

We've been studying some very interesting properties of Electromagnetic Radiation

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gave us Intensity
Pressure
Power / energy
Polarization

we studied the speed of light in material
& discovered Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

& this led us to studying lenses

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Next, we studied consequences of the wave like properties of light

Interference & Diffraction

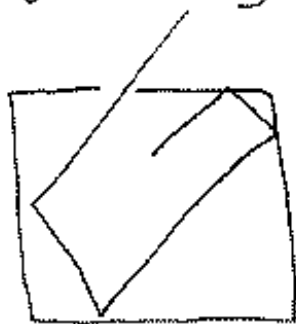
Now, I want to show you a case where this model breaks down

consider a very contrived case of a conductor say that exists like a box and has only a tiny hole at the top

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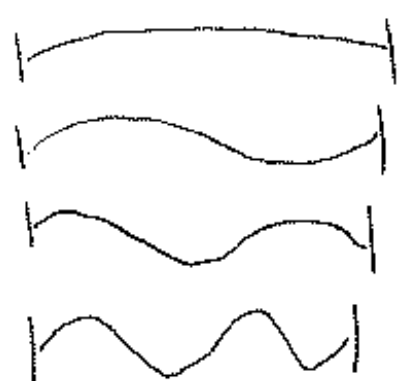
becomes a standing wave in the "box"

and you'd expect wavelengths based on the sides of the box



radiation getting inside gets trapped (pretty much if the hole is small) & what ever gets out, tends to be a function of temperature

like you'd expect waves on a string



in one dimension

and so on

now, we dealt with energies in an ideal gas by giving each degree of freedom an equal share of the energy

$$\left(\frac{1}{2} m v^2 = \frac{3}{2} kT \right) \quad \text{3 degrees of freedom}$$

Now, if we do the same here, the smaller wavelengths will carry away the lions share of the energy since we can fit more of those ("degrees of freedom") into the box

This is called the "ultraviolet catastrophe"

here's what we really see,

As I pass more and more current through this quartz lamp, it gets hotter and whiter so $\lambda \propto \frac{1}{T}$ (more T shorter)

In Fact the peak λ is predicted (for a black body) to obey Wiens law

$$\lambda_{\max} T = 0.2898 \times 10^{-2} \text{ m K}$$

predict for a light bulb

$$\lambda_{\max} = 550 \text{ nm}$$

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Clearly, we need to amend our model. In order to fit his Black body (Puzzling) data, Max Planck used a trick of allowing the possible energy states to have energies proportional to their frequencies

$$E = n h f \quad \begin{array}{l} \text{some proportionality} \\ \text{constant} \end{array}$$

and that the light we are seeing is due to transitions from one of these energy states to another

$$E_{\text{light}} = (n_{\text{start}} - n_{\text{finish}}) h f$$

$$\text{since } c = \lambda f$$

$$E_{\text{light}} = (n_{\text{start}} - n_{\text{finish}}) \frac{h c}{\lambda}$$

Couple of cool things to notice

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The light is emitted in discrete energy bundles
There is a lowest energy $\frac{hc}{\lambda}$ when $n=1$ (quanta)

and you can no longer let $\lambda \rightarrow 0$ or $E \rightarrow \infty$

Plank admittedly claimed he was lucky that everything just worked out.

There is another problem with our classical view of EM radiation

You'll notice that if I charge a piece of zinc up with negative charge (Fur on a rubber rod) and shine ultra violet light on the zinc, the charge quickly drains away.

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Now, with our old model,
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a few 10^{-19} J (called the
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Convenient unit eV

↑ short hand for 1.6×10^{-19} C

we just use eV instead of J

So we need to absorb a few eV
how long does this take?

100 W light bulb

— lets say I ^{intensity} @ surface is $1 \frac{\text{W}}{\text{m}^2}$

— each electron or atom if you like
takes up 10^{-10} m of space

— Energy for each atom

$$\sim \frac{\text{W}}{\text{m}^2} (10^{-10} \text{ m})^2 \sim 10^{-20} \frac{\text{J}}{\text{s}}$$

- takes a few seconds to energize
- doesn't depend on the light source
- we put more power in, get stuff faster
- if we wait long enough, should absorb enough energy to come off

What we see no matter what the charge is

- 1) No electrons are emitted if f is below some value called the cutoff frequency
- 2) ^{max} Energy of the ejected electrons is a function of the frequency of the incoming light not the intensity
- 3) Electrons get ejected right away.

