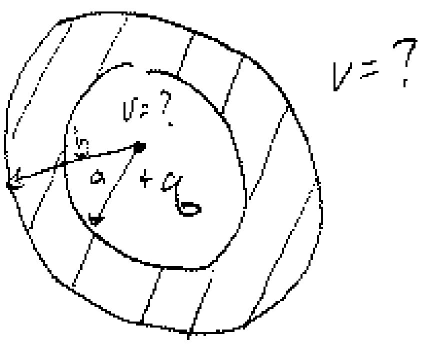


Know the electric field every where how do we get potential

lets make $V_I = r$ inside

$V_f = \infty$

and well take $V_F \Rightarrow 0$

to get to ∞ we'll take a charge a charge from $a \to b$, then from $b \to \infty$

$$V_F - V_I = (V_\infty - V_b) + (V_b - V_a) + (V_a - V_r)$$

$$= -\int_b^\infty \vec{E}\cdot d\vec{s} - \int_a^b \vec{E}\cdot d\vec{s} - \int_r^a \vec{E}\,d\vec{s}$$

$$= -\int_b^\infty \frac{kq_b}{r^2}\,dr - \int_a^b 0\cdot ds - \int_r^a \frac{kq_b}{r^2}\,dr$$

$$= -\left[-\frac{kq_b}{r}\Big|_b^\infty\right] - 0 - \left[-\frac{kq_b}{r}\Big|_r^a\right]$$

$$-V_I = \left(-\frac{kq_b}{b} + \frac{kq_b}{a} - \frac{kq_b}{r}\right)$$

$$V_{\text{inside}} = \left(\frac{kq_b}{r} - \frac{kq_b}{a} + \frac{kq_b}{b}\right)$$

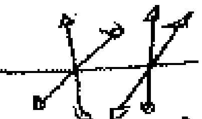
Lets continue our discussion of Gauss's Law LECTURE 3
Sept 7 ①

(the relationship between surface shapes, Electric fields & charge)

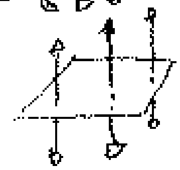
$$\oint \vec{E}\cdot d\vec{A} = q_{enclosed}/\varepsilon_0$$

point charge 

$E(4\pi r^2) = q/\varepsilon_0$

line of charge 

$E(2\pi r \ell) = q/\varepsilon_0$

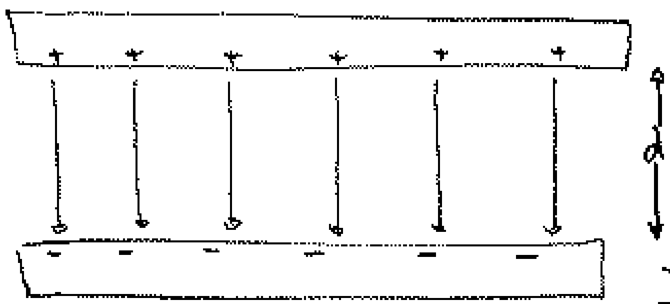
sheet of charge 

$E(2x^2) = q/\varepsilon_0$

Our goal is to connect it to something real. How do we make E?

Consider 2 conducting sheets separated by a distance, d

In between the sheets the field looks like ⇒



On the top sheet I have a charge $+q$

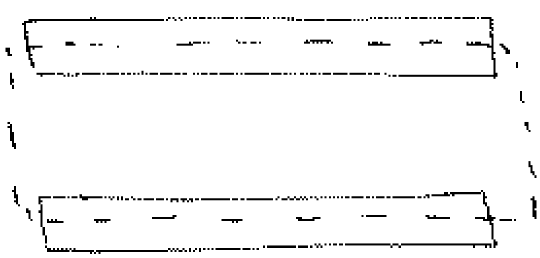
on the bottom sheet I have a charge $-q$

What can we glean from Gauss's Law (I see many ways)

- Choose a shape that mimics the E field (box)

