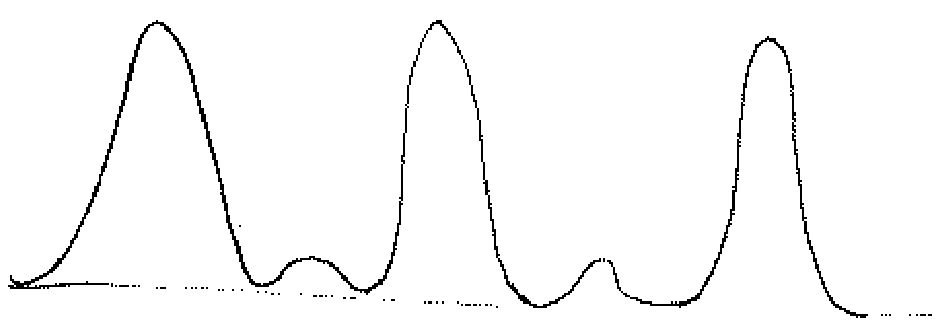

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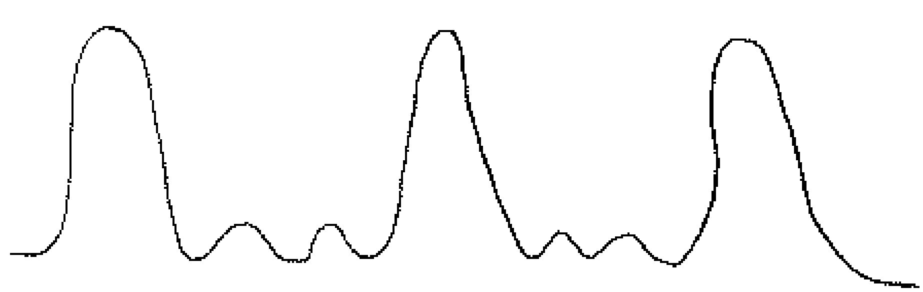

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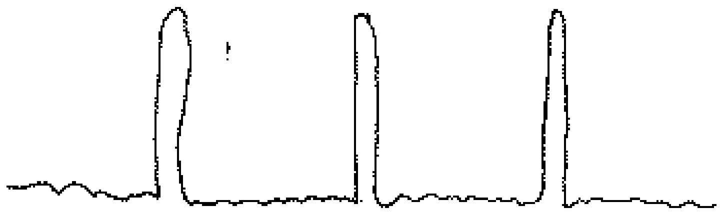

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a device made with many slits is called a diffraction grating.

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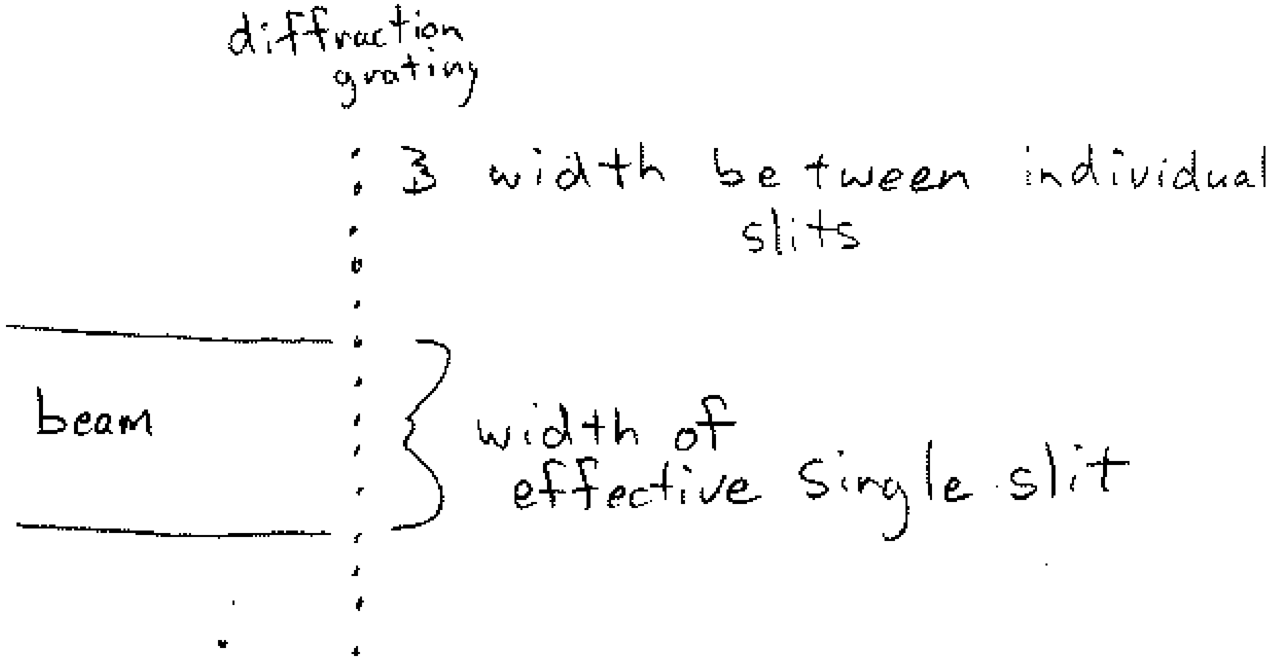