Now, Einstein said, and this is where he got his 1st nobel prize, that this was happening because light is composed of those discrete bundles of energy γ described by Plank, & individual photons were colliding with individual e^- s

It takes a certain amount of energy γ to kick out an electron

(due to the image charge if you like) that is characteristic of the material. He called this ϕ the work function

To just begin to kick out electrons you need

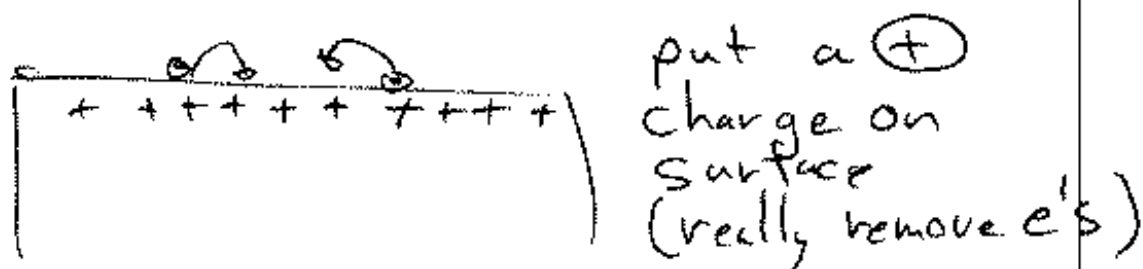
$$E_{\text{photon}} \geq \phi$$

$$h f_{\text{cutoff}} = \phi$$

$\uparrow$ Plank's constant $6.626 \times 10^{-34} \text{ Js}$
(also useful $hc = 1240 \text{ eVnm}$)

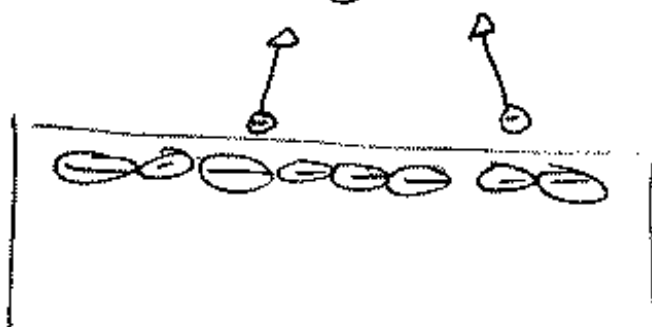
$$\text{maximum kinetic energy } K_{\text{max}} \text{ of ejected electrons} = hf - \phi$$

So, we can understand what is happening with our zinc (must be clean!)



put a $\oplus$ charge on surface (really remove e^- 's)

electrons can't get out



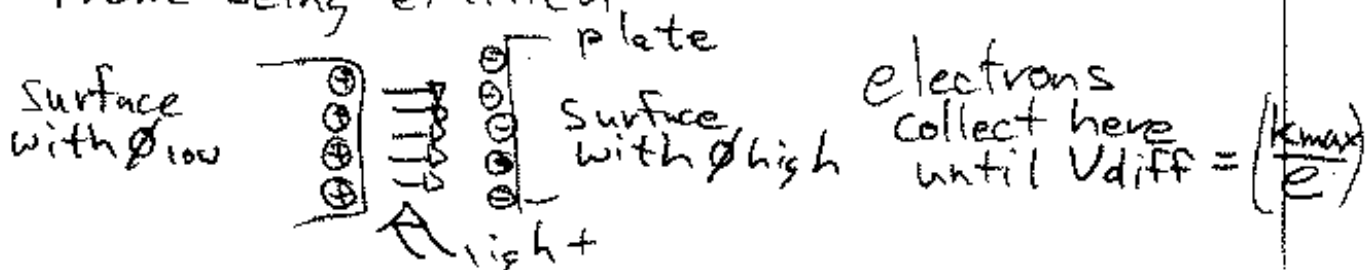
put more e^- 's on, e^- 's more likely to come out (less reverse potential)

