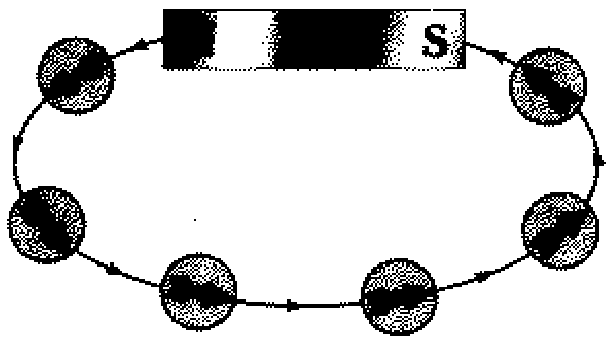
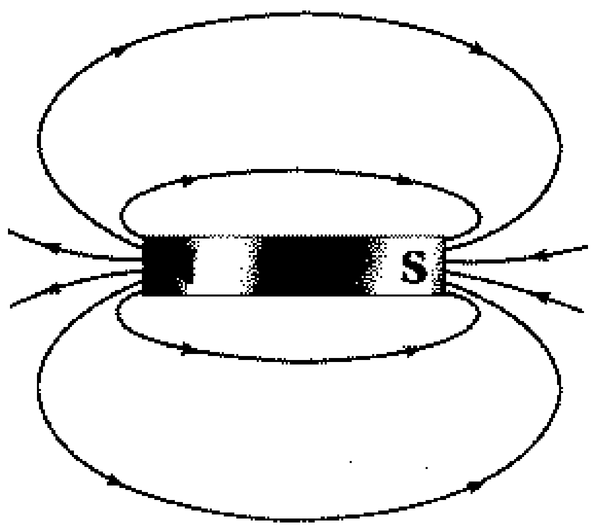
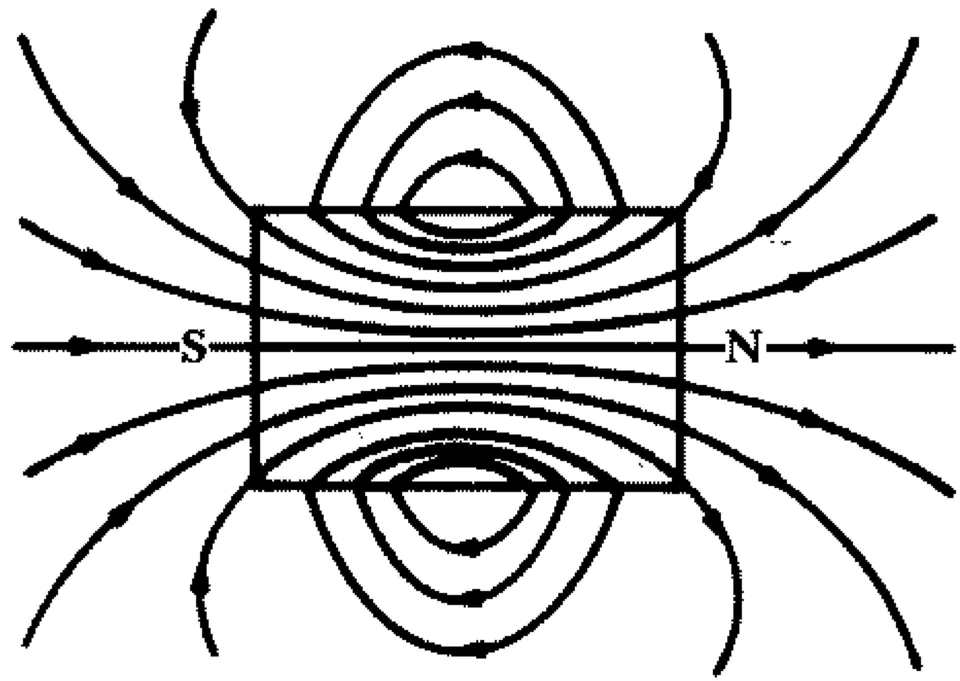


(a)



(b)

Figure 22.1



(a)

