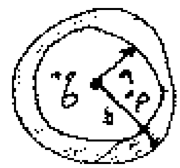
Lets finish our discussion from last time.

We found out $\Delta V = -\int_I^F \vec{E}\cdot d\vec{s}$ $\qquad \frac{\partial V}{\partial x} = -E_x$

among other things & were trying to calculate the electric potential for



$\vec{E} = \frac{kq}{r^2}\hat{r}$ which we determined using Gauss's Law

My Faviorite way to calculate the Electric Potential is to consider the work to move a positive charge from one point to another. Lets move a charge from a point inside the sphere to infinity

work from P to a + a to b + b to ∞

$$\cancel{q_0}\overset{\to 0}{V_F} - \cancel{q_0}V_I = \left[-(\cancel{q_0})\int_p^a \frac{kq}{r^2}dr\right] + \left[-\cancel{q_0}\int_a^b 0\,dr\right] + \left[-\cancel{q_0}\int_b^\infty \frac{kq}{r^2}dr\right]$$

$$= \frac{kq}{r}\Big|_p^a + 0 + \frac{kq}{r}\Big|_b^\infty = \left(\frac{kq}{a} - \frac{kq}{p}\right) + \left(\cancel{\frac{kq}{\infty}}^{\,0} - \frac{kq}{b}\right)$$

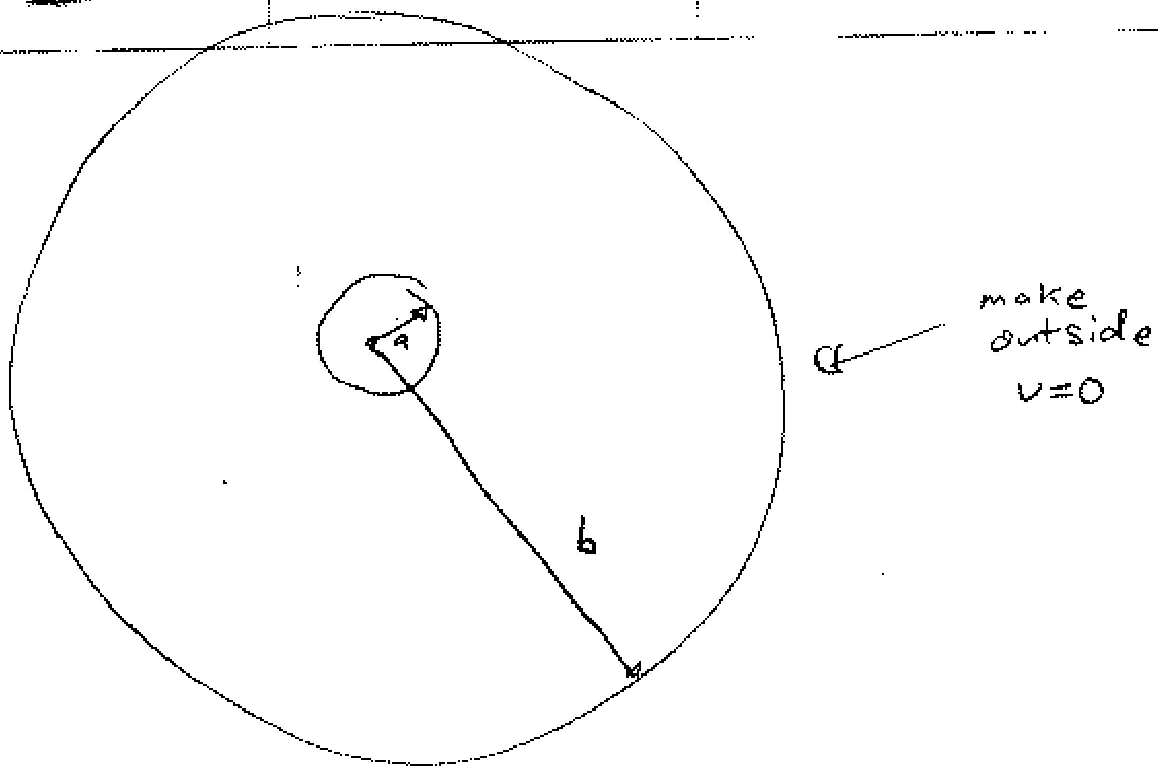
$$-V_I = \frac{kq}{a} - \frac{kq}{p} - \frac{kq}{b}$$

$$\text{or } V_{inside} = \frac{kq}{p} + \frac{kq}{b} - \frac{kq}{a} \qquad @\ p = a \quad V = \frac{kq}{b}$$