but still need energetic enough photons to do the job

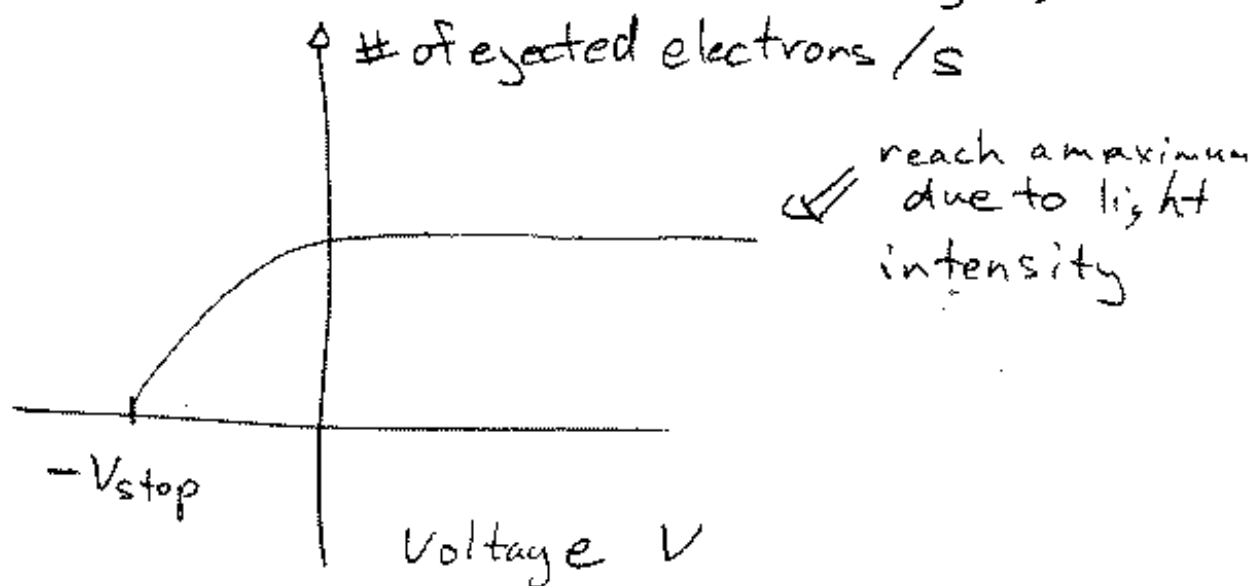
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K_{max} by finding what the potential difference is to try and stop electrons from being emitted



or, if you maintain a potential difference across the plates, you can prevent some of the electrons from being ejected



none get out when $V_{\text{stop}} = \frac{K_{\text{max}}}{e}$

choose this because it is easier to measure

~~ex~~ I have several lasers on hand

and can determine how much voltage

I need to stop electrons being ejected
for each one

f (Hz)	1.5×10^{15}	1.0×10^{15}	7.5×10^{14}	6.0×10^{14}	5.0×10^{14}
λ nm	200	300	400	500	600
V_{stop}	4.20	2.06	1.05	0.41	0.03

now if

$$K_{\text{max}} = hf - \phi = eV_{\text{stop}}$$

I should be able to plot K_{max} vs f
and find both h & ϕ (eV_{stop} vs f)

$f (10^{14} \text{ Hz})$

15
10
5

about $4.9 \times 10^{14} \text{ Hz}$

-2

electrons
no longer
ejected

eV_{stop}

at each spot

$$eV_{\text{stop}} = hf - \phi$$

expect if $f = 0$ $eV_{\text{stop}} = -\phi$

$$\phi \approx -(-1.8 \text{ eV}) \Rightarrow 1.8 \text{ V}$$

$\{ h$ is slope of line

$$\frac{1}{h} \approx \frac{1.5 \times 10^{15} \text{ Hz} - 5 \times 10^{14} \text{ Hz}}{4.20 \text{ eV} - 0.03 \text{ eV}}$$