Figure 29.12a

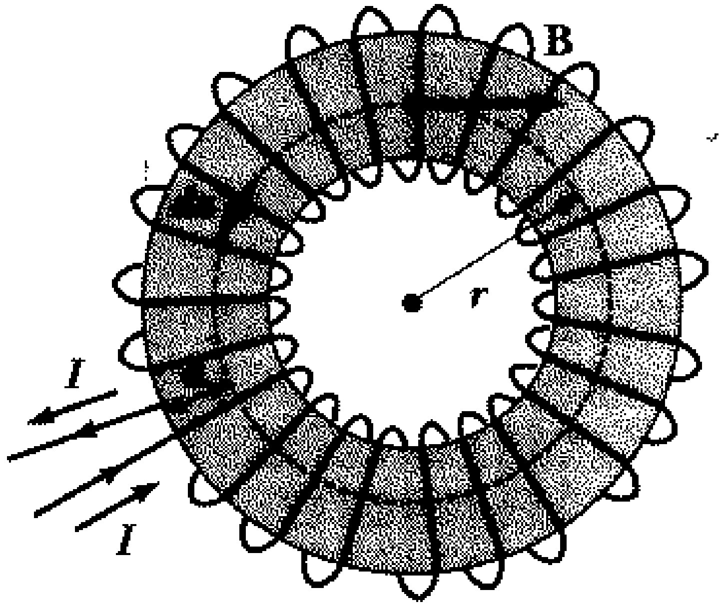


Figure 22.21

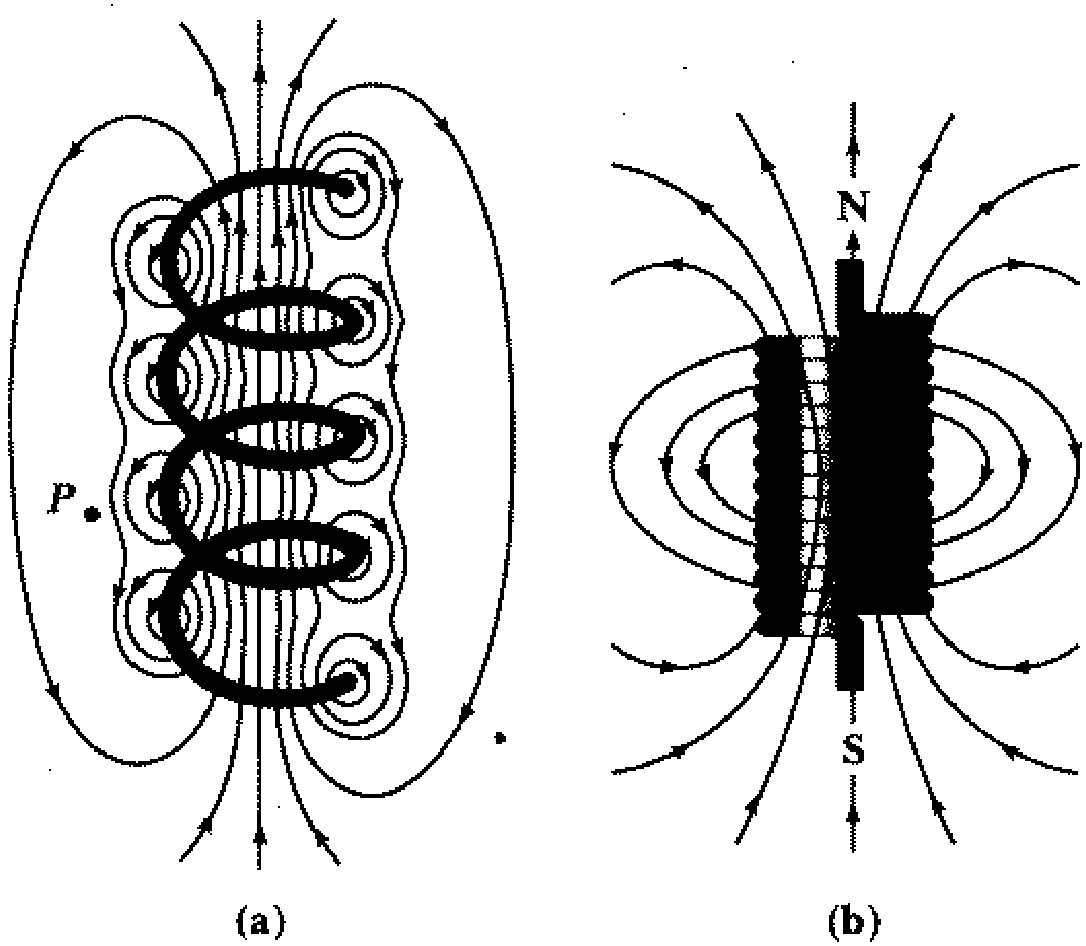


Figure 22.23

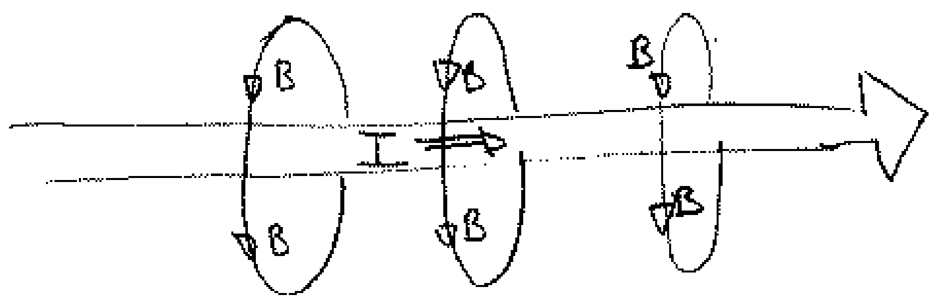
About 180 years ago, A teacher was preparing a lecture demonstration and noted the following:

A compass needle is deflected by a wire carrying current.

- A current produces a magnetic field In this case $\perp$ to the wire

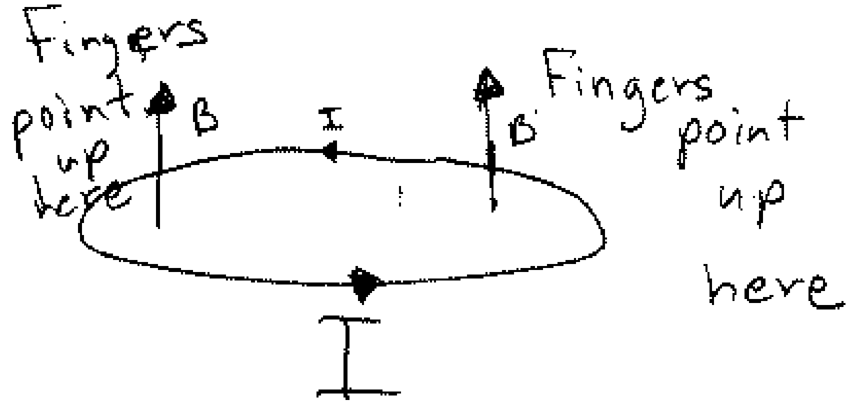
The way we deal with this is a lot like we dealt with electric fields, except "perpendicular."

