

Magnets

①

You've probably played with them, or seen them in cartoons. What do we know from playing?



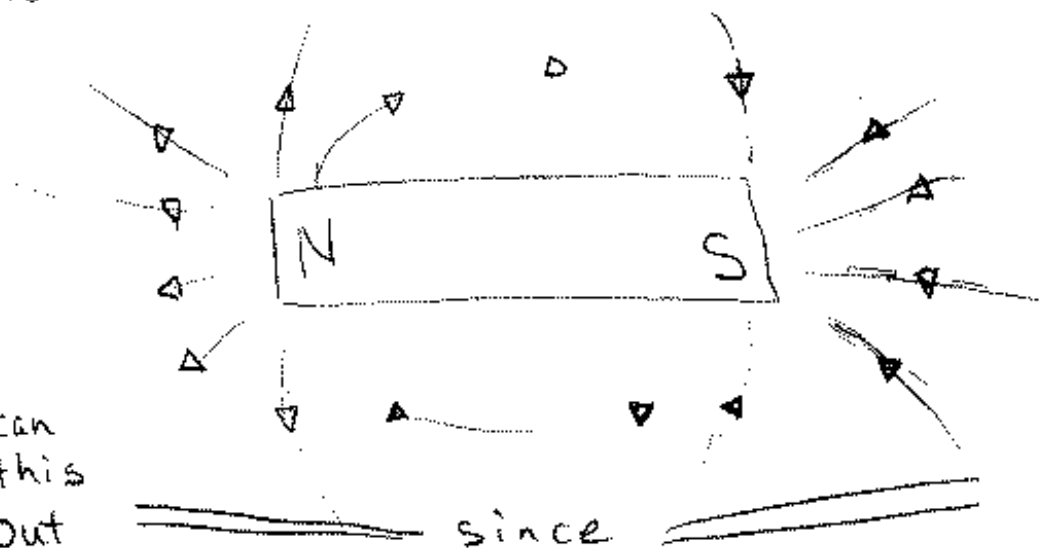
magnets have 2 poles a north pole
& a south pole

like poles repel
opposites attract

Even "Flat" magnets behave this way.

One way you can stack them and they stick
one way they repel each other.

In fact, every magnet has a north and a south pole. The magnetic field looks a lot like that for 2 charges



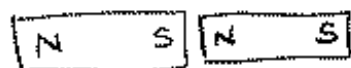
outside
the magnet,
lines look
like
this

a compass
points along
the lines

You can
map this
out

since

Break a magnet, you get 2 magnets, an isolated north or south pole has never been seen.



North pole attracted to south pole.

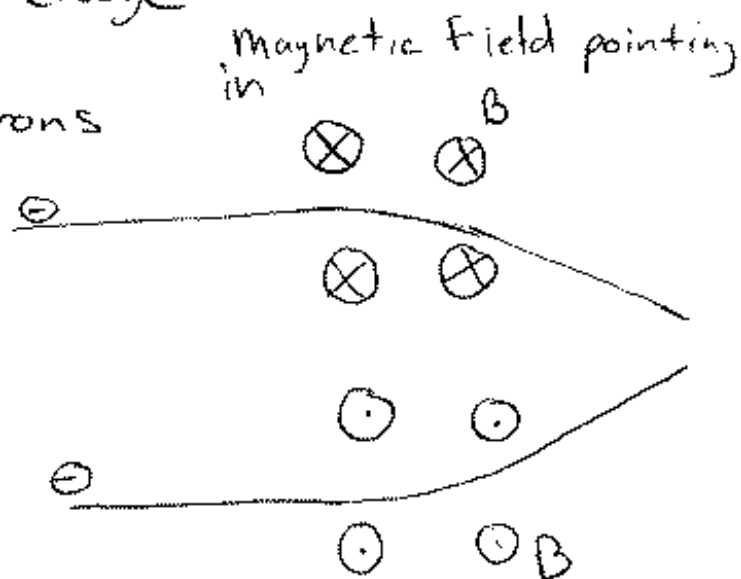
North pole of a compass is attracted to earth's south magnetic pole (which is what we call north!) Gets a torque, but how?

Lets investigate more.

