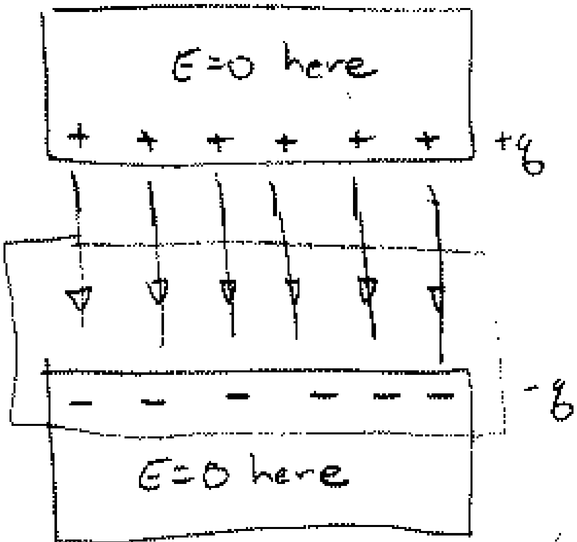
Consider 2 plates charged oppositely

E=0 here



$$|E|(\text{area of plate}) = \frac{|q|}{\varepsilon_0}$$

$$E = \sigma/\varepsilon_0$$

$$\boxed{\text{true at surface of any conductor}}$$

E=0 here

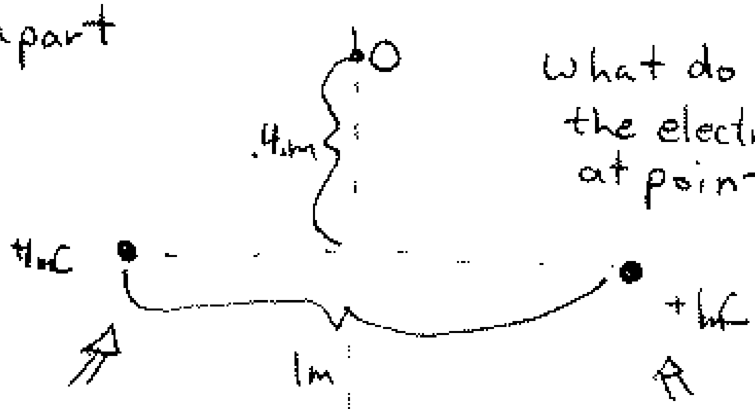
LECTURE 2 - Sept 05, 2000

Last time we talked about the properties of charge ⊕ ⊖, like repel, opposites attract, $|F| = k\frac{q_1 q_2}{r^2}$, $E = F/q$ and performed a calculation. I also told you that the study of electrostatics was important because modern electronic devices can be sensitive to Electro-Static-Discharge.

Turns out there are powerful tools that allow us to calculate the effect of distributions of charge, especially if the charge displays great symmetry. Let me show you what I mean.

Suppose we take 2 charges, each 1mC and place them a meter apart

What do you suppose the electric field is at point O



this charge has an electric field that points ↗ at O

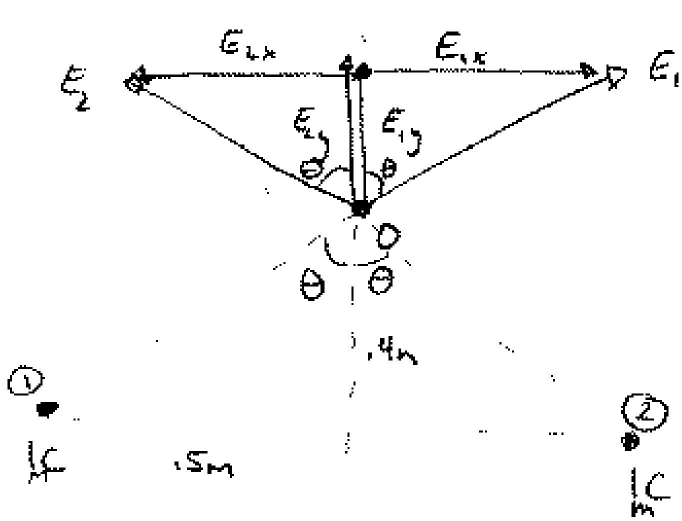
this charge has an electric field that points ↖ at O

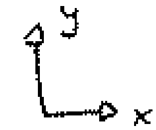
we can add Electric fields like we used to add forces

