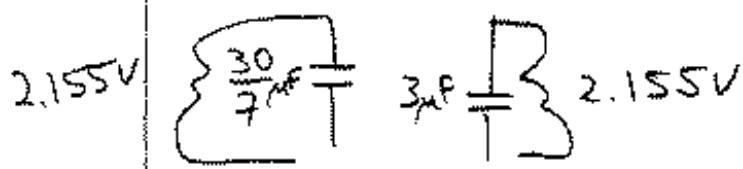


(10)

now, we examine the  $\frac{30}{7}\mu F$  combo in parallel with the  $3\mu F$  Combo. The voltage across each is  $2.155V$ .



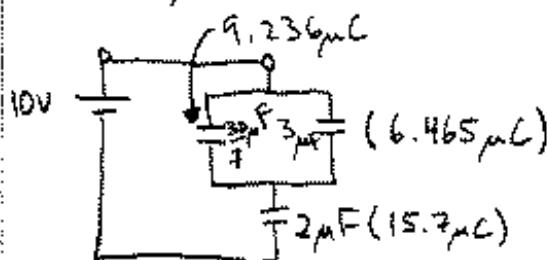
$$Q = CV$$

$$Q_{3\mu F} = (3\mu F)(2.155V) = 6.465\mu C$$

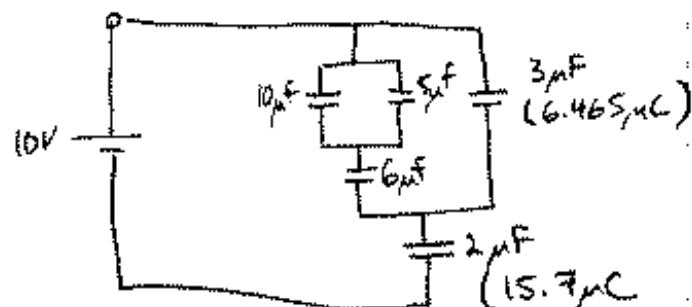
$$Q_{\frac{30}{7}\mu F} = \left(\frac{30}{7}\mu F\right)(2.155V) = 9.236\mu C$$

check  $Q_{tot} = 6.465\mu C + 9.236\mu C = 15.7\mu C$   
great!

so, here's what we have so far



where we want to



notice that  $Q$  on the  $\frac{30}{7}\mu F$  combo is the same as the charge on the  $6\mu F$  cap

Recall

$$\frac{1}{\frac{30}{7}\mu F} = \frac{1}{6\mu F} = \frac{1}{10\mu F} = \frac{5\mu F}{6\mu F}$$

$$\left. \right\} 2.155V$$

so  $q$  on  $6\mu F \Rightarrow 9.236\mu C$

And we can use the same trick as before

$$\begin{aligned}
 2.155V &= \text{voltage across } 6\mu F + \\
 &\quad \text{voltage across } 15\mu F \text{ combo} \\
 &= \frac{9.236\mu C}{6\mu F} + \frac{9.236\mu C}{15\mu F} \\
 &= 1.54V + 0.6157V = 2.156V \\
 &\quad \text{close}
 \end{aligned}$$

so, the voltage across the  $15\mu F$  combo is

$$0.6157V$$



Final

$$\begin{aligned}
 \text{so } Q_{5\mu F} &= 5\mu F(0.6157V) \\
 &= 3.079\mu C
 \end{aligned}$$

$$\begin{aligned}
 Q_{10\mu F} &= 10\mu F(0.6157V) \\
 &= 6.157\mu C
 \end{aligned}$$

$$\text{check } Q_{5\mu F} + Q_{10\mu F} = 9.236\mu C$$

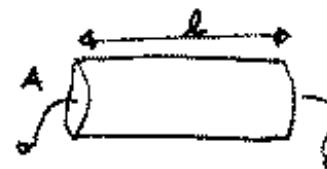
$$\begin{aligned}
 \frac{1}{2} (3.079\mu C + 6.157\mu C) &= 9.236\mu C \\
 &\text{cool!}
 \end{aligned}$$

now, as a check, the potential difference across the  $10\mu F + 6\mu F + 3\mu F = 10V$

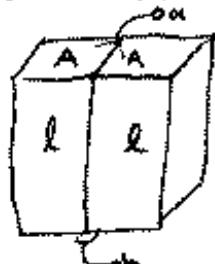
$$0.6157V + 1.54V + 7.85V = 10.005V \text{ good}$$

We've been talking about resistance.