Magnets effect other magnets, a magnetic field has an effect on a moving charge

demo beam of electrons

Notice



Force is $\propto$ velocity,

Force is $\perp$ to the magnetic field. B is defined

So that

$$\vec{F} = q \vec{v} \times \vec{B}$$

B has units of $\frac{N}{C} \frac{1}{(m/s)}$

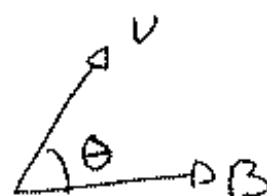
or Tesla

$$1T = 1Ns/cm$$

Mathematically

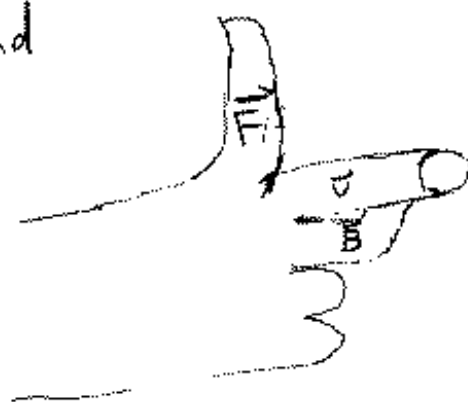
$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$



$$\begin{aligned} \vec{F} &= q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i} (v_y B_z - B_y v_z) \\ &\quad + \hat{j} (v_z B_x - B_z v_x) \\ &\quad + \hat{k} (v_x B_y - B_x v_y) \\ &= |\vec{v}| |\vec{B}| \sin \theta \end{aligned}$$

and you can find the direction with your right hand



F is always $\perp$ to v
 $\perp B$

you can think of the v & B vectors defining the xy plane, then F is always pointing in the $(\pm z)$ direction

Couple of cool things

- 1) can get particles to go in a circle
- 2) can get particles to go in a helix

ex what B is required to keep an electron, moving 10^6 m/s in a 10cm circle?

Force needed to keep electron moving in a circle

$$\begin{aligned} \frac{mv^2}{r} &= qvB \\ \frac{mv}{qr} &= B = \frac{9.1 \times 10^{-31} \text{ kg} (10^6)^2}{1.6 \times 10^{-19} \text{ C} (0.1 \text{ m})} = 5.7 \times 10^{-10} \text{ T} \\ &\text{weak} \end{aligned}$$

(important for TV's?)

By comparison, huge particle accelerators use many small magnets operating at several tesla to keep protons moving in a circle. (4)

#7 $B = \frac{4.8 \times 10^{-16} \text{ kg m/s}}{(1.6 \times 10^{-19} \text{ C})(1000 \text{ m})} = 3 \text{ T}$ ✓ very close to c.

Earth's magnetic field is $\frac{1}{20,000} \text{ T}$ or so.

now, since moving charges are effected by magnetic fields, we expect that current is too.

$$T = \frac{N}{C \cdot m/s} = \frac{N}{(\frac{C}{s})m} = \frac{N}{A \cdot m}$$

or

$$N = A \cdot m \cdot T$$

{ maybe we can think of bunches of charges moving around in material as little loops of current }

usually, as we did for a little piece of charged wire, we'll consider a little piece of current

$$d\vec{F} = I d\vec{\ell} \times \vec{B}$$

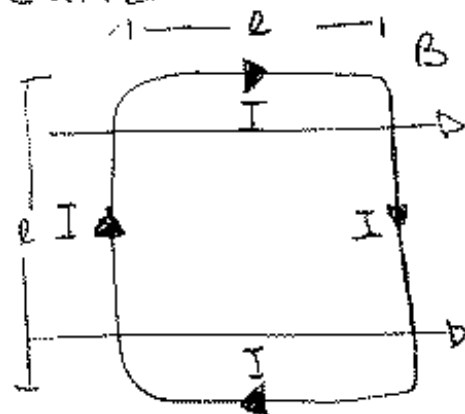
(direction of current)

demo wire jumps out of a magnet.