$$h = 4.17 \times 10^{-15} \frac{\text{eV}}{\text{s}}$$

$$\uparrow$$

$$1.6 \times 10^{-19} \text{ C}$$

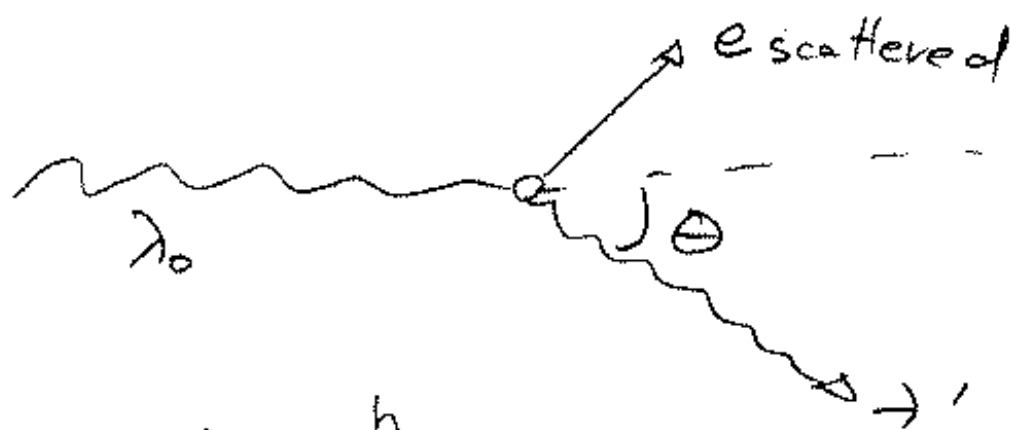
$$= 6.67 \times 10^{-34} \text{ Js} \quad \text{ok!}$$

$\{$ this is how h was measured about 10 years
after Einstein's prediction

The real nail in the coffin occurred when it was observed that some times x-rays bounced off electrons in a material as if the x-ray and the electron were billiard balls.

Using relativistic kinematics you find

$$\lambda' - \lambda_0 = \lambda_c (1 - \cos\theta)$$



$$\lambda_c = \frac{h}{mc} \quad \leftarrow \text{foreshadowing!}$$

† this is what was seen

now recall from your relativity

$$E^2 = p^2 c^2 + m^2 c^4 \quad (\text{need to add in } E_0 = mc^2)$$

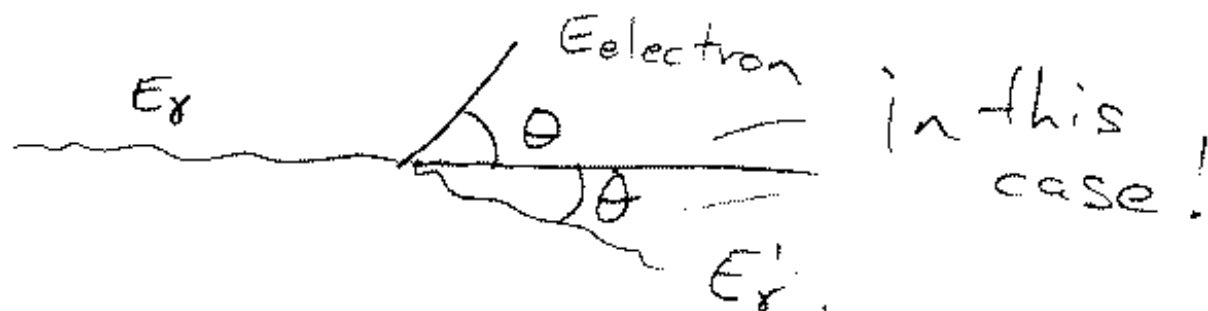
$$\uparrow (KE + mc^2)$$

Suppose you could measure the angle where the θ of the emitted photon is the same as the angle the electron was emitted.

What angle should you have if

$$E_\gamma = \frac{hc}{\lambda_0} = 0.880 \text{ MeV} \quad ? \quad \begin{cases} E_\gamma' \\ E_{\text{electron}} \end{cases} ?$$

know momentum must be conserved & Energy must be conserved



Careful!

