Last time talked about capacitance

Parallel plate

$$C = \frac{\epsilon_0 A}{d} \quad \text{units farads}$$

$$CV = Q$$

said you could increase C with a dielectric material

$$C \to C' \qquad K \geq 1 \text{ dielectric constant}$$

Parallel plate $C' = \frac{(K\epsilon_0) A}{d}$

Now, when you charge up a capacitor, you are storing energy. It is as if you are removing charge from one plate and putting it on the other when you charge up a capacitor

ΔU = energy it took to move $\Delta Q = \Delta Q V$

but as we put more and more charge on, V change

$$\Delta U = \Delta q \left(\frac{q_{tot}}{C} \right)$$

$$U_{tot} = \sum \Delta U = \sum \Delta q \frac{q_{tot}}{C}$$

$$= \int_0^{q_{final}} \frac{q}{C} dq = \frac{1}{2} q^2 / C$$

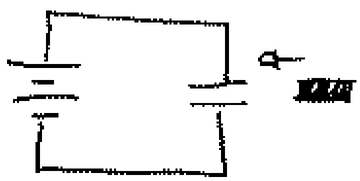
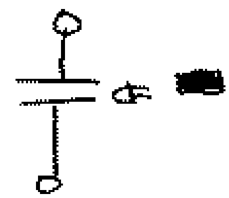
total energy stored in a capacitor

since $q = CV$ $\quad U_{tot} = \frac{1}{2} C V^2$ too.

now, a curious thing happens if you take a charged capacitor & insert a dielectric

2 ways

reocurring theme!

hooked to battery (V same)	charged up (Q same)
	
$U_{before} = \frac{1}{2} C V^2$	$U_{before} = \frac{1}{2} \frac{q^2}{C}$
$U_{after} = \frac{1}{2} C' V^2$	$U_{after} = \frac{1}{2} \frac{q^2}{C'}$
$U_{after} > U_{before}$	$U_{after} < U_{before}$
took work to move in dielectric	dielectric gets sucked in

tough to see

We also mentioned that the time it takes for a capacitor to charge up will depend on the conducting properties of our set up

A allows charge to move freely

lots of $\frac{\Delta Q}{\Delta t}$ flow in wire

B charge doesn't move so freely

small $\frac{\Delta Q}{\Delta t}$ in wire

what's happening?

Think about a wire, there are a certain number of charges that are free to move around

$$n = \frac{\#\ q\text{'s free to move}}{\text{volume}}$$

now, when we put an electric field across a wire, charges move, experience a force

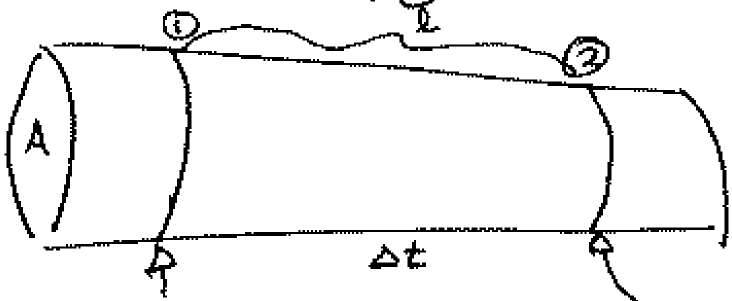
so

$$v_D = \underbrace{\frac{qE}{m}}_{\text{acceleration a charge feels}} \underbrace{\tau}$$

time on average between collisions of atoms

(property of the material)

here's our wire



in time Δt, charge from

moved to

so, a total volume of charge moved

from 1 to 2

the distance between 1 & 2 is

$v_0 \Delta t$

So the total amount of charge that went from 1 to 2 is

$$\Delta q = (q n)(v_0 \Delta t) A$$

$$\cancel{\Delta q = q n v_0}$$

so $\frac{\Delta q}{\Delta t} = (q n v_0 A)$

we moved charges in an electric field so there is a potential difference beteen 1 & 2

$$\frac{\Delta q}{\Delta t} = (qnA)\left(\frac{qE}{m}\tau\right) = \left(\frac{q^2 n\tau A}{m}\right)E$$

$$\frac{\Delta q}{\Delta t}\left(\frac{m}{q^2 n\tau A}\right) = E$$

$$V_2 - V_1 = \Delta V = -\int_{1}^{2} \vec{E}\cdot d\vec{s} = -E\ell$$ (so a potential drop)

$$\text{or } |\Delta V| = \frac{\Delta q}{\Delta t}\underbrace{\left(\frac{m}{q^2 n\tau}\right)}_{\substack{\text{property of material} \\ \text{called resistivity}}}\frac{\ell}{A}$$