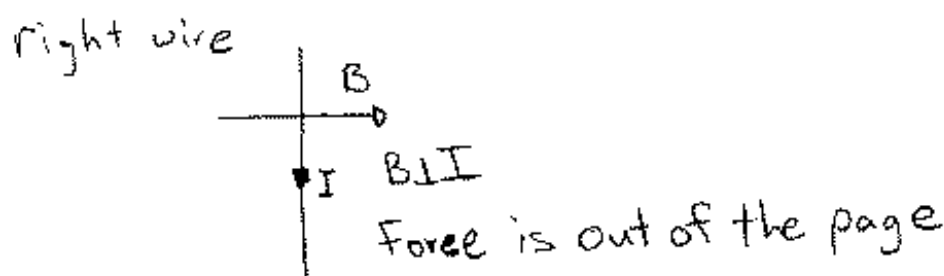
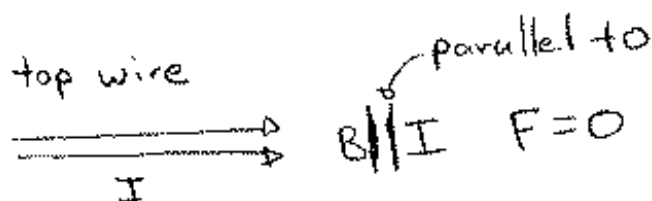
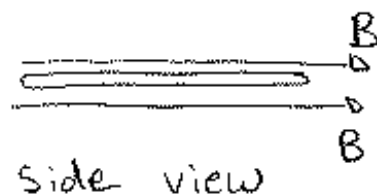
consider the following case:

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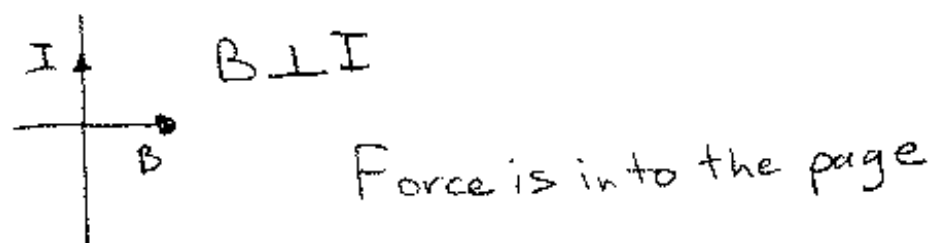
Suppose we have a loop of wire carrying a current



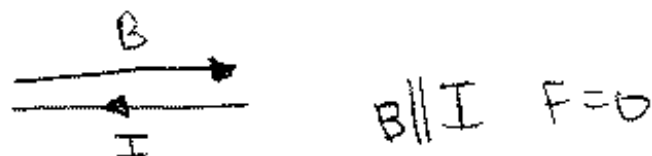
Where are the forces directed?



left wire



Bottom wire



loop rotates! $\hookrightarrow$ we've got our torque!

(6)

Here's a trick to remember which way the loop rotates.

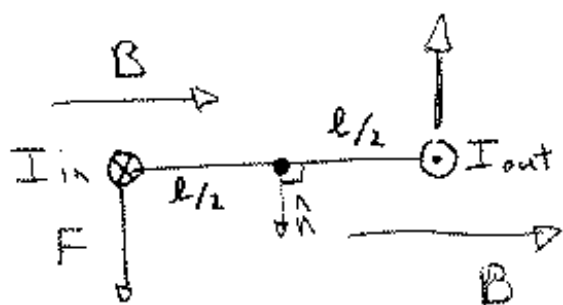
1) Put your fingers along the current direction, your thumb defines a vector, in this case a normal to the loop.

2) This normal wants to point in the direction of the B field.

Lets calculate the torque. Torque is Force times a distance

$$\tau = \text{Force} \times \frac{\text{lever arm}}{\perp \text{ distance from rotation axis}}$$

Since our loop rotates about it's center, we have $\frac{l}{2}$ as a distance, and our force is up in one spot and down at another, looked at from the side



If you recall, counter-clockwise torque points out of the page.

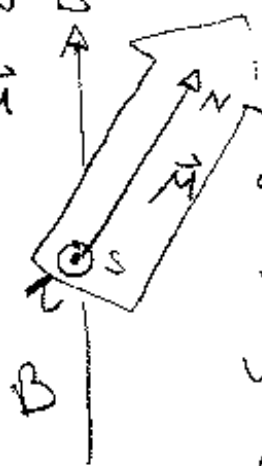
Easier to see here that the magnetic force produces a rotation

$$\begin{aligned} |F| &= |I l B| \text{ for both sides} \\ |\tau| &= |I l B| \frac{l}{2} + |I l B| \frac{l}{2} \quad \left\{ \begin{array}{l} \text{Both} \\ \text{rotate} \\ \text{same} \\ \text{way} \end{array} \right\} \\ &= |I l^2 B| = |I A B| \end{aligned}$$

In general, the quantity IA is called the magnetic moment, μ , and the direction is defined as stated. $\vec{\tau} = \vec{\mu} \times \vec{B}$ ⑦