Recall that

$$R_{ab} = \rho \frac{l}{A}$$


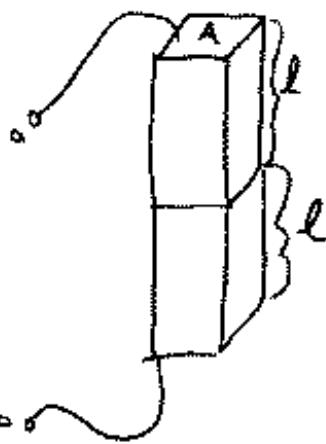
Now, like capacitors, we will have to deal with combinations of resistors. Again, consider 2 identical resistors



side by side we get 2x Area

$$\overline{R}_{\text{new}} = \frac{l}{\rho} (A + A) = \frac{2A}{\rho}$$

$$\overline{R}_{\text{new}} = \frac{\rho l}{2A} \quad \frac{1}{2} R \text{ of 1}$$



like

$$\overline{R}_{\text{new}} = \frac{\rho}{A} (l + l) = 2 R_{\text{one}}$$

Backwards from capacitors!

Resistor

$$\text{eqn} \quad \frac{\rho l}{A}$$

Capacitor

$$\frac{\epsilon_0 A}{d}$$

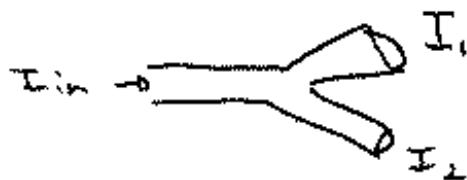
add areas	parallel	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$	$C = C_1 + C_2 + C_3$
add gaps	series	$R = R_1 + R_2 + R_3$	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

So how do we motivate this. Very similar to capacitors. We'll use voltage and charge. This time though charge conservation becomes current conservation.

If we have current flow in a conductor

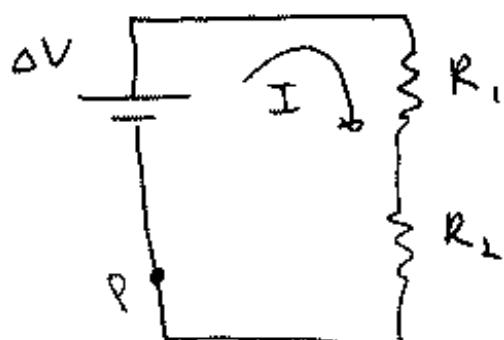
$$I_{in} = I_{out}$$

& the current is constant  
(likewise evaluate charge  
at equilibrium)



$$I_{in} = I_1 + I_2$$

so, in the series combination



$I$  through  $R_1$  is  
the same as  $I$   
through  $R_2$

now, if I pick a point on the circuit and follow it in the direction I drew the current, I know: { $\Delta V$  is Potential across  $R_1$  &  $R_2$ }

Charges get a boost in potential as they pass through the battery

Recall, those resistors get hot as current passes through.

(3)

So, if we call the point where we started  $\Delta V$

get  $+\Delta V_{\text{batt}}$  through the battery

lose  $\Delta V_1$  as we pass through  $R_1$

$\Delta V_2$  as we pass through  $R_2$

$$\Delta V = \Delta V_{\text{batt}} - \Delta V_1 - \Delta V_2$$

$$= \Delta V_{\text{batt}} - IR_1 - IR_2$$

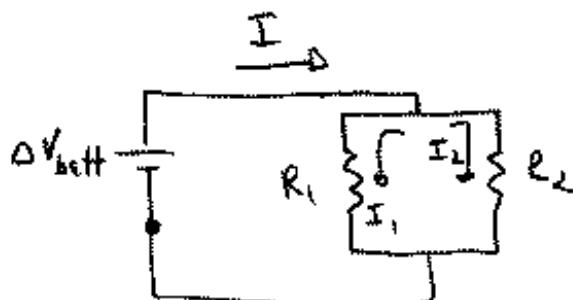
$$IR_1 + IR_2 = \Delta V_{\text{batt}}$$