Here's what the shape of a B field looks like around a wire

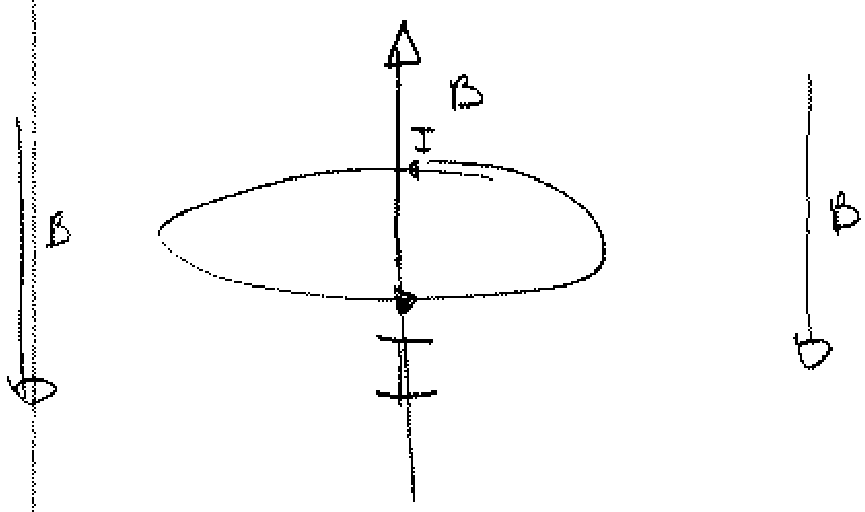


Put your right thumb in the direction of the current, your fingers point in the direction of the B field.

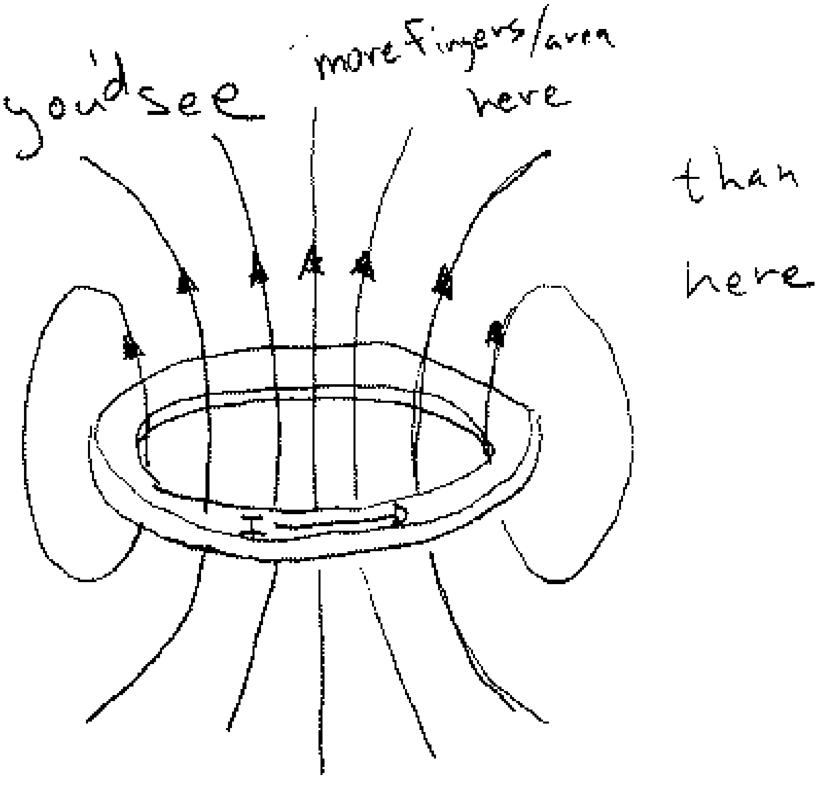
This works great. Try a loop of wire



expect



in fact you'd see



How do we deal with this quantitatively?
Turns out its very similar to Coulombs law & Electric Fields

