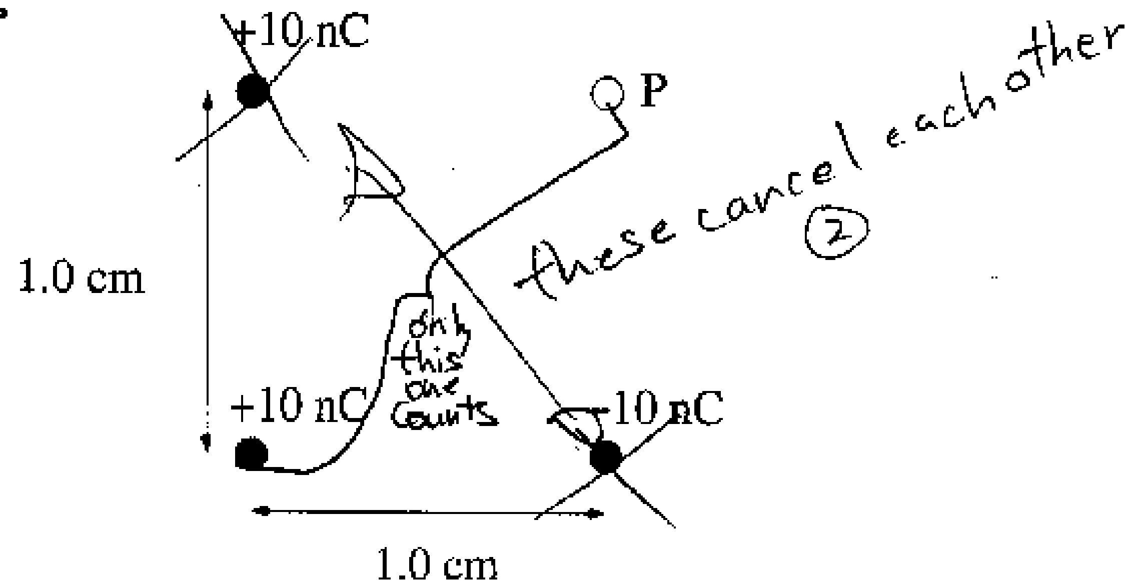
1) Short Answer (4 points each, cont'd)(SHOW YOUR WORK)

d) Calculate the electric potential at point P for the collection of charges shown in the figure below.

$V = \sum \frac{kq}{r}$



② $V = \frac{(9 \times 10^9 N\frac{m^2}{C^2})(10nC)}{(0.01m^2 + 0.01m^2)^{1/2}} = 6363.96$

$= 6.4 \text{ kV}$

e) An isolated capacitor in air has a stored energy of 1.0×10^{-9} J. If we now insert a dielectric material of dielectric constant $\kappa = 1.3$ into the gap of the capacitor, what is the new stored energy of the capacitor?

② isolated ≡ charge conserved

$[U_{new} = 0.77nJ]$ ①

① $U = \frac{1}{2}\frac{q^2}{C}$ $U_{new} = \frac{1}{2}\frac{q^2}{C_{new}}$

$\frac{U_{new}}{U} = \frac{\frac{1}{2}\frac{q^2}{C_{new}}}{\frac{1}{2}\frac{q^2}{C}} = \frac{C}{C_{new}}$

$U_{new} = U\frac{C}{C_{new}} = 1.0 \times 10^{-9} J \left(\frac{C}{1.3C}\right) = 0.77 \times 10^{-9} J$

works either way

1) Short Answer (4 points each)(SHOW ~~YOUR~~ WORK)

a) A +3.0 nC (positive) charge and a +1.0 nC (negative) charge are located 0.80 m apart from each other. What is the force on the ~~+3.0~~ nC (positive) charge due to the +1.0 nC (negative) charge?

(1) $|F| = \frac{k q_1 q_2}{r^2}$ (opposites attract)

(2) $= \frac{(9\times10^9\ N\frac{m^2}{C^2})(3\times10^{-9}C)(1\times10^{-9}C)}{(0.8m)^2} = 4.2\times10^{-8}N$

(1) $[F = 4.2\times10^{-8}N]$

b) Two widely separated spheres, one with a radius of 3.0 cm and charge on its outer surface +q (Q_{small}=+q), and one sphere with a radius of 7.0 cm and ~~big~~ charge on its outer surface (Q_{big}=+q), are connected by a thin wire, i.e. the wire forces them to be at the same potential. The charge on the big (r=7.0 cm) sphere *after* the spheres are connected by a thin wire is:

? give credit either way.

Charge is conserved ⇒ $Q_{big} + Q_{small} = Q_{big}^{After} + Q_{small}^{After}$ (1)

⇒ $\frac{kQ_{big}^{After}}{r_{big}} = \frac{kQ_{small}^{After}}{r_{small}}$ (2)

(1) $[Q_{big} = \frac{7}{5}(+q)]$

$Q_{small}^{After} = \frac{3cm}{7cm} Q_{big}^{After}$ so $(+q)+(+q) = \frac{7}{7}Q_{big}^{After} + \frac{3}{7}Q_{big}^{After} \Rightarrow Q_{big}^{After} = 2(\frac{7}{10})(+q) = \frac{7}{5}+q$

c) The electric potential in a certain region of space can be described by the equation:

$$V(x,y,z) = 10x - \frac{16y^2}{(1+z^2)}$$

What is the electric field at the origin (0,0,0)?