Since same  
I passes through  
everything

$$\underbrace{I(R_1 + R_2)}_{\text{Equivalent}} = \Delta V_{\text{batt}}$$

Equivalent

Lets do the same with parallel



$$\Delta V_{\text{batt}} - I_1 R_1 = 0$$

$$\frac{\Delta V_{\text{batt}}}{R_1} = I_1$$

$$\Delta V_{\text{batt}} - I_2 R_2 = 0$$

$$\frac{\Delta V_{\text{batt}}}{R_2} = I_2$$

$$I_1 R_1 = I_2 R_2$$

$$I = I_1 + I_2$$

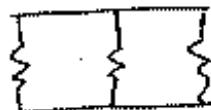
$$= \frac{\Delta V_{\text{batt}}}{R_1} + \frac{\Delta V_{\text{batt}}}{R_2}$$

$$= \Delta V_{\text{batt}} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$= \Delta V_{\text{batt}} \left( \frac{1}{R_{\text{equiv}}} \right)$$

(4)

So, how do we deal with a complicated branch of Resistors? Look at a small piece & is current the same through these? Is voltage across the same



voltage across  
is same

or



Current through  
is same

You can replace these with equivalent resistances

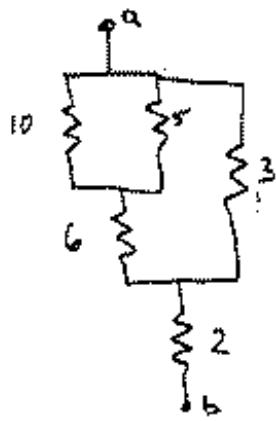
If you see



for instance,  
you need  
a higher  
technique

lets go through an example of equivalent resistance that can be done with our replacements

(5)



10 &amp; 5 in parallel

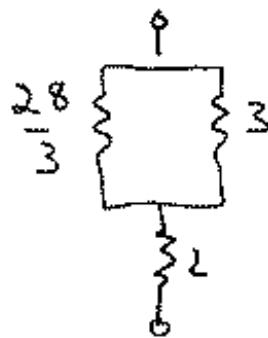
$$\frac{1}{R} = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$

$$R = 10/3 \quad (3.33\Omega)$$



10/3 &amp; 6 in series

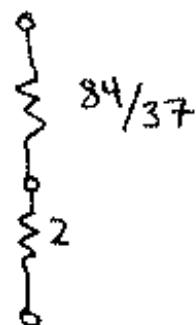
$$R = \frac{10}{3} + 6 = \frac{28}{3} \quad (9.33\Omega)$$



28/3 &amp; 3 in parallel

$$\frac{1}{R} = \frac{1}{3} + \frac{3}{28} = \frac{28}{84} + \frac{9}{84} = \frac{37}{84}$$

$$R = \frac{84}{37} \quad (2.27\Omega)$$



$$R_{\text{series}} = 2 + \frac{84}{37} = \frac{74}{37} + \frac{84}{37}$$

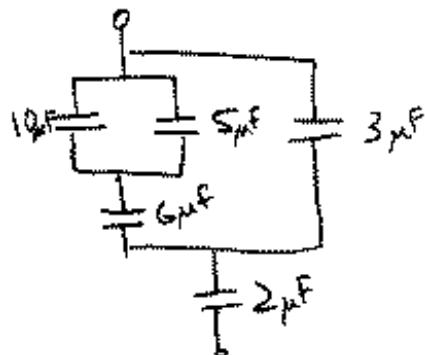
$$= \frac{158}{37}$$

$$\approx 4.27\Omega$$

(6)