$(r<a)$

as we expected

Lets try another. At the back of your TV there is probably a cable. This cable has a characteristic property that we can calculate



$$E_{inside} \text{ from Gauss's law } = \frac{q}{2\pi\varepsilon_0 \ell}\,\frac{1}{r}$$

$$V_F - V_I = -\int_I^F \vec{E}\cdot d\vec{s}$$

$$\cancel{V_b}^{\,0} - V_a = -\int_a^b \frac{q}{(2\pi\varepsilon_0\ell)}\,\frac{dr}{r} = \frac{q}{2\pi\varepsilon_0\ell}\left(-\ln r\Big|_a^b\right)$$

$$V_a = \frac{q}{2\pi\varepsilon_0\ell}\left(\ln(b)-\ln(a)\right)$$

$$= \frac{q}{2\pi\varepsilon_0\ell}\ln\left(b/a\right)$$

again like the parallel plates we can relate V to q via a constant that depends on the physical characteristics of the conductor.

This constant is called Capacitance and (capacity for object to have q for V) we usually write $q = CV$

Bigger Capacitance $\equiv$ more charge for given voltage

units of Capacitance $\rightarrow$ Farads (C/V)

$\varepsilon_0 = 8.85\times10^{-12}$ f/m

(One reason it's so convenient)

so, for a length, l, of this coaxial cable

$$V\left(\underbrace{\frac{2\pi\varepsilon_0}{\ln(b/a)}\, l}_{\text{Capacitance}}\right) = q$$

& since this stuff usualy comes in bunches of 100's of ft

usually quote a C/l for this stuff $\frac{2\pi\varepsilon_0}{\ln(b/a)} = \frac{1}{2k}\frac{1}{\ln(b/a)}$

typically $b \sim 1/8$ in, $a \sim 1/64$ in $\quad C/l = \frac{1}{18\times10^9 N\frac{m^2}{C^2}}\frac{1}{\ln(8)} \approx 27\times10^{-12}\, F/m$

& as before units of capacitance are C/V or $C\Big/\frac{Nm}{C} = \left(\frac{C^2}{Nm}\right)$

called a farad

Recall for a parallel plate $V = q\,\frac{d}{\varepsilon_0 A}$

where $C = \frac{\varepsilon_0 A}{d}$ $\left\{\begin{array}{l}\text{important to keep in}\\ \text{mind, what is constant}\\ \text{Voltage} \leftrightarrow \text{Charge too}\end{array}\right\}$

This is useful for visualizing what happens when start building complicated circuits with capacitors

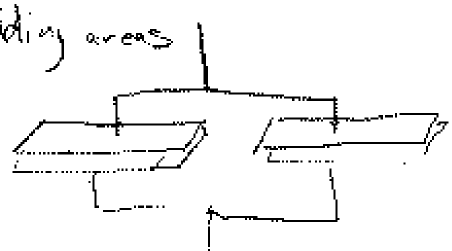
<u>consider</u> increase area $\longleftrightarrow$ increace capacitance

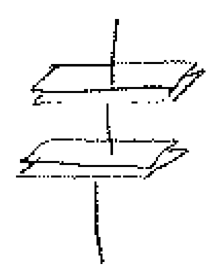
increase gap $\longleftrightarrow$ decrease capacitance

Adding areas $C_{tot} = C_1 + C_2 + C_3 + C_4 \cdots$ $\quad$ capacitors exactly the same $C_{tot} = \frac{\varepsilon_0}{d}(A_1 + A_2 + A_3 \cdots)$

gaps $\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots$ $\quad \frac{1}{C_{tot}} = \frac{1}{\varepsilon_0 A}(d_1 + d_2 + d_3)$