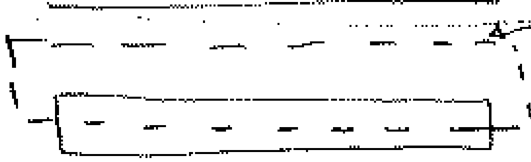
$q_{enc} = 0$

$E = 0$ inside a conductor



$\vec{E}\cdot\hat{n} = 0$ at sides

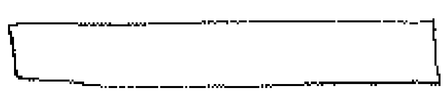
$0 = 0$



Same story

E must be 0 here

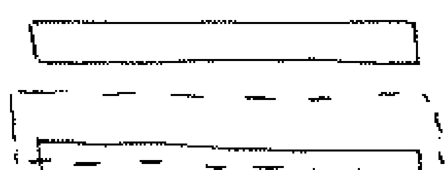
Since $q_{enc} = 0$ & $E = 0$ in cond & $\vec{E}\cdot\hat{n} = 0$ at sides



$EA = q_{enc}/\varepsilon_0$

$E = \dfrac{q_{enc}}{A\varepsilon_0}$

so what?



$q_{enc} = -q$

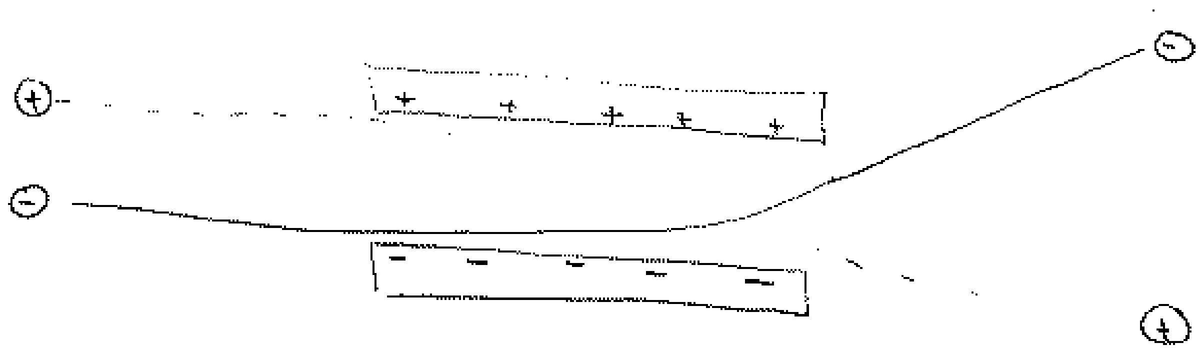
$\vec{E}\cdot\hat{n} = -E$

$A = x^2$

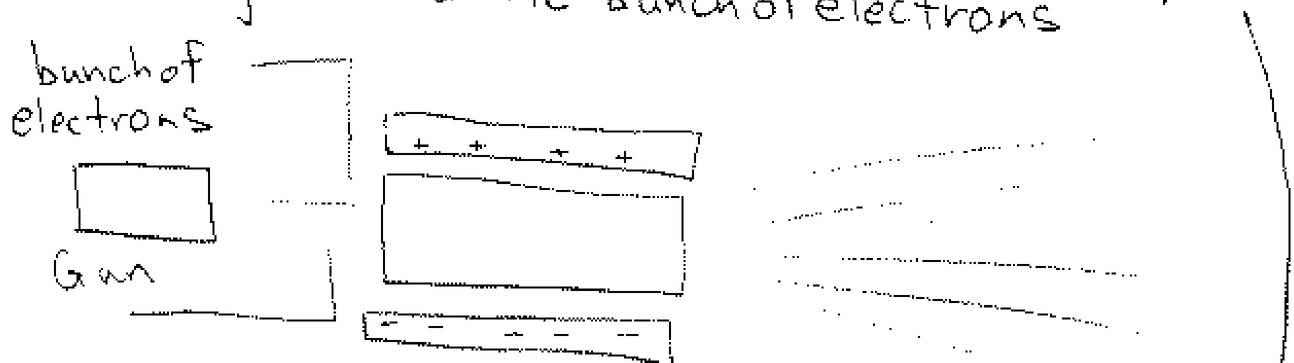
$Ex^2 = q_{enc}/\varepsilon_0$

Check this out

what do you suppose happens if I shoot an electron between the plates? It bends up!



now, imagine a whole bunch of electrons

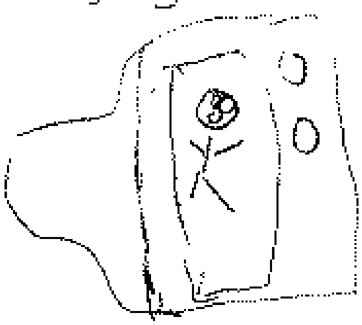


and sheets on front back top & bottom that you could control the amout of charge on

and a way to record the spot where the electron hit

Could be useful?