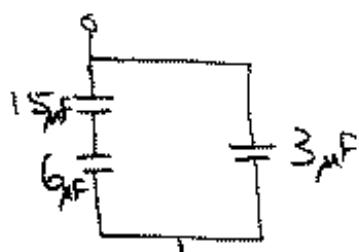
for capacitors you can ask is q same on each  
V across same

lets do the same one  
with caps! (all in  $\mu\text{F}$ )



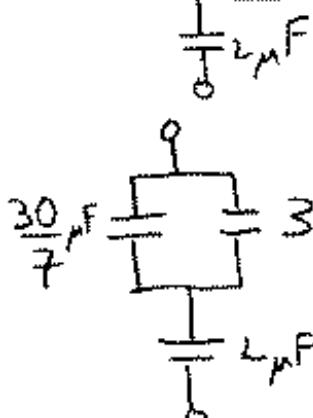
$10 \frac{1}{4} 5$  in parallel

$$C = 10\mu\text{F} + 5\mu\text{F} = 15\mu\text{F}$$



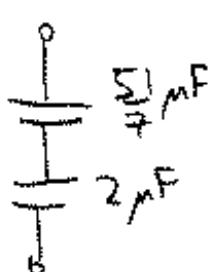
$15 \frac{1}{4} 6$  in series

$$\frac{1}{C} = \frac{1}{15\mu\text{F}} + \frac{1}{6\mu\text{F}} = \frac{2}{30} + \frac{5}{30} = \frac{7}{30}$$



$\frac{30}{7} + 3$  in parallel

$$C = \frac{30}{7} + 3 = \frac{51}{7}\mu\text{F}$$



$\frac{51}{7} \frac{1}{2}$  in series

$$\frac{1}{C} = \frac{7}{51\mu\text{F}} + \frac{1}{2\mu\text{F}} = \frac{14}{102} + \frac{51}{102} = \frac{65}{102\mu\text{F}}$$

$$C = \frac{102}{65}\mu\text{F}$$

different

$$= 1.57\mu\text{F}$$

Smallest resistor dominates in parallel

Smallest capacitor dominates in series

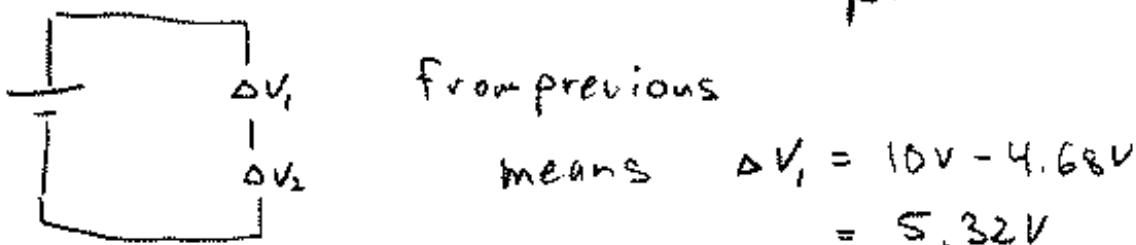
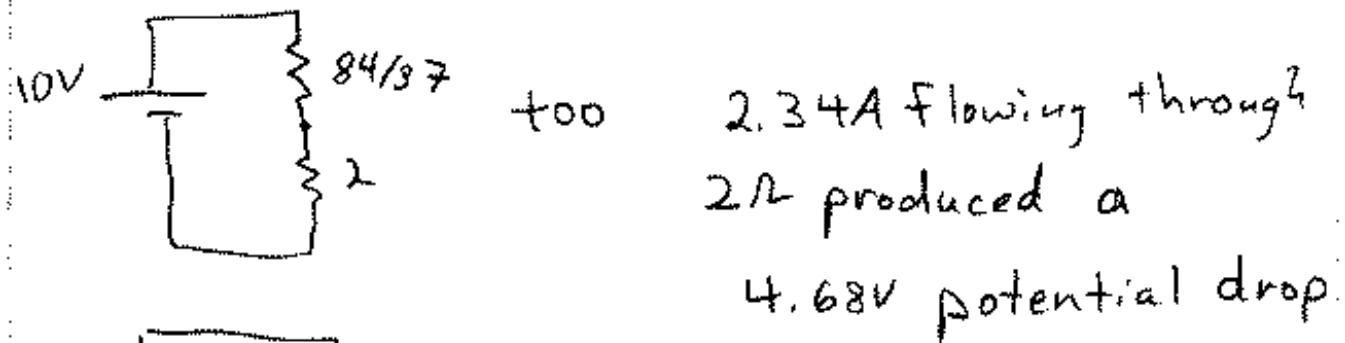
Now, let's put a voltage across our combination and try to determine the current in each element

