Lecture 7

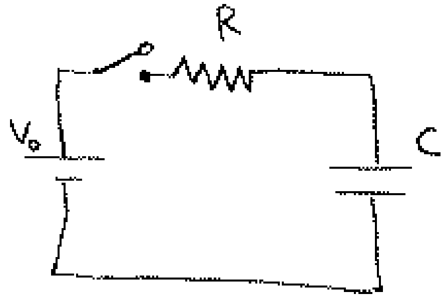
Capacitors in a real circuit

Up till now we've considered 2 extreme cases

- Electrostatic Equilibrium (charges have stopped moving)
- Steady State Current I is constant

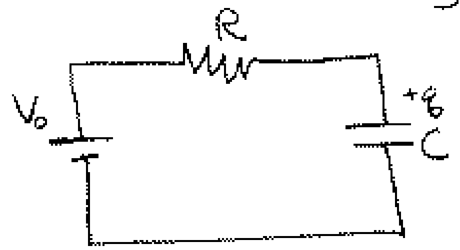
But what happens in between?

Before



Initially, the switch is open, no charge is on the capacitor

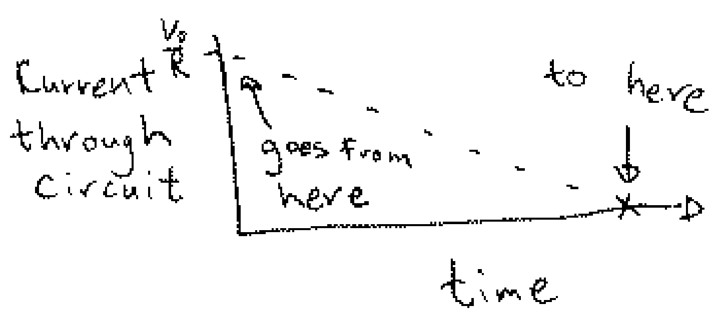
After a long time



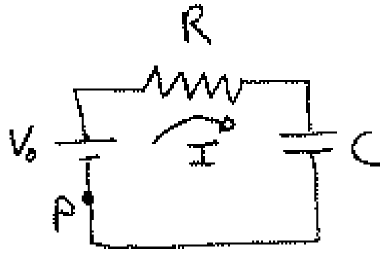
Charge has flowed and stopped, $I=0$. This means that the potential difference across C is the same as the battery

As soon as we close the switch, charge will flow, $I \neq 0$, in fact since $q(\text{on } C) = 0$, $I = V_0/R$

So, whatevery we do, our mathematical solution should represent the extreme points



Lets take a point on the circuit and look at the Potential.



gain potential V_0

lose IR through resistor

lose q_{cap} on Capacitor

$$V_0 = IR + \frac{q_{cap}}{C}$$

now, q_{cap} goes from 0 to CV_0

so $\frac{dq_{cap}}{dt}$ is positive and we can define

current $I = \frac{dq_{cap}}{dt}$

or
$$V_0 = \frac{dq_{cap}}{dt}R + \frac{q_{cap}}{C}$$

$$\frac{V_0}{R} = \frac{dq_{cap}}{dt} + \frac{q_{cap}}{RC}$$

trick 1 Rearrange (Charge on one side time on the other)

$$\frac{V_0}{R} - \frac{q_{cap}}{RC} = \frac{dq_{cap}}{dt}$$

$$dt = \frac{dq_{cap}}{\frac{V_0}{R} - \frac{q_{cap}}{RC}}$$

$$\int -dt = RC \int \frac{dq_{cap}}{q_{cap} - CV_0}$$

$$-t = RC \ln(q_{cap} - CV_0) + \text{const}$$

trick 2 $$\frac{-t}{RC} = \ln(q_{cap} - CV_0) - \ln(k)$$

some constant

$$-\frac{t}{RC} + \ln(k) = \ln(q_{cap} - CV_0)$$

$e \qquad e$

$$e^{-\frac{t}{RC}} e^{\ln k} = k e^{-t/RC} = q_{cap} - CV$$

now, let's take a look at our extreme cases to figure out the constant

@ $t = 0$ $q_{cap} = 0$

$$k e^{0} = -CV_0$$

$$k = -CV_0$$

$$-CV_0 e^{-t/RC} = q_{cap} - CV_0$$

$$CV_0 - CV_0 e^{-\frac{t}{RC}} = q_{cap}$$

$$CV_0(1 - e^{-t/RC}) = q_{cap}$$

at $t = \infty$

$$e^{-t/RC} = e^{-\infty} = 0$$

$$CV_0 = q_{cap}$$

look at current

$$I = \frac{dq_{cap}}{dt} = \frac{d}{dt}(CV_0)(1 - e^{-t/RC})$$

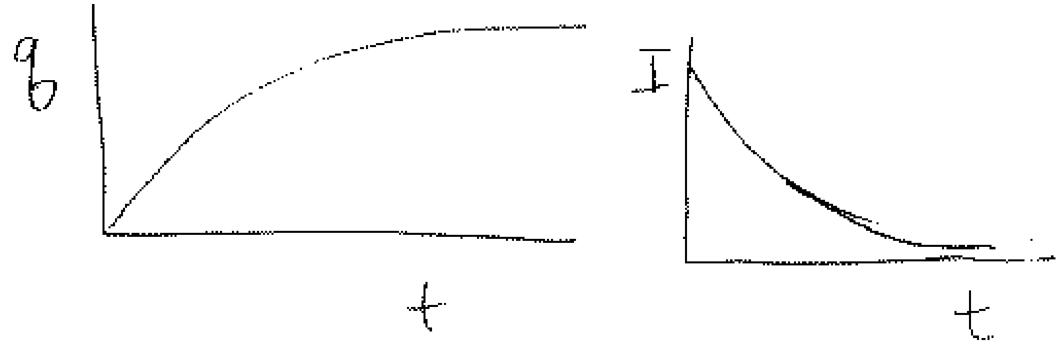
$$= -CV_0 \frac{d}{dt} e^{-t/RC} = -CV_0 \left(-\frac{1}{RC}\right) e^{-t/RC}$$