So, if one knows the magnetic moment, one can calculate torque, if you know $\vec{B}$

Ex a compass has $\vec{\mu}$



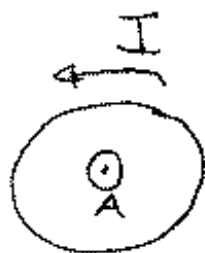
← rotates this way as you'd expect.

recap

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$= I d\vec{\ell} \times \vec{B}$$

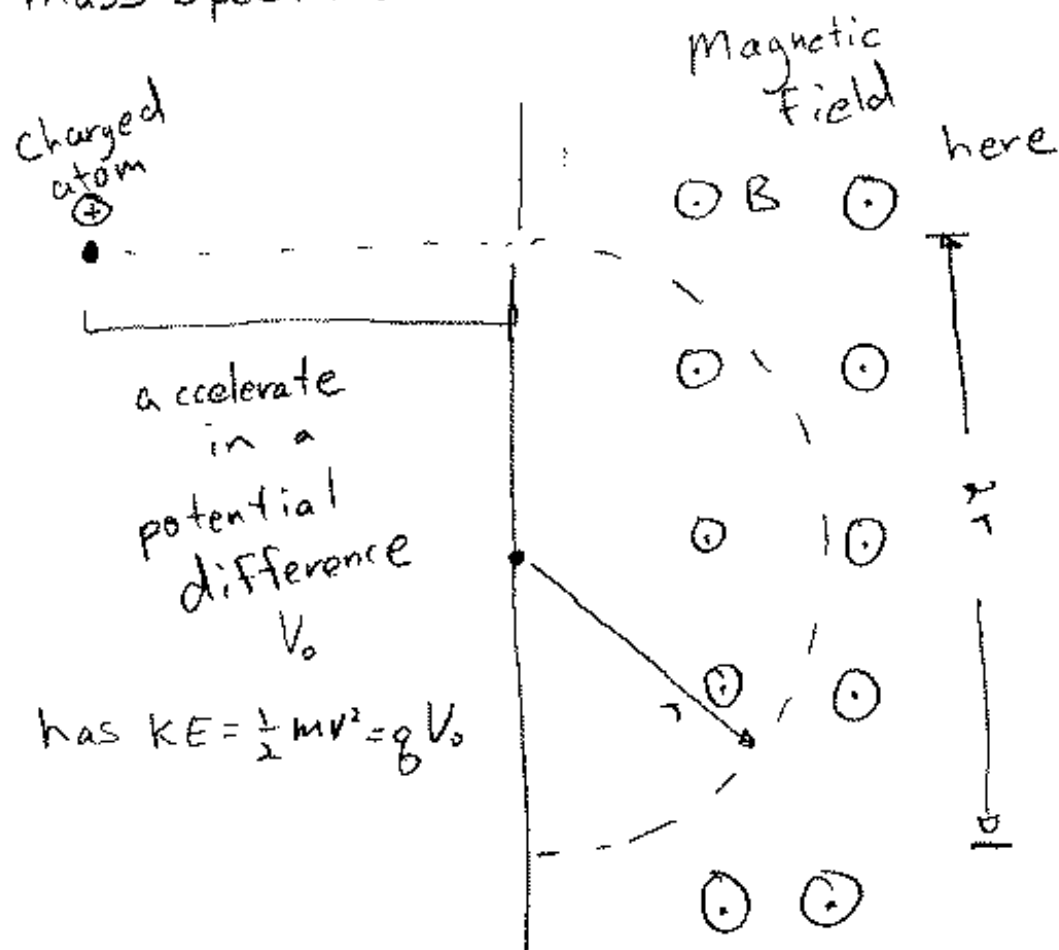
$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \vec{\mu} = I\vec{A}$$



one application of magnetic field

Mass Spectrometer

8



$$\left(\frac{r}{2}\right) \frac{mv^2}{r} = qvB \left(\frac{r}{2}\right)$$

$$\frac{1}{2}mv^2 = \left(qvB \frac{r}{2}\right) = qV_0$$

$$v = \frac{2V_0}{Br}$$

$$\frac{1}{2}mv^2 = qV_0$$

$$m = \frac{2qV_0}{v^2} = \frac{2qV_0}{\left(\frac{2V_0}{Br}\right)^2} = \frac{B^2 r^2}{2V_0}$$