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$$a = N(\text{width between gratings})$$

or, if you like, the angular width of a peak, $\Delta\theta = \frac{\lambda}{a} \approx \left(\frac{\lambda}{Nd}\right)$

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small angles

$$\theta_1 \sim \frac{\lambda_1}{d}, \quad \theta_2 \sim \frac{\lambda_2}{d}$$

$$\Delta\theta = \left(\frac{\lambda_1}{d} - \frac{\lambda_2}{d}\right)$$

interesting case when λ_1 & λ_2 are close

to satisfy Rayleigh's criteria

this $\Delta\theta$ must be $\geq$ the angular width we calculated before

$$\frac{\lambda_1}{d} - \frac{\lambda_2}{d} \geqslant \frac{\lambda_1}{Nd}$$

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Excited atomic mercury
Excited atomic sodium has 2 "lines" that are very close $\lambda_1 = 589.59\,nm$ $\lambda_2 = 589.00\,nm$

$$R \approx \frac{589.59\,nm}{589.59 - 589.00}$$ ⇐ most conservative (some use average)

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$$3mm = (999)d$$

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$3\,\mu m$

2nd order $6\,\mu m$

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if $d = 3\,\mu m$ $\sin\theta = \frac{3(589\,nm)}{3000\,nm} \approx \frac{1.8}{3}$

can go no higher than 5th order

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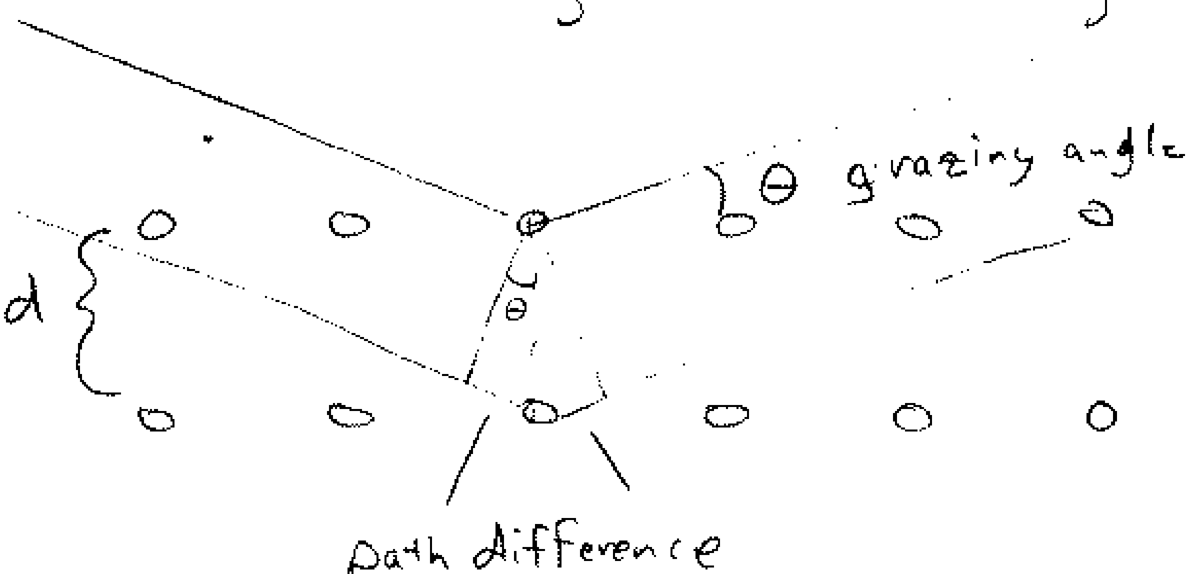
$$\sin\theta = \frac{3(589.6\,nm)}{9\,\mu m} \sim \frac{1.8}{9} \quad OK$$