Usually you see these things in objects like this

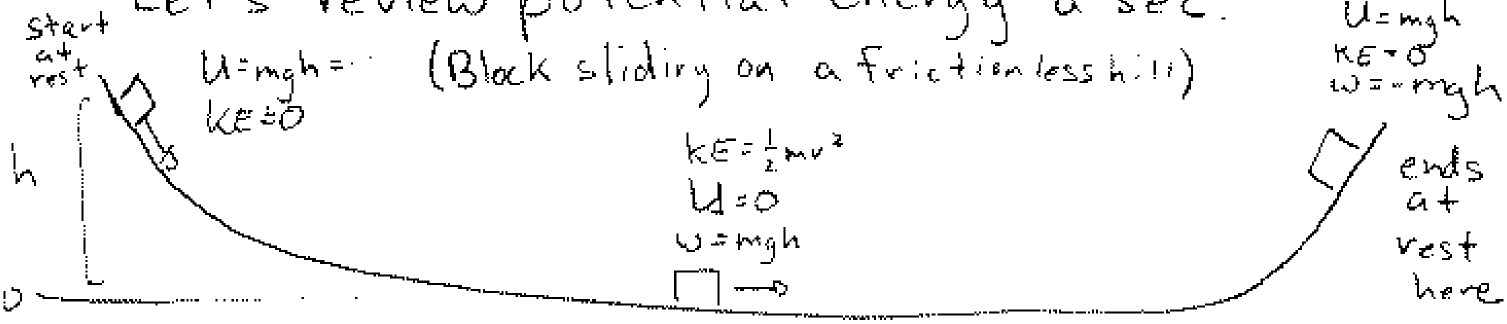


a TV (more likely an oscilloscope)

So that Electric field can be pretty handy sometimes

You notice that, in between the plates, work is being done on the charge. The charge is moving from a place with high potential energy to a place with lower potential energy because the electric field is doing work on it. (3)

Let's review potential energy a sec.



$$\Delta U = U_F - U_I = 0 - mgh$$
$$W = mgh$$
$$\Delta U = -W$$

$$\Delta U = U_F - U_I = mgh - 0$$
$$W = -mgh$$
$$\Delta U = -mgh$$

$$\text{or } W = \int_I^F \vec{F} \cdot d\vec{s}$$

$$\Delta U = -\int_I^F \vec{F} \cdot d\vec{s}$$

in terms of the electric field

$$\Delta U = -q_0 \int_I^F \vec{E} \cdot d\vec{s}$$

& usually, instead of potential energy, we talk about Electric potential, $V = U/q_0$

& as with potential energy, only differences are meaningful

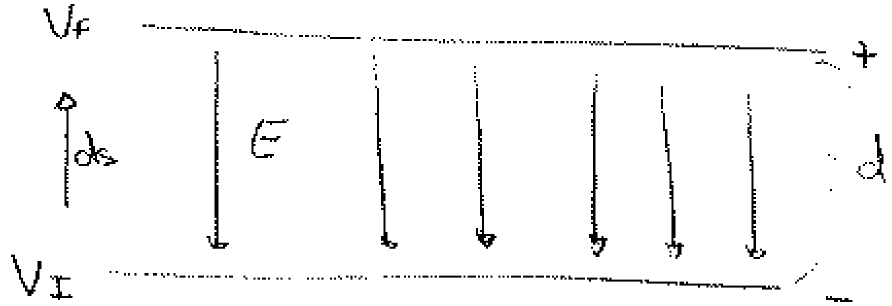
OK, so, we've got this difference in electric potential related to the Electric field

$$\Delta V = -\int_I^F \vec{E} \cdot d\vec{s}$$

What's so cool about it is that the units for Electric potential are called volts. The same thing you find on a battery. You can use a potential difference to make an electric field! We can't do that very easily with Gravity....

Oscilloscope is a great demonstration of this.

between the plates (Separate them by a distance d)



higher potential here than here

What would a positive charge do?

$$\Delta V = V_F - V_I = -\int_0^d \vec{E} \cdot d\vec{s} = Es\Big|_0^d = Ed$$

Confirms what we thought

& recall our expression for E, $E = q/\varepsilon_0 A$

$$Ed = \frac{qd}{\varepsilon_0 A} = \Delta V$$

So, the amount of charge on the plates depends on the ΔV & geometry of the conductor; $q = \frac{\varepsilon_0 A}{d} \Delta V$

There are a few tricks that come in handy for electric potential.