$$= \frac{V}{R} e^{-t/RC}$$

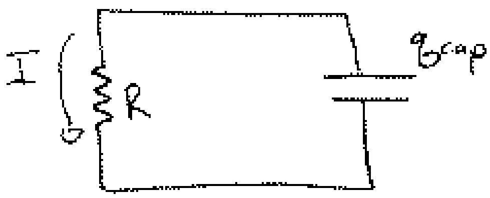
$@ t = 0 \quad I = \frac{V}{R}$

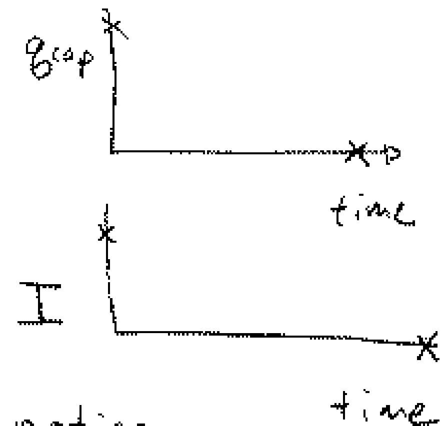
$@ t = \infty \quad I = 0$

as you have seen in the lab



Now you should see how the discharging capacitor works





notice

$$V_{\substack{across\\cap}} = V_{across_R}$$

$$\frac{q_{cap}}{C} = IR$$

but $I = \frac{-dq_{cap}}{dt}$ (since Δq_{cap} is negative)

$$\frac{q_{cap}}{C} = -\frac{dq_{cap}}{dt}R$$

$$\frac{-dt}{RC} = \frac{dq_{cap}}{q_{cap}}$$

$$-\frac{t}{RC} = \ln(q_{cap}) - \ln(k)$$

$$ke^{-t/RC} = q_{cap}$$

$$k = Q_0$$

$$q_{cap} = Q_0 e^{-t/RC}$$

energy consumed by resistor

$$P = I^2 R$$

$$= \left(\frac{Q_0^2}{(RC)^2} e^{-2t/RC}\right) R$$

$$U = \int_0^\infty P\,dt$$

$$= \frac{Q_0^2}{(RC)^2} R \int_0^\infty e^{-2t/RC}\,dt$$

$$= \frac{Q_0^2}{(RC)^2} R \left[-\frac{RC}{2} e^{-2t/RC}\Big|_0^\infty\right]$$

$$= \frac{R^2 C}{2}\frac{Q_0^2}{(RC)^2}\left[-(0-1)\right]$$

$$= Q^2/2C\ !$$

We live in a time of exponential growth

It is only recently that population growth has stopped its exponential march.

We live in a time of exponential growth in the computing industry.

Compute power (# transistors on a chip) has tended to double every 18 mo's, since about 1960 (seen some papers that say it has been exponential for even longer than that) This is called Moores Law

If this continues, how powerful will computers be in 20 years?

$$CP(t) = CP_0 e^{bt}$$

know $2 = e^{b\,1.5yr}$

$$\ln(2) = b\,1.5 \,, \quad b = 0.462$$

in 20 years

$$\frac{CP(20)}{CP_{now}} = e^{(0.462)20} = 10{,}321 \text{ times}$$

Seems incredible, probably we'll have computers in and on us!

Will we need to supply power for all that, can we? We're already having problems.

I read a while ago that at present consumption, the oil will last 45 years.

I also read that world oil consumption is increasing 2% / year

so $$\frac{\Delta \text{Consumption}}{\Delta \text{time}} = \left(\text{Consumption at the time}\right)\frac{.02}{\text{year}}$$

$$\frac{d\,\text{Con}}{dt} = \text{Con}\,(.02/\text{yr})$$

$$\frac{d\,\text{Con}}{\text{Con}} = dt\left(\frac{.02}{\text{yr}}\right)$$ Very similar (exponential again)

$$\text{Consumption}(t) = \text{Consumption}_0\, e^{t\left(\frac{.02}{\text{yr}}\right)}$$

$$\text{total oil consumed} = \int_0^{t_0} \text{Consumption}_0\, e^{t\left(\frac{.02}{\text{yr}}\right)}dt = \left[\frac{\text{Consumption}_0\,\text{yr}}{.02}\right]\left(e^{.02t_0/\text{yr}} - 1\right)$$

So what is t_0? If 45 years = 45 big barrels of oil

$$45\,bb = \left[\frac{\left(\frac{1\,bb}{yr}\right) yr}{.02}\right] e^{.02 t_0/yr} - 1$$

$$.9 = e^{.02 t_0/yr} - 1$$

$$1.9 = e^{.02 t_0/yr}$$

$$\ln(1.9) = .02 t_0/yr$$

$$t_0 = 32.1 \text{ years}$$

bit less than 45 years!

(better start conserving)

if we conserve 2% / year

$$.9 = \left(1 - e^{-0.02 t_0/yr}\right)$$

$$.1 = e^{-0.02 t_0/yr}$$

$$t_0 = 115 \text{ years}$$

.... what about shale ...