Diffraction gratings are all over the place

CD's for instance are a diffraction grating

Can have diffraction of x rays too

Consider a crystal that has layers



$\delta = 2\,(d \sin\theta)$

get constructive interference when

$\delta = \lambda$

$2d\sin\theta = m\lambda$

λ xrays $\sim 0.01\,nm$

so can measure d's if

$(2d > \lambda)$ $d > \frac{\lambda}{2}$

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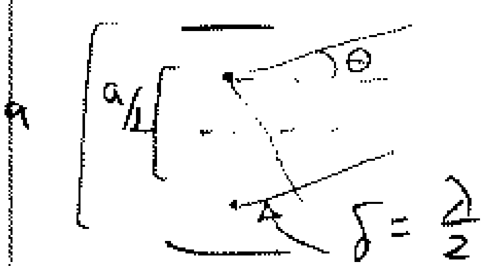

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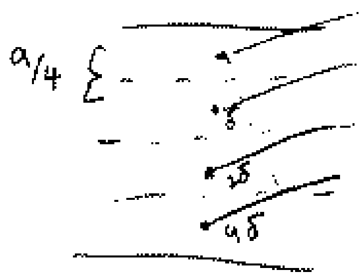

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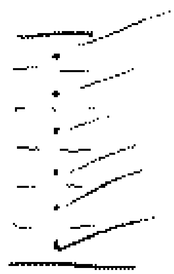

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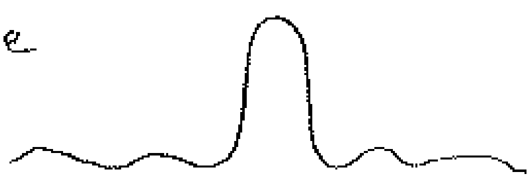

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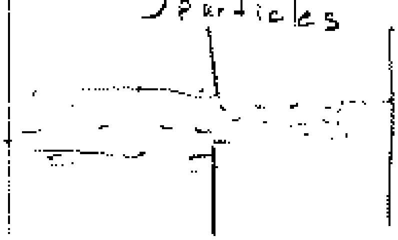