(For materials where this doesn't change as we change E we can make a linear relationship between

$\underbrace{\frac{\Delta q}{\Delta t}}_{\text{current } I}$ and ΔV

$$I\underbrace{\left(\rho\frac{\ell}{A}\right)}_{\text{called resistance}} = \Delta V$$

has units of Ohms or $V/(C/s)$

$$IR = \Delta V$$

Lets take a look at a wire

a good conductor typically has a resistivity of $2\times10^{-8}\ \Omega\,m$ copper is $1.7\times10^{-8}\ \Omega m$

A 1m long piece of ~~square~~ copper wire has a side length of .1mm, how much current flows when we attach it to a 1.5v battery

$$\Delta V = IR$$

$$\frac{\Delta V}{R} = I$$

need to calculate R

$$R = \frac{(1.7 \times 10^{-8}\,\Omega m)(1m)}{(0.0001m)^2}$$

$$= 1.7\,\Omega$$

$$I = \frac{1.5V}{1.7\,\Omega} \approx 1A \text{ or so}$$

I we cut the wire in half and try again

$$R = 3.4\,\Omega$$

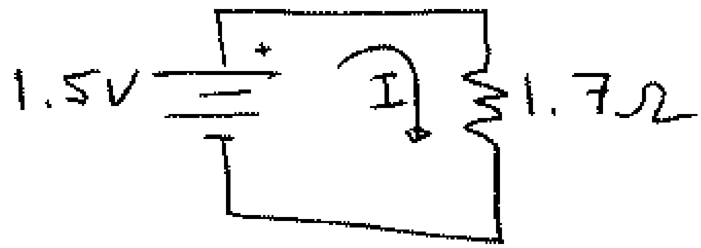
$$I \sim \frac{1}{2}A$$

For convenience, we usually indicate a resistor with the symbol 

a battery with 

higher potential

and try to solve problems symbollicaly



You are right, as the current flows and potential drops we generate heat

$$\Delta \text{Energy} = \Delta Q \, V$$

$$\frac{\Delta \text{Energy}}{\Delta \text{time}} = \text{Power} = \frac{\Delta Q}{\Delta t} V = VI$$
$$= I^2 R$$
$$= \frac{V^2}{R}$$

Can imagine that if R is small enough in our example, it will get so hot that the metal will melt.

demo Fuses

couple of wierd things

what is v_0?

we said $I = (nq) v_0 A$

$$v_0 = \frac{I}{nqA}$$

copper ~~63.5 g/mol · 6.02×10²³ atoms/mol~~

$$\left(6.02\times10^{23}\frac{\text{atoms}}{\text{mol}}\right)\left(8.95\frac{\text{g}}{(\text{cm})^3}\right)\left(\frac{1\text{ mol}}{63.5\text{ g}}\right)$$

$$n = 8.5\times10^{22}\ \text{atoms}/(\text{cm})^3$$
$$= 8.5\times10^{28}\ \text{atoms}/\text{m}^3$$

$q = 1.6\times10^{-19}\,\text{C}$ $A = (.0001\,\text{m})^2$

$$v_0 = \frac{1\text{A}}{\left(8.5\times10^{28}\frac{1}{\text{m}^3}\right)(.0001\text{m})^2(1.6\times10^{-19}\text{C})}$$

$$= \frac{1}{(136)}\ \text{m/s}$$

small? how does this work?

Fuses burn out at a given current

if we assume it takes the same power

to melt a metal

$$P_{30A} = P_{20A}$$

$$(30A)^2 \cancel{\rho} \frac{l_{30}}{A_{30}} = (20A)^2 \cancel{\rho} \frac{l_{20}}{A_{20}}$$

if $\rho = \rho$ & $l_{30} = l_{20}$

$$A_{20} = A_{30} \left(\frac{20}{30}\right)^2$$

so one dimension of the area will be

about 40% as big

20A fuse is thinner

look at fuse, pretty good