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \ldots$$

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \ldots$$

You guessed it we use vectors to find E_{tot} ②





$$\cos\theta = \frac{|E_{1y}|}{|E_1|} = \frac{.4}{\sqrt{.4^2+.5^2}}$$

the x parts cancel and we're left only with the y parts, lets calculate the y part for charge ② & double it

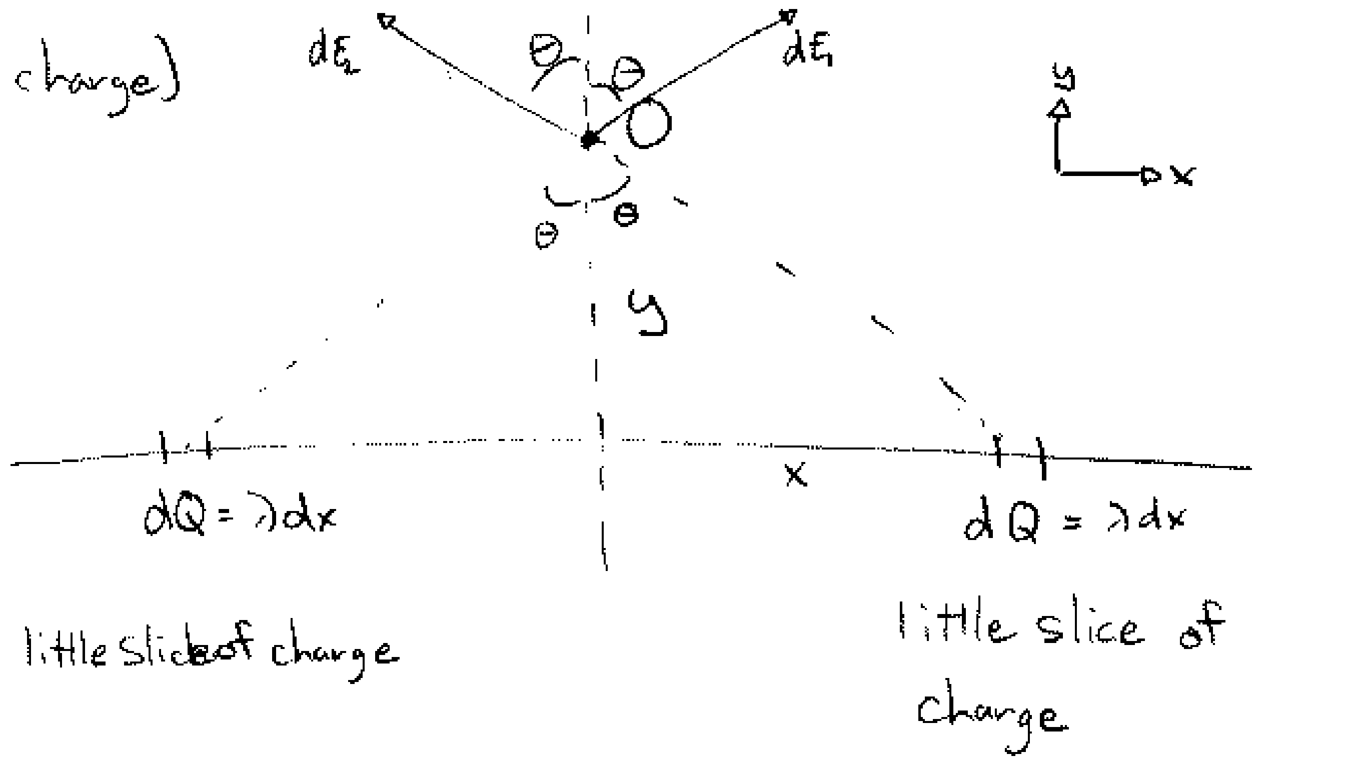
$$E_{2y} = \frac{K q_2}{r_2^{\,2}}\cos\theta = 8.99\times10^9 \,N\frac{m^2}{C^2}\frac{1C}{(.5m^2+.4m^2)}\frac{.4m}{\sqrt{.4^2+.5^2}}$$

$$= 1.37\times10^4 \,N/C \text{ in } \hat{j}$$

$$\vec{E}_{tot} = 2.74\times10^4 \,N/C \;\hat{j}$$

now, suppose we are interested in a whole line of charge. We can proceed in the same way. We just use the result of the calculation we just made again and again in an infinite sum. Here's what I mean. Suppose we have a very long (infinite) line of charge. this line of charge

has a charge per unit length of $\lambda = 1\,\text{mC}/\text{m}$ (3)
(if we slice out a ~~metch~~ meter of this line, we'll get 1 mC of charge)