1) Electric potential decreases in the direction of the electric field vector

2) Potential differences are meaningful.

3) challenge is to find where to set $V=0$

ex point charge, what's the potential difference between $r=\infty$ & $r=R_0$?

for $+q$

$$\Delta V = V_F - V_I = -\int_{R_0}^{\infty} \vec{E} \cdot d\vec{s} = -\int_{R_0}^{\infty} \frac{kq}{r^2} dr$$

$$= -\left(-\frac{kq}{r}\Big|_{R_0}^{\infty}\right) = -\left[-0-\left(-\frac{kq}{R_0}\right)\right]$$

$$= -\frac{kq}{R_0}$$

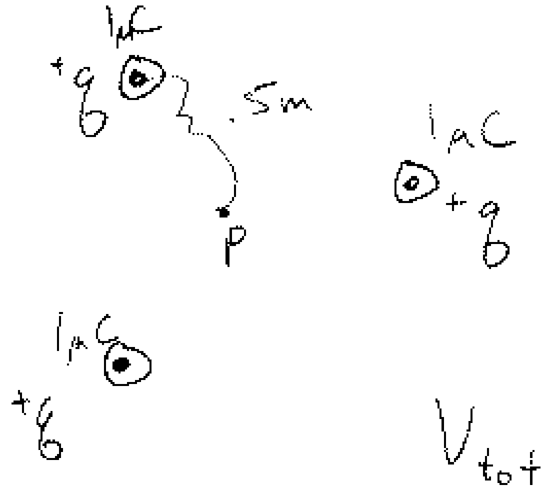
Usually, define $V_\infty = 0$

$$\Delta V = -V_I = -\frac{kq}{R_0} \quad \text{or} \quad V_{at\,R_0} = \frac{kq}{R_0}$$

& $V_{at\,R_0}$ is called the electric potential at R_0 even if it is the difference between $r=\infty$ & R_0

(easier to do calculations with)

Example



what is the potential at P?

$$V_{tot} = \sum \frac{kq}{r} = \frac{k(1\mu C)}{.5m} + \frac{k(1\mu C)}{.5m} + \frac{k(1\mu C)}{.5m}$$

$$= \frac{3\left(8.99\times10^{9}\,N\frac{m^2}{C^2}\right)\left(10^{-6}C\right)}{.5m}$$

$$\sim 5.40\times10^{4}\,\frac{Nm}{C}$$

lots easier than all those integrals!

4) notice how the integral is set up

$$V = -\int \vec{E}\cdot d\vec{s} = -\int\left[E_x\,dx + E_y\,dy + E_z\,dz\right]$$

looks like

$$dV = -E_x dx - E_y dy - E_z dz$$

$$= -\left(\frac{\partial V}{\partial x}\right)dx - \left(\frac{\partial V}{\partial y}\right)dy - \left(\frac{\partial V}{\partial z}\right)dz$$

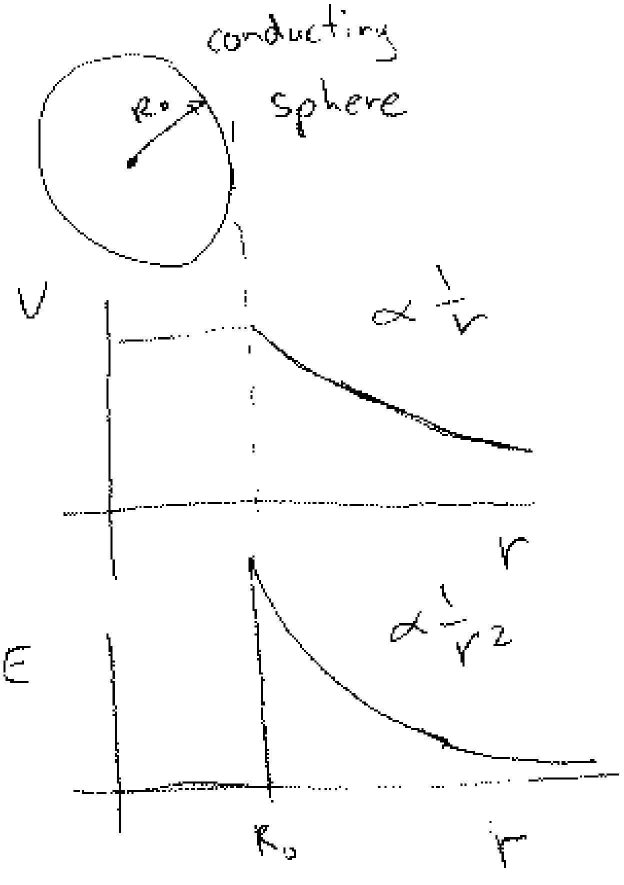
IF you know V, you can calculate E

<u>ex</u> We've said $E=0$ in conductor at equilibrium. Implies that V is constant