$$P_{\text{electron}} \sin \theta = P_\gamma' \sin \theta$$

$$P_\gamma = P_{\text{elec}} \cos \theta + P_\gamma' \cos \theta = 2P_\gamma' \cos \theta$$

$$E_{\text{before}} = E_\gamma + m_e c^2$$

$$E_{\text{after}} = E_\gamma' + KE_{\text{elec}} + m_e c^2$$

$$KE_{\text{elec}} = E_\gamma - E_\gamma'$$

$$\begin{cases} \text{Since } \lambda' - \lambda_0 = \lambda_c (1 - \cos \theta) \\ \lambda' = 2\lambda_0 \cos \theta \end{cases} \quad \begin{cases} \frac{h}{\lambda_0} = 2 \frac{h}{\lambda'} \cos \theta \\ P_\gamma = 2P_\gamma' \end{cases}$$

$$\Rightarrow 2\lambda_0 \cos \theta - \lambda_0 = \lambda_c (1 - \cos \theta)$$

$$\lambda_0 (2 \cos \theta - 1) = \lambda_c (1 - \cos \theta)$$

$$2 \cos \theta - 1 = \frac{\lambda_c}{\lambda_0} (1 - \cos \theta)$$

$$2 \cos \theta + \frac{\lambda_c}{\lambda_0} \cos \theta = 1 + \frac{\lambda_c}{\lambda_0}$$

$$\cos \theta = \frac{1 + \frac{\lambda_c}{\lambda_0}}{2 + \frac{\lambda_c}{\lambda_0}}$$

$$\frac{\lambda_c}{\lambda_0} = \frac{\frac{K}{m_e c}}{\frac{K_c}{E}} = \frac{E}{m_e c^2}$$

$$\theta = \cos^{-1} \left(\frac{1 + \frac{.880 \text{ MeV}}{.511 \text{ MeV}}}{2 + \frac{.880 \text{ MeV}}{.511 \text{ MeV}}} \right)$$

$$= 43^\circ$$

$$\frac{hc}{\lambda'} = \frac{hc}{\lambda_0} \frac{1}{2 \cos \theta} = \frac{.880 \text{ MeV}}{2 (\cos 43^\circ)}$$

$$= .602 \text{ MeV}$$

$$KE_{\text{electron}} = 0.880 \text{ MeV} - 0.602 \text{ MeV} = .278 \text{ MeV}$$

$$\text{check } p_e = p_{\gamma c} \quad p_e c = \sqrt{(.278 + .511)^2 - .511^2} = .601 \text{ MeV} \text{ pretty good!}$$