As in your homework, we try to work backwards

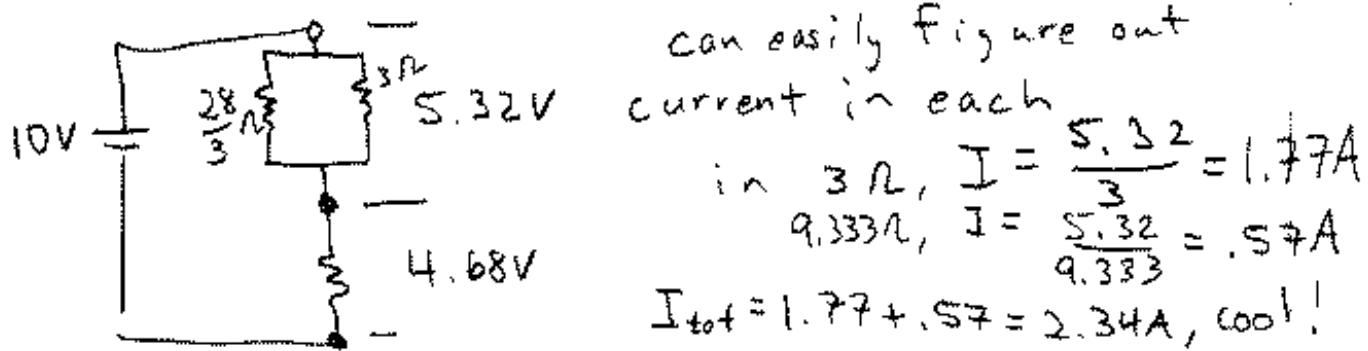
Our equivalent resistor looks like

$$10V \quad \boxed{I} \quad 4.27\Omega \quad I = \frac{\Delta V}{R} = \frac{10V}{4.27\Omega} = 2.34A$$

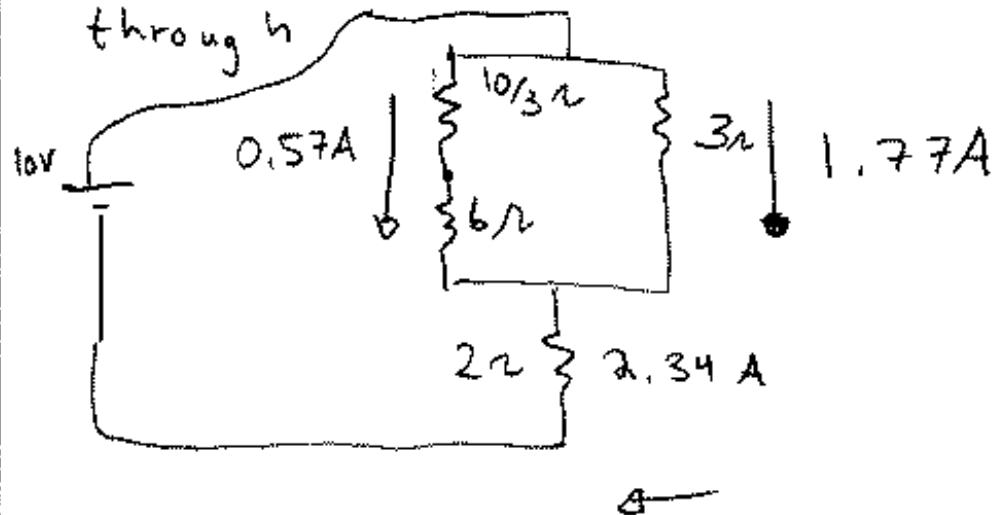
this means we have 2.34A flowing through