$$d\vec{E}_{tot} = \frac{2k\lambda dx}{(x^2+y^2)}\cos\theta$$

bit of $\bar{E}$ tot due to this bits of charge

now, a bit of substitution

$$\frac{x}{y} = \tan\theta$$

$$x = y\tan\theta$$

$$dx = \frac{y}{\cos^2\theta}d\theta$$

$$dE_{tot} = \frac{2k\lambda\left(\frac{y}{\cos^{\cancel{2}}\theta}d\theta\right)\cancel{\cos\theta}}{y^2\tan^2\theta + y^2}$$

$$y^2\frac{\sin^2\theta}{\cos^2\theta} + y^2\frac{\cos^2\theta}{\cos^2\theta} = \frac{y^2}{\cos^2\theta}$$

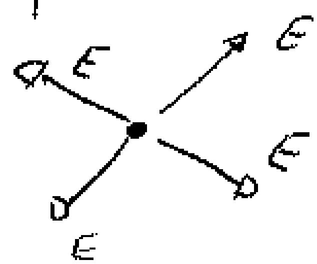
$$= 2k\lambda\frac{\frac{y\,d\theta}{\cos\theta}}{(y^2/\cos^2\theta)} = \frac{k\lambda}{y}\cos\theta\, d\theta$$

to get E_{tot}, just sum up all the bits of charge from $\theta = 0$ to $\theta = \pi/2$

$$E_{tot} = \int_0^{\pi/2} \left(\frac{2k\lambda}{y}\right) \cos\theta \, d\theta = \frac{2k\lambda}{y} \sin\theta \Big|_0^{\pi/2}$$

$$= \frac{2k\lambda}{y}$$

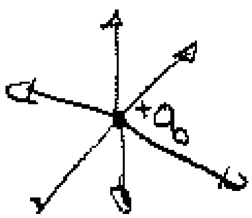
this is true if we picked O above or below the line, and E always points out radially. If we look at the end of a wire



like spokes on a wheel

now, you could imagine calculating what the electric field is due to a bunch of infinite wires placed side by side to make an infinite sheet. But there's an easier way, using graphical methods for this calculation.

now, lets take our drawing of the Electric field a little more seriously. How useful is it



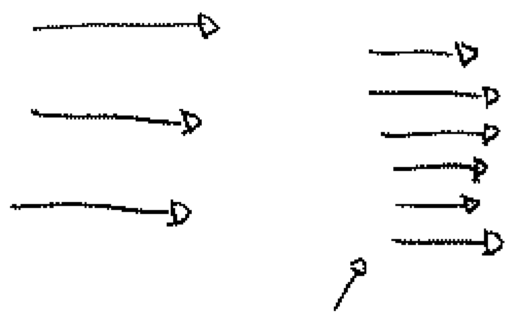
- lines should be symmetric like the field
- closer the lines are the stronger the field is
- same number of lines go through a big or a small sphere if it is centered on the charge

now look at $E = \frac{kq}{r^2}$ like $\frac{\text{Constant}}{\text{area}}$

infact, if we call $k = \frac{1}{4\pi\varepsilon_0}$

$$E = \frac{q}{\varepsilon_0}\frac{1}{4\pi r^2} \propto \frac{\#\text{ lines}}{\text{Area}}$$

<u>example</u>

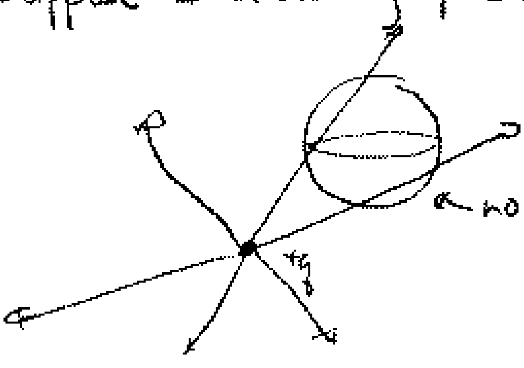


E is stronger here

but we can't quite say

$\frac{q}{\varepsilon_0} \longleftrightarrow$ # lines

Suppose I draw my sphere outside the charge



no charge in there

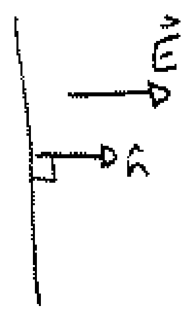
same number of lines entering as leaving

If there are more lines leaving than entering ↳ charge inside ⑥
Some connection between complete closed surfaces, lines and charge.