$$\frac{\partial V}{\partial x}=0 \text{ inside a conductor}$$

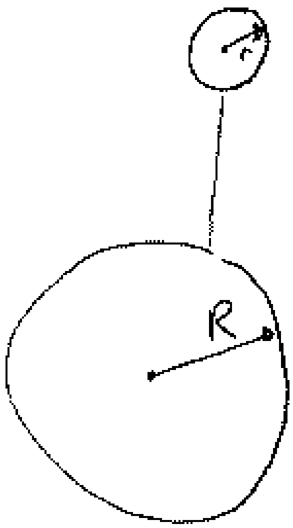
makes sense, recall Franklin found no force on the ball inside the conductor (no change in potential)

Also implies every point on the surface of a conductor is at the same potential as the inside. $\frac{\partial}{\partial x}(\text{constant})=0$



This can be very useful!

Suppose we have 2 spherical conductors connected by a thin wire



This means their surfaces are at the same potential.

So, if there's any Charge in there

$$\frac{Kq_R}{R} = \frac{Kq_r}{r}$$

what about the electric field

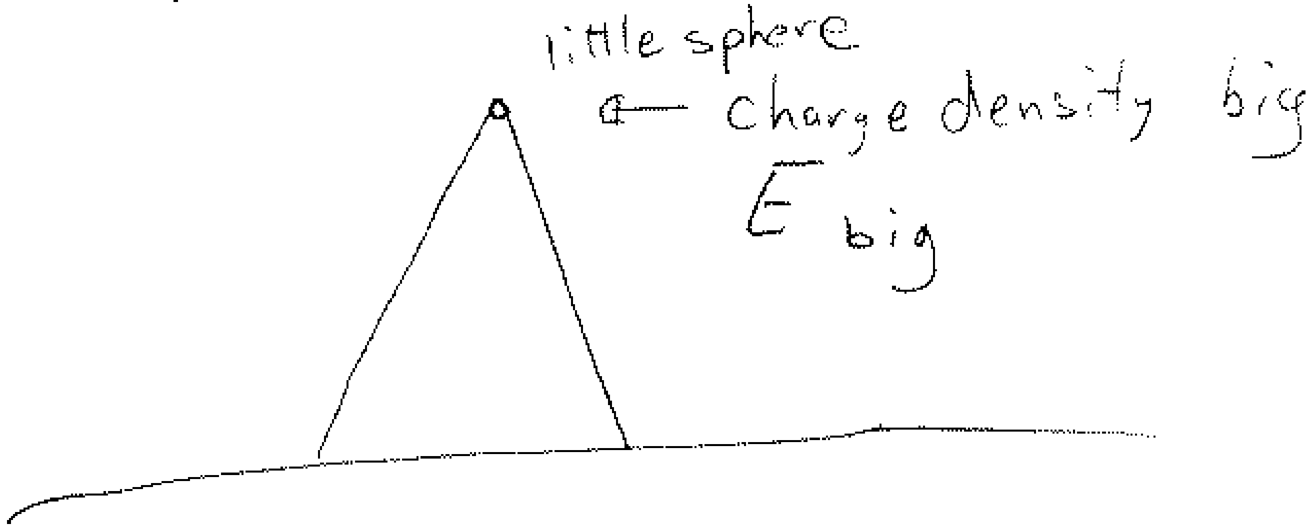
$$\frac{Kq_R}{R^2} = ?$$

lets look at the ration of electric Fields

$$\frac{E_r}{E_R} = \frac{\frac{Kq_r}{r^2}}{\frac{Kq_R}{R^2}} = \frac{\cancel{\frac{Kq_r}{r}}\,\frac{1}{r}}{\cancel{\frac{Kq_R}{R}}\,\frac{1}{R}} = \frac{\frac{1}{r}}{\frac{1}{R}} = \frac{R}{r}$$

Electric field is bigger on the small sphere = surface charge density bigger on small sphere

Application lightning rods



big sphere

Electric field gets so strong it starts to break down the air (like the little ball & the Van De Graff generator)