so we really simplify the problem



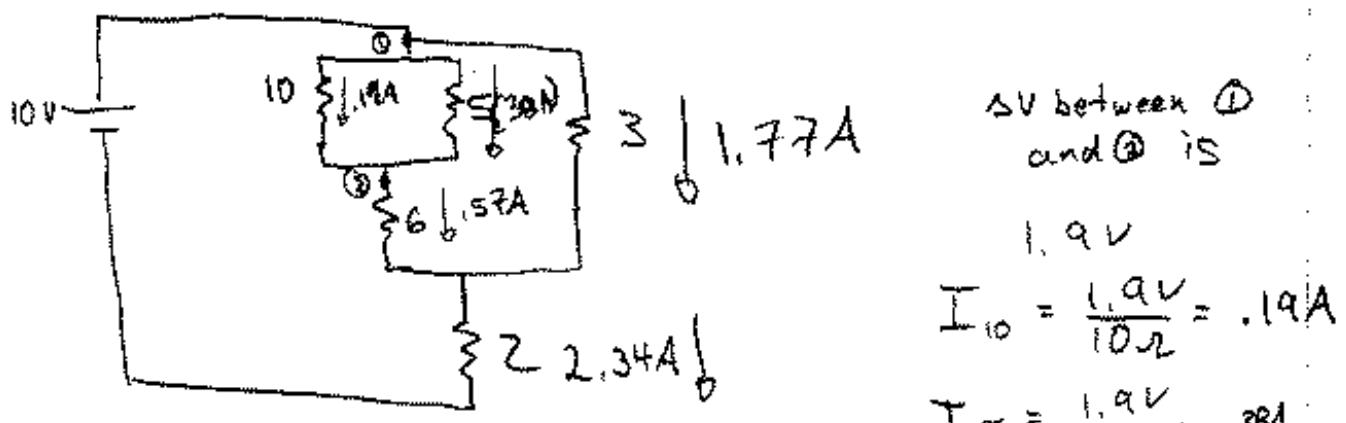
So, we now know that 0.57A flows through h



So the voltage drop across the  $\frac{10}{3}\Omega$  is  $\frac{10}{3} \cdot .57 = 1.9V$

check Voltage across  $6\Omega$  is  $6\Omega(.57A) = 3.42V$

$2.34A$   $\Delta V_{tot}$  across  $\frac{10}{3} \& 6\Omega = 5.32V$  good!



hottest!  
resistor

$$I_{tot} = 0.19A + 0.38A + 1.77A$$

$$= 2.34A \text{ all checks}$$

$\Delta V$  between ① and ② is

$$1.9V$$

$$I_{10} = \frac{1.9V}{10\Omega} = .19A$$

$$I_5 = \frac{1.9V}{5\Omega} = .38A$$

(a)

How about the capacitor system if we put a voltage on it

- look for series  $\leftrightarrow$  same charge

- look for parallel  $\leftrightarrow$  same voltage

Our equivalent circuit

$$10V \quad \boxed{\text{---}} \quad \frac{1}{\text{T}} \quad \frac{1}{\text{T}} \quad 1.57\mu F \quad Q = CV \\ = 1.57\mu F (10V) = 15.7\mu C$$

$$\frac{1}{\text{T}} \quad 1.57\mu F \Rightarrow \frac{1}{\text{T}} \quad \frac{51}{7}\mu F \quad \left. \begin{array}{l} \text{charge} \\ \text{is} \\ \text{same as } Q \text{ above} \end{array} \right\} \quad \text{so } q \text{ on } 2\mu F \text{ is} \\ \frac{1}{\text{T}} \quad 2\mu F \quad (15.7\mu C)$$

We should be able to check this, add up the potential differences on each, we get 10V

Should

$$10V = \frac{15.7\mu C}{(51/7)\mu F} + \frac{15.7\mu C}{2\mu F}$$

$$= 2.155V + 7.85V = 10.0005V$$

pretty close!

So, we know the potential difference across the  $\frac{51}{7}\mu F$  combination is 2.155V.