(2) $E_x = -\frac{\partial V}{\partial x} = -10\ N/C$

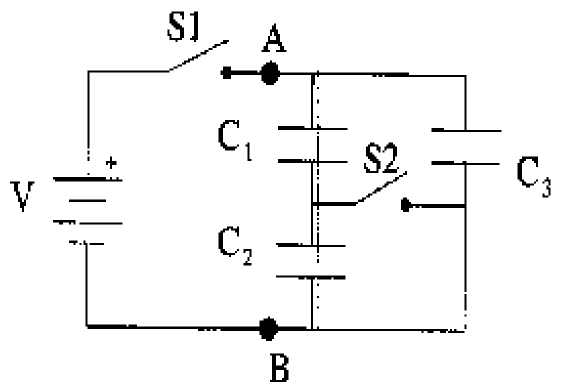
$E_y = -\frac{\partial V}{\partial y} = -\frac{32y}{(1+z^2)}$ @ $y=0$ $E_y = 0$

$E_z = -\frac{\partial V}{\partial z} = \frac{(-2z)(-16y^2)}{(1+z^2)^2}$ @ $y=0$ or $z=0$ $E_z = 0$

Only x component (2) $\vec{E} = -10\frac{N}{C}\hat{x}$

2) Capacitors(Show your work)(4 pts. each part)

Consider the circuit shown below. All capacitors are initially uncharged and both switches, S1 and S2, are open. We are going to close and open switches and calculate the outcome.



$C_1 = 10$ pf
$C_2 = 20$ pf
$C_3 = 30$ pf
V = ?

a) We close only switch S1 and notice that a long time later (at electrostatic equilibrium) the charge on $C_3 = 7.0pC$. What is the potential difference of the battery?

S1 closed $V_{ab} = V = V_{across}\ C_3$ (1)

(1) $Q = CV \quad V = \frac{Q}{C} = \frac{7.0pC}{30pf} = 0.23V$ [$V_{batt} = 0.23V$] (2)

b) C_3 is a parallel plate capacitor with a an area of $3.0 \times 10^{-4} m^2$. Calculate the magnitude of the electric field between the plates of of C_3, assuming the volume between the plates of C_3 is filled with a material of dielectric constant $\kappa = 2$.

(1) $V = Ed$ inside a Parallel plate cap

$C = \frac{\epsilon_0 A}{d}$ or $\frac{\kappa \epsilon_0 A}{d}$ w/dielectric

$E = \frac{V}{d}$

(1) [$E = 1.3\ k\frac{N}{C}$]

(2) $E = V\left(\frac{C}{\kappa \epsilon_0 A}\right) = \sigma/\kappa\epsilon_0$!

$\left(\frac{7.0 \times 10^{-12} C}{3.0 \times 10^{-4} m^2}\right) / 2(8.85 \times 10^{-12} \frac{C^2}{m^2 N}) = 1318 \frac{N}{C}$

2) Capacitors(cont'd)

c) How much energy is stored by capacitor C_1? (1)

Capacitance of C_1 & $C_2 = \left(\frac{1}{10pf} + \frac{1}{20pf}\right)^{-1}$

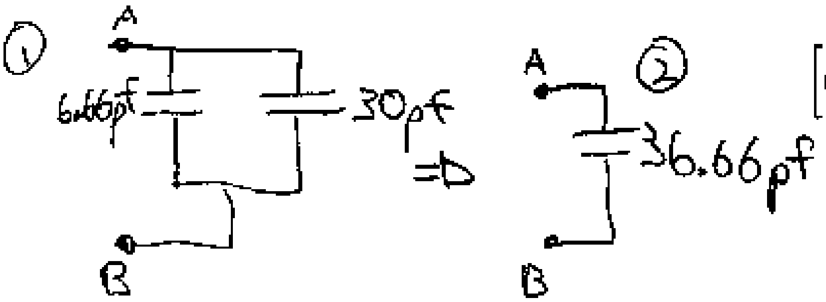
$= 6.66 pf$

$[U_1 = 0.12 pJ]$ (1)

(2) Q on C_1 or $C_2 = 0.233V \times 6.66pF$

$= 1.55 pC$ $\quad U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(1.55pC)^2}{10pf} = 0.1$

d) Find the effective capacitance between points A and B.



(1) $[C_{AB} = 36.66pF]$

e) Now switch S1 is opened and S2 is closed *in that order*.

1st: S1 opened
2nd: S2 closed

After S2 is closed and the circuit has reached electrostatic equilibrium, calculate the potential difference across C_1.