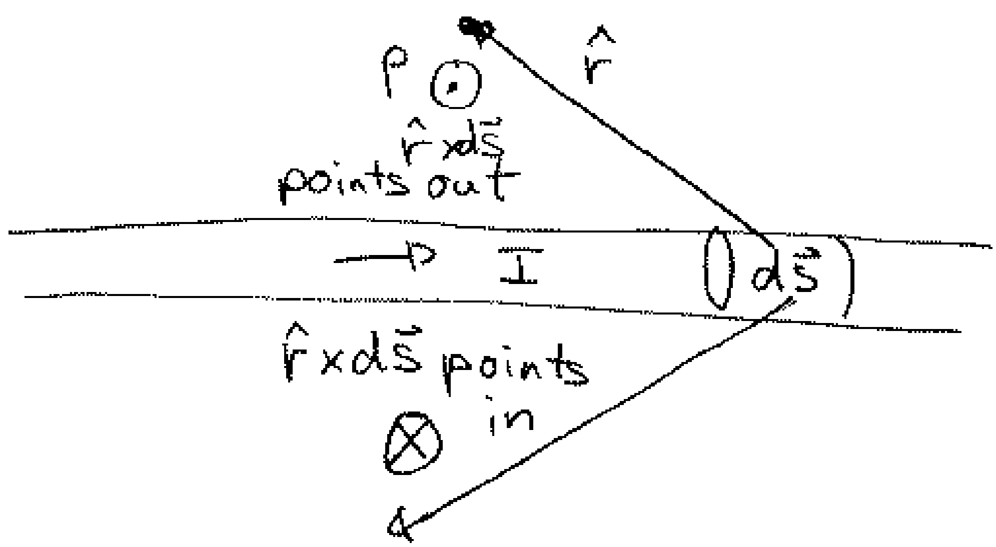
Biot - Savart

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{s} \times \hat{r}}{r^2}$$

μ_0 = permeability of free space

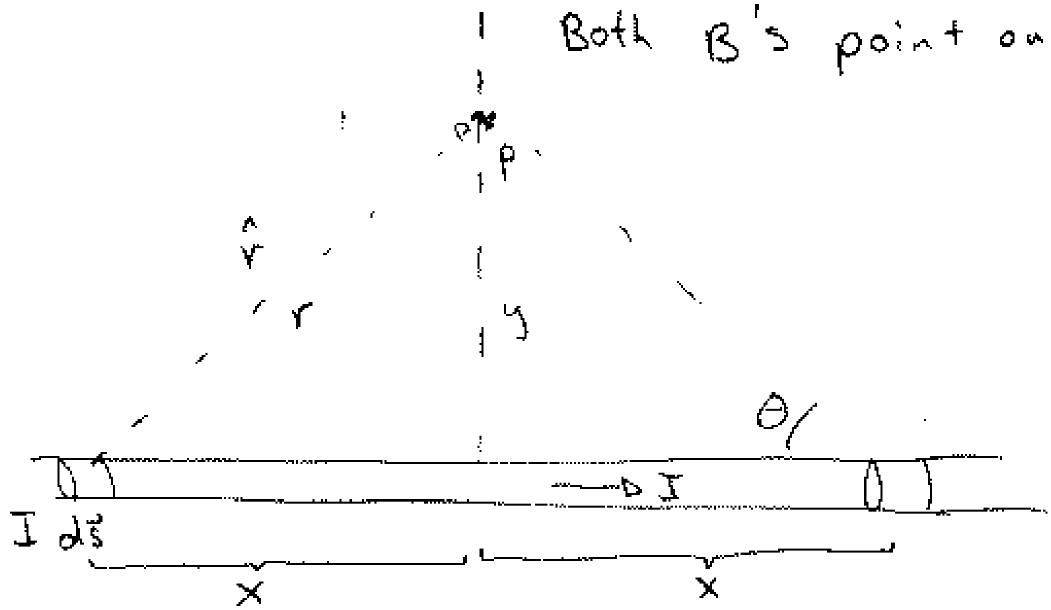
$4\pi \times 10^{-7} \frac{Tm}{A}$

Heres how it works



if we evaluate what the field strength is at point P, we proceed as we did for an infinitely charged wire

Both B's point out



$$dB = 2\left(\frac{\mu_0}{4\pi}\right)\frac{I\,dx}{x^2+y^2}\sin\theta$$

$$x = y/\tan\theta$$
$$dx = -y\left[(\sec^2\theta)/\tan^2\theta\right]d\theta$$
$$= -y\,d\theta/\sin^2\theta$$

$$x^2+y^2 = y^2/\sin^2\theta$$

$$\int dB = 2\frac{\mu_0}{4\pi}I\int_{\pi/2}^{0}\frac{\left(-\frac{y\,d\theta}{\cancel{\sin^2\theta}}\right)\sin\theta}{(y^2/\cancel{\sin^2\theta})}$$

$$B = \frac{2\mu_0}{4\pi}I\left(-\frac{1}{y}\right)\int_{\pi/2}^{0}\sin\theta\,d\theta$$