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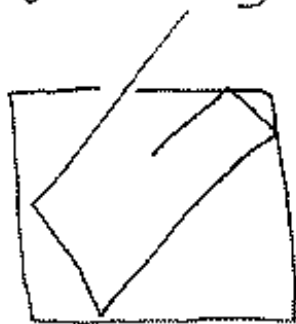
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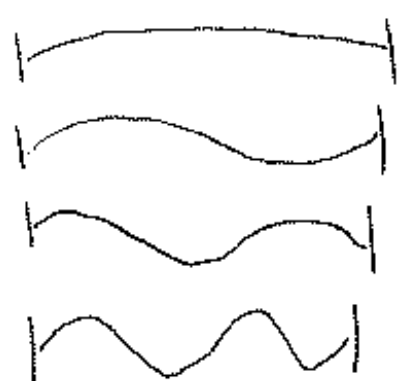
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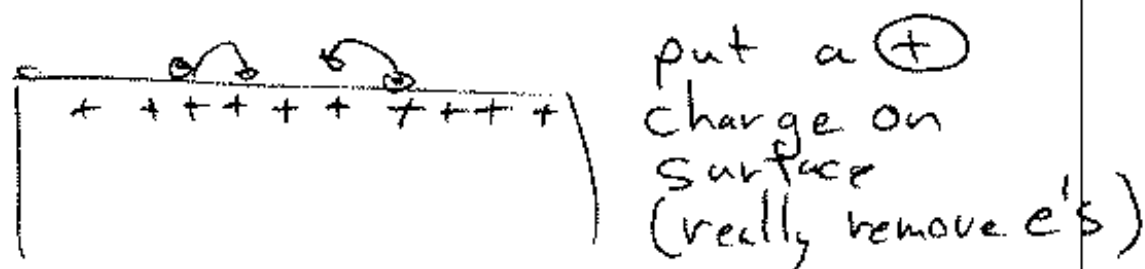
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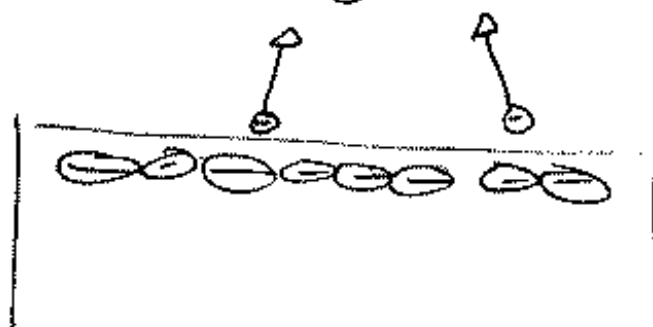
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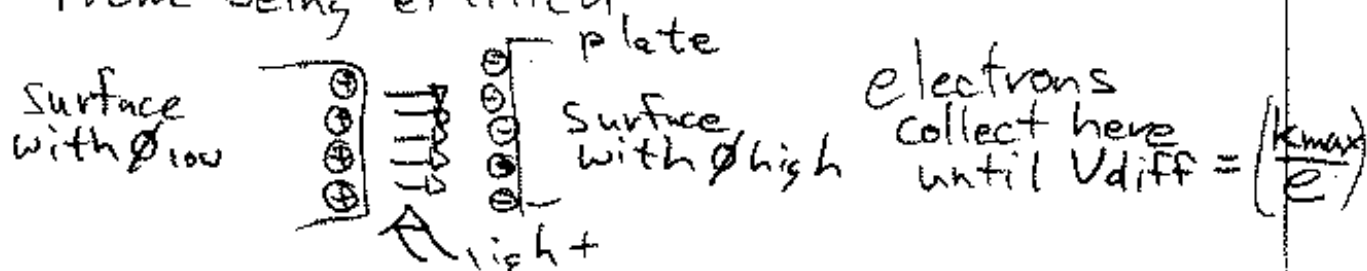
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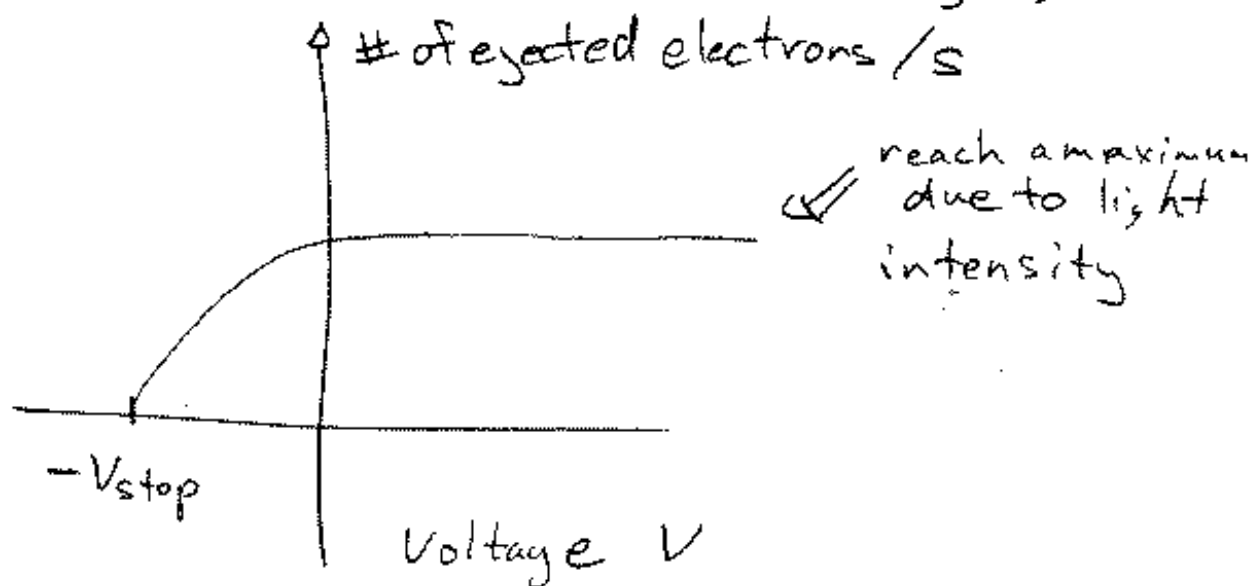
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