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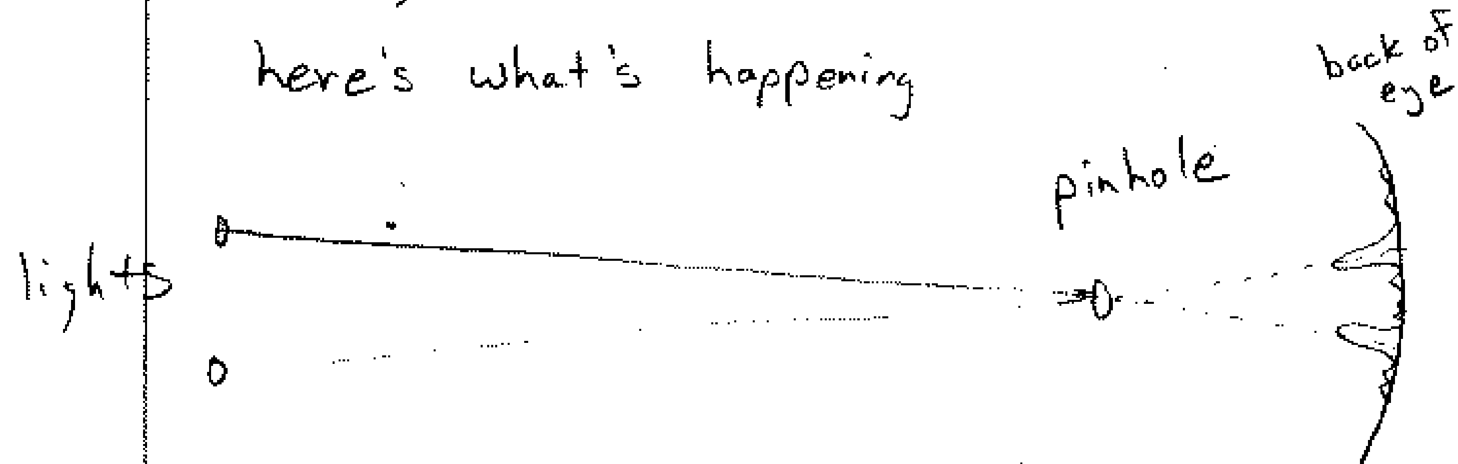

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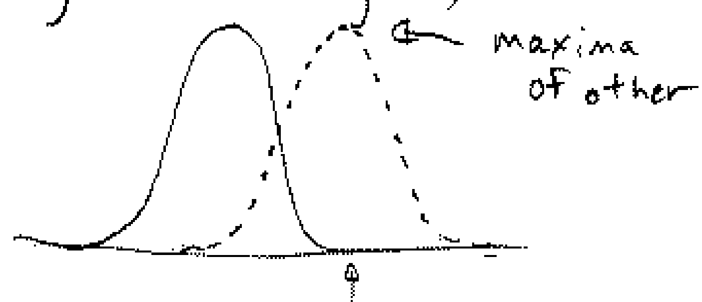

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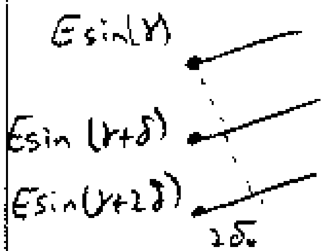

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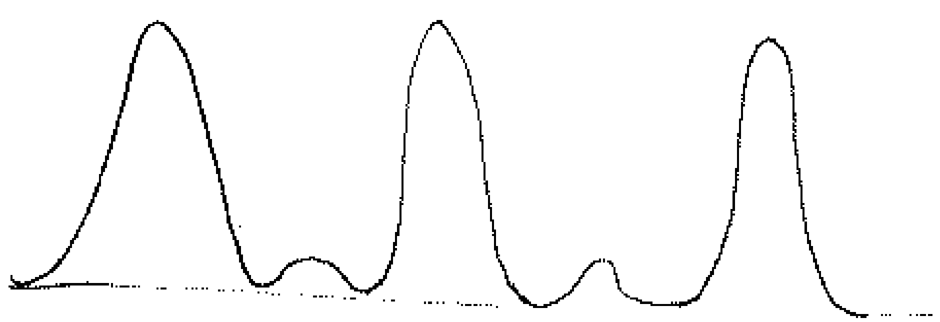

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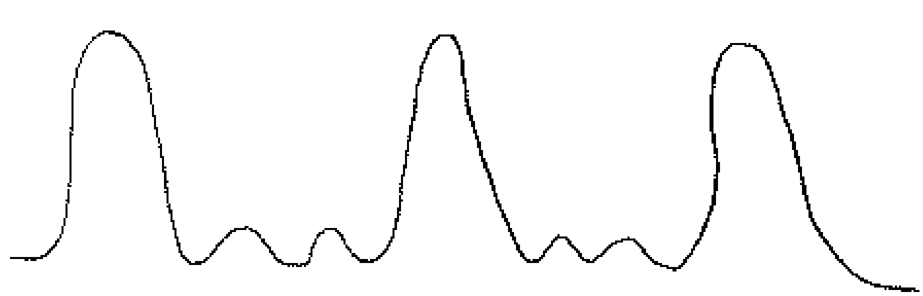