$$B = \frac{\mu_0}{2\pi y}I\,\underbrace{-\left[-\cos\theta\right]_{\frac{\pi}{2}}^{0}}_{1} = \frac{\mu_0 I}{2\pi y}$$

now, there's a trick with this one like there was a trick for gauss's law

I want you to notice a couple things

1) Circular field around the wire

$$B = \frac{\mu_0 I}{2\pi y}$$

Circumference of a circle

Generalization

Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

completeline integral

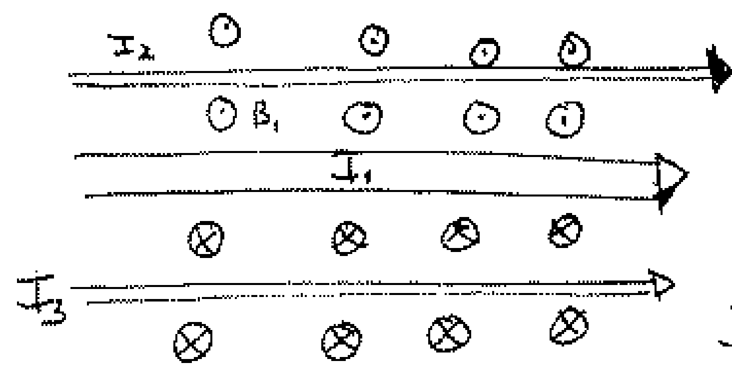
useful for problems with great symmetry

⇒ you try to choose problems where B & ds are the same

(makes remembering the B field a piece of cake for lots of stuff)

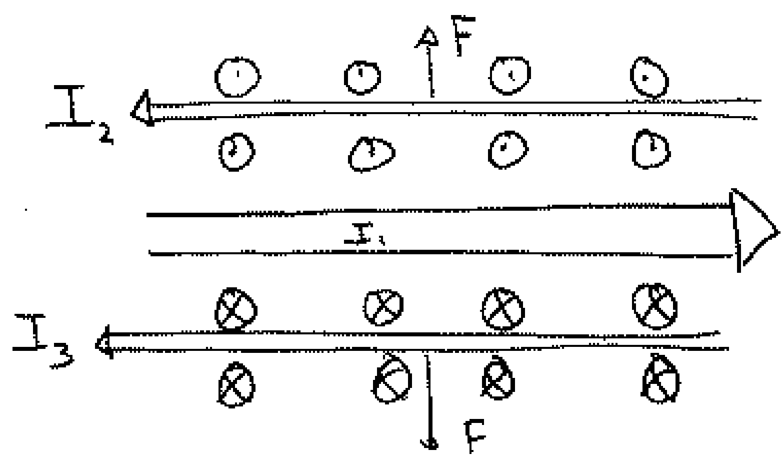
2) A current carrying wire near another one will exert a force on it.

which way? pick one wire to produce the B field, the other wire to calculate the force



$I_2 d\vec{l}_2 \times \vec{B}_2$ is towards I_1

I_3 is attracted to I_1 too



repelled!

demo

Are we at a huge risk from these B fields

consider a conductor carrying 20A of current in your home, 1m away you feel

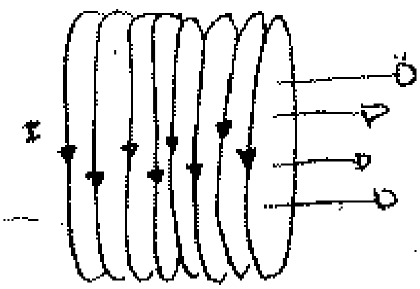
$$B = \frac{\mu_0 I}{2\pi y} = \frac{4\pi \times 10^{-7} \frac{Tm}{A}\, 20}{2\pi \quad 1m} = 4 \times 10^{-6}\, T$$

(less than earth's)

Now, let me show you the other 2 applications of Ampere's Law

The solenoid:

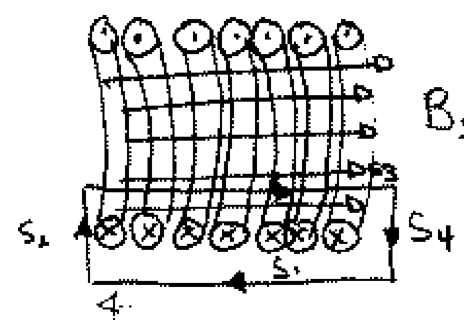
Imagine stacking a bunch of current loops in a line (you can do the calculation this way)
Current all going in same direction



B field inside will begin to get pretty straight and strong

B field outside gets more and more stretched (weak compared to inside

If we could cut this in half we'd see



B_{strong}

B_{weak}

Choose our path in ampere's Law

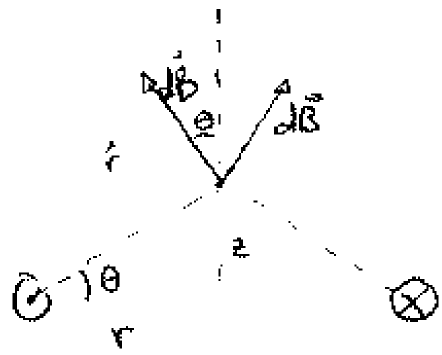
$$B_{strong} S_3 + 0 S_4 + \cancel{B_{weak}} S_1 + 0 S_2 = \mu_0 I_{enc}$$

0

$$I_{enc} = \Big(\underbrace{\frac{\#\text{of turns}}{\text{unit length}}}_{n} \cdot I_{in\,each} \Big) S_3$$

$$B_{inside} S_3 \sim \mu_0 n I_0 S_3$$

$$B_{inside} \sim \mu_0 n I_0$$



only have $\cos\Theta$ component

$$B = \frac{\mu_0}{4\pi} \frac{I\,\underbrace{(2\pi r)}_{S_{tot}}}{(r^2+z^2)} \cos\Theta$$

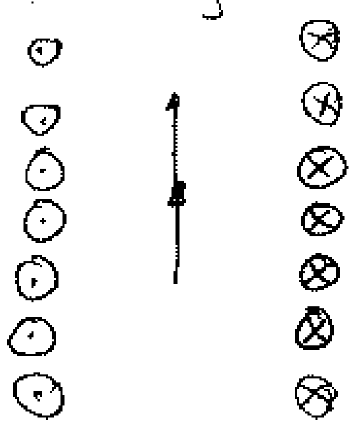
in $\hat{z}$

usual way to write this

$$B = \frac{\mu_0 I}{2} \frac{r^2}{(r^2+z^2)^{3/2}}$$

@ large z $B \sim \frac{\mu_0 I}{2} \frac{r^2}{z^3}$

now suppose we have a bunch of along the axis these rings



distance between rings is ℓ, have a current density of $\frac{I}{\ell}$ $dB = \frac{\mu_0}{2} \frac{I}{\ell} dz \frac{r^2}{(r^2+z^2)^{3/2}}$

- choose point in the center

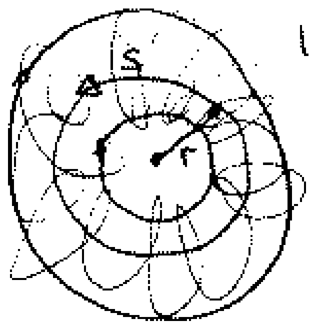
Field contributions are symmetric

$$B = \cancel{2} \int_0^\infty \frac{\mu_0}{\cancel{2}} \frac{I}{\ell} R^2 \frac{dz}{(R^2+z^2)^{3/2}}$$

$$= \mu_0 \frac{I}{\ell} \cancel{R^2} \left[\frac{y}{\cancel{R^2}(R^2+y^2)^{1/2}} \Big|_0^\infty \right] = \mu_0 \left(\frac{\#\text{coils}}{\text{length}}\right) I$$

as before

The toroid
(A solenoid that has swallowed its own tail



looks like a spring

path encloses all the current

Field has circular symmetry

$$B\, 2\pi r = \mu_0 I_0 N$$ # of turns

$$B = \frac{\mu_0 I N}{2\pi r}$$

gets stronger towards the middle

Inside & outside the donut $B \sim 0$