(1) C_2 discharges, has $V=0$

(1) $[V_{new} = 0.21 V]$

(1) C_1 & C_2 combine charge

(1) & are at same pot

treat as parallel with $Q_{tot} = 1.55pC + 7pC$

$C_{tot} = 10pf + 30pf = 40pF$

$V = \frac{Q_{tot}}{C_{tot}} = \frac{8.55pC}{40pF} = 0.213 V$

or use charge conservation w/

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$, $Q_1 + Q_2 = Q_{tot}$

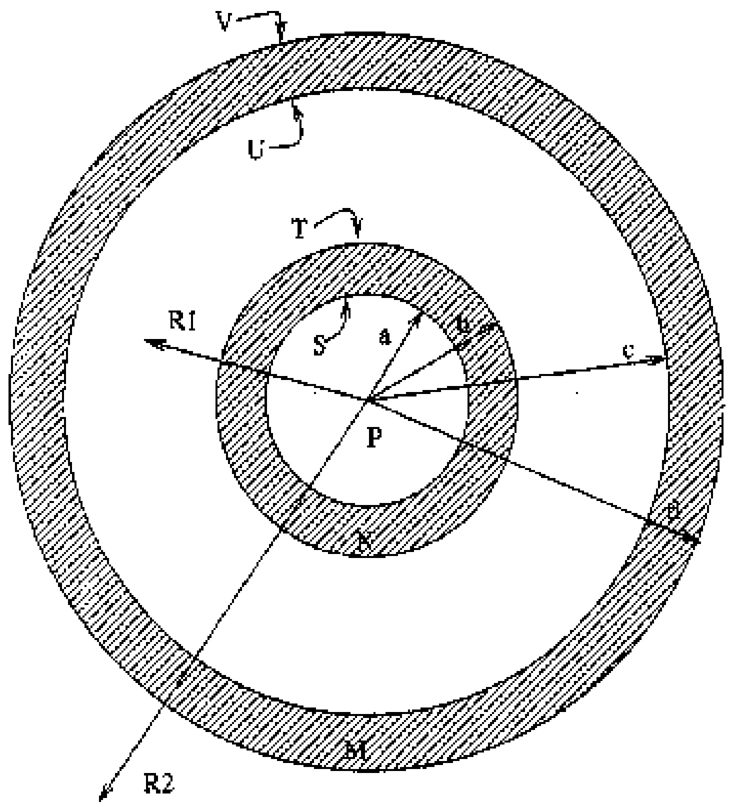
$Q_1 + Q_1\frac{C_2}{C_1} = Q_{tot}$

$Q_1 = \frac{Q_{tot}}{(1 + \frac{C_2}{C_1})}$

$= \frac{8.53pC}{(1 + \frac{30}{10})} = 2.13pC$

3) (Gauss Law) A conducting spherical shell, of inner radius a and outer radius b, is inside of and concentric with another conducting spherical shell, of inner radius c and outer radius d. The inner shell has a net charge of $^{-}2q$ and the outer shell has a net charge of ^{-}q. A charge of $-3q$ is located at the center of the 2 spheres at point P.

Charge at Center (P) = $^{-}3q$
inner shell net charge = $^{-}2q$
outer shell net charge = ^{+}q



Answer the following questions. (No need to show your work)

a) What is the magnitude of the electric field at the point a/2, inside the spherical shells?

Only $q_{enc} = -3q$

$E(4\pi r^2) = -3q/\varepsilon_0$

$E = \frac{-3q}{4\pi\varepsilon_0}\frac{1}{(a/2)^2}$

$|E| = |-3q/\pi\varepsilon_0|$

[$|E| =$ $3q/\pi\varepsilon_0$] (3 pts)

3) (Gauss Law)(No need to show your work)(cont'd)

b) What is the magnitude of the electric field at the point N, inside the inner shell?

inside conductor

[$|E| = 0$] (2 pts)

c) What is the magnitude of the electric field at R1, between the inner and outer shells?

$q_{enc} = -3q$ @center + $-2q$ on inner shell

$|E(4\pi(R1)^2)| = |-5q/\varepsilon_0|$

[$|E| = \frac{5q}{(4\pi\varepsilon_0)R1^2}$] (2 pts)

d) What is the magnitude of the electric field at point M, inside the outer shell?

inside cond. $E = 0$

[$|E| = 0$] (2 pts)

e) What is the magnitude of the electric field at R2, outside the outer shell?

$q_{enc} = -3q + (-2q) + (+q) = -4q$

[$|E| = \frac{4q}{4\pi\varepsilon_0(R2)^2}$] (3 pts)

f) How much charge is on the inside surface (surface S) of the inner sphere?

in order to get $q_{Enclosed} = 0$

need $-3q + Q = 0$ $Q = 3q$

[$Charge = 3q$] (2 pts)

g) How much charge is on the outside surface (surface T) of the inner sphere?

inner shell has $q_{net} = -2q$

$-2q = 3q_{\text{inner surface}} + Q_{\text{outer surface}}$

[$Charge = -5q$] (2 pts)

same tot!

h) How much charge is on the inside surface (surface U) of the outer sphere?

has to cancel $q_{enc} = -5q + Q$

$= 0 =$

$Q = 5q$

[$Charge = 5q$] (2 pts)

same tot

i) How much charge is on the outside surface (surface V) of the outer sphere?