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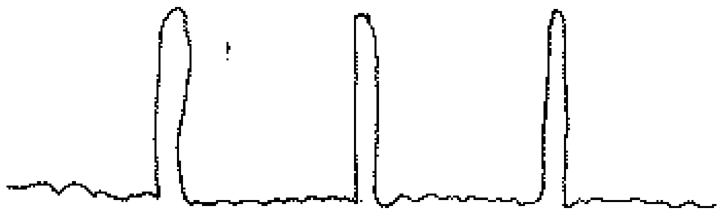

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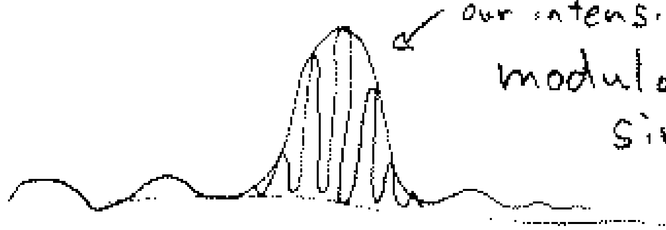

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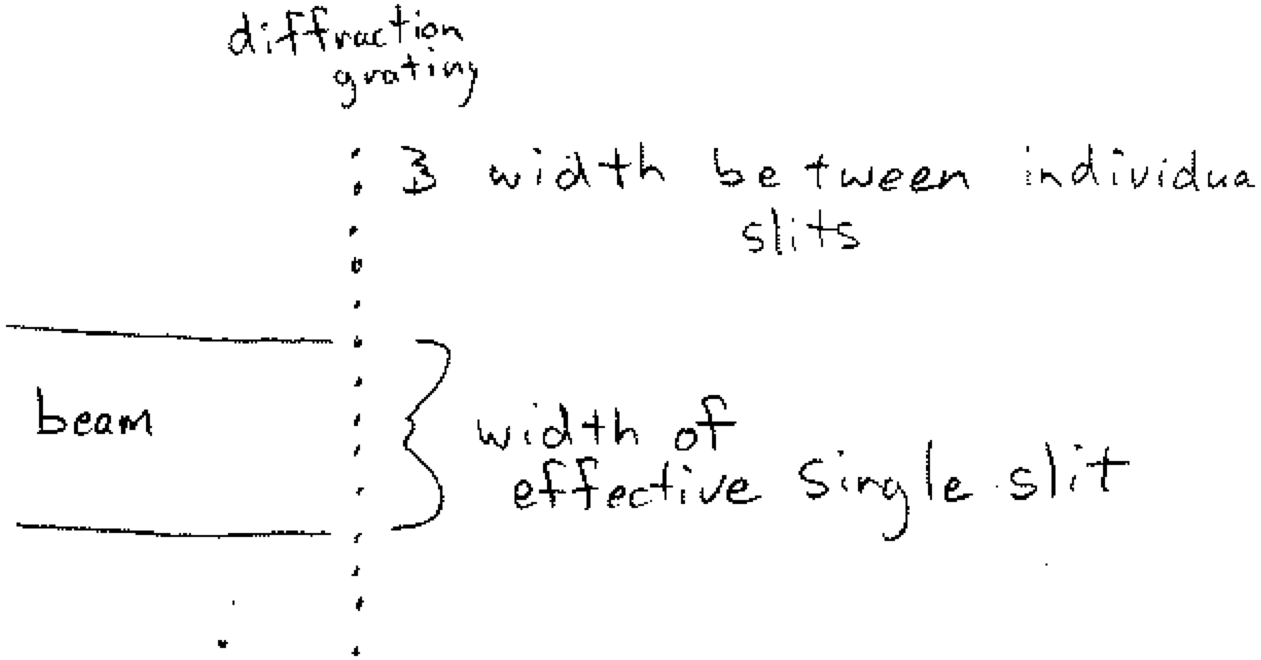

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