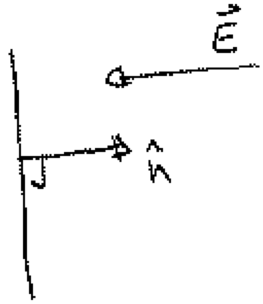
presumably if we wanted to be general, we should be able to draw any complete closed surface around a charge and get the same answer. to do this we need to discuss the concept of electric flux.

Simply put, the electric flux is just the Electric field perpendicular to a surface times the area of the surface.

normal to a surface



flux is positive



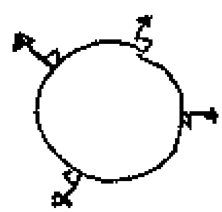
flux is negative

flux through these surfaces is the same

in general

$$\Phi = \vec{E} \cdot \hat{n} A$$

& if we sum up all the flux over a whole surface



$$\oint \vec{E} \cdot \hat{n} dA = \frac{q_{inside}}{\varepsilon_0}$$

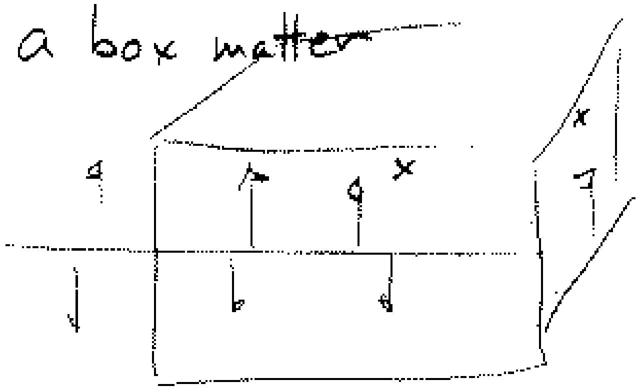
We can use this to great advantage

recall our cylinder, the E field pointed out like spokes on a wheel. If we choose a cylindrical surface to enclose our wire, the ~~Ends~~ caps of the cylinder have no flux in them so $\Phi_{tot} = E(2\pi r)\ell = q/\varepsilon_0$

$$\text{or } E = \frac{q}{2\pi r \ell \varepsilon_0} = \frac{2k(q/\ell)}{r} = \frac{2k\lambda}{y}$$

as before

a sheet of charge is just as easy, only 2 sides of a box ~~matter~~



$$2Ex^2 = q/\varepsilon_0$$

$$E = \frac{q}{x^2}\frac{1}{2\varepsilon_0}$$

$$= \frac{\sigma}{2\varepsilon_0}$$ ← charge/area

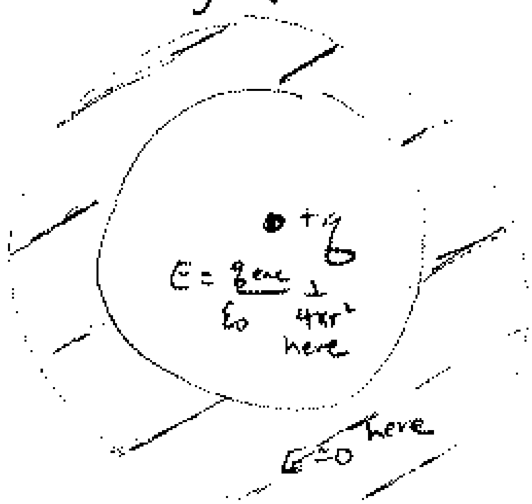
To complete our bag of tricks for this unit, note that inside a conductor, $E=0$ as we said before, this means that all the charge sits on the outside of the conductor at equilibrium. A useful concept and consequence of the inverse square law (in fact a very sensitive test of it)

Can this really be so? Lets try an example. At high frequencies, radio will act alot like charge at equilibrium and you can effectively shield equipment in a cage.

You can have lots of fun with gauss's law and conductors. Consider the following (9)

we put a charge inside of an unchanged conducting sphere

What is E?



$E = ?$

Consider that $E = 0$ inside the conductor

$Q_{tot} = 0$ on the conductor

the +q induces charge on the inside of the sphere equal to $-q$ so $E = 0$ inside the conductor since q enclosed by a complete closed surface is 0. Since $Q_{tot} = 0$ on the sphere $+q$ is on the outside surface of the sphere so E outside is as you'd expect for a point charge

$$\frac{q}{\varepsilon_0} = \frac{+q - q + q}{\varepsilon_0} = q/\varepsilon_0$$