Adding areas

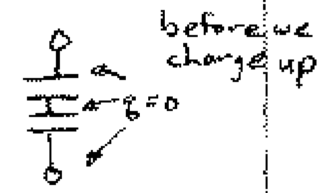




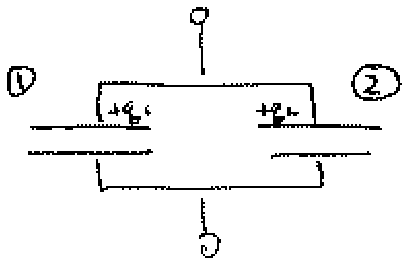
adding gaps

what if capacitors are different

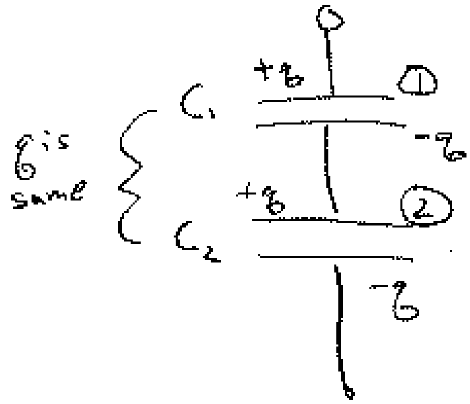
2 tricks Voltage & charge



Parallel



Series



Potential is the same across each

$q_{①} = V_{same} C_1$

$q_{②} = V_{same} C_2$

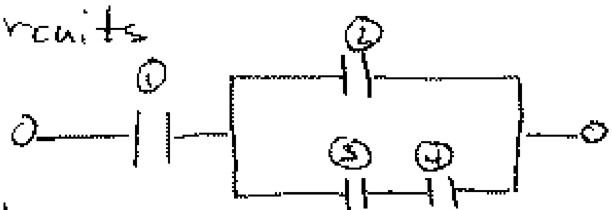
$q_{tot} = q_{①} + q_{②} = V_{same}(C_1 + C_2)$
$= V_{same}(C_{tot})$

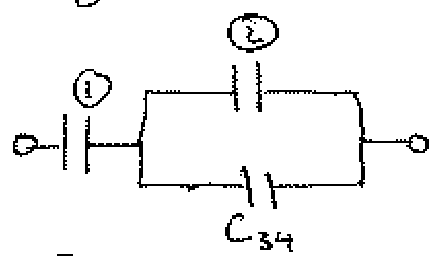
$V_{tot} = V_1 + V_2$
$= \frac{q}{C_1} + \frac{q}{C_2} = q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$
$= q\left(\frac{1}{C_{tot}}\right)$

V same

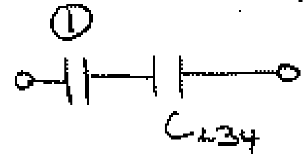
q same

and you can divide & conquer (many) complicated circuits



= 

$\frac{1}{C_{34}} = \frac{1}{C_{③}} + \frac{1}{C_{④}}$

= 

$C_{234} = C_2 + C_{34}$

$= C_{1234} = C_1 + C_{234}$

Capacitor demo

> charge up 1 capacitor
hook up to another
use conservation of charge to explain

> charge up 2 capacitors separately
show voltages add

> Charge up 2 together
show how each has voltage

> show there's energy stored in capacitor
(how your computer can hold info if no battery)

Questions ① how do we make capacitors
② how much energy is stored

Each bit of charge will take some energy
& alter voltage

$$dW = V\,dq = \frac{q}{C}\,dq$$

$$W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2$$

Typically capacitors are made with some insulating material put between plates

That stuff has more use than just to keep the plates separate

① Increase dielectric strength (Voltage before you get a spark)

② Increase capacitance
(modify's value of ε_0

Curious Consequences

In principle.

Capacitor in air, insert dielectric Q constant

$$U_{before} = \frac{1}{2}\frac{q^2}{C}$$

$$U_{after} = \frac{1}{2}\frac{q^2}{C'} \qquad C' > C$$

dielectric get's sucked in!

Capacitor hooked to battery V constant

$$U_{before} = \frac{1}{2}CV^2$$

$$U_{After} = \frac{1}{2}C'V^2$$

$$C' > C$$

have to put energy in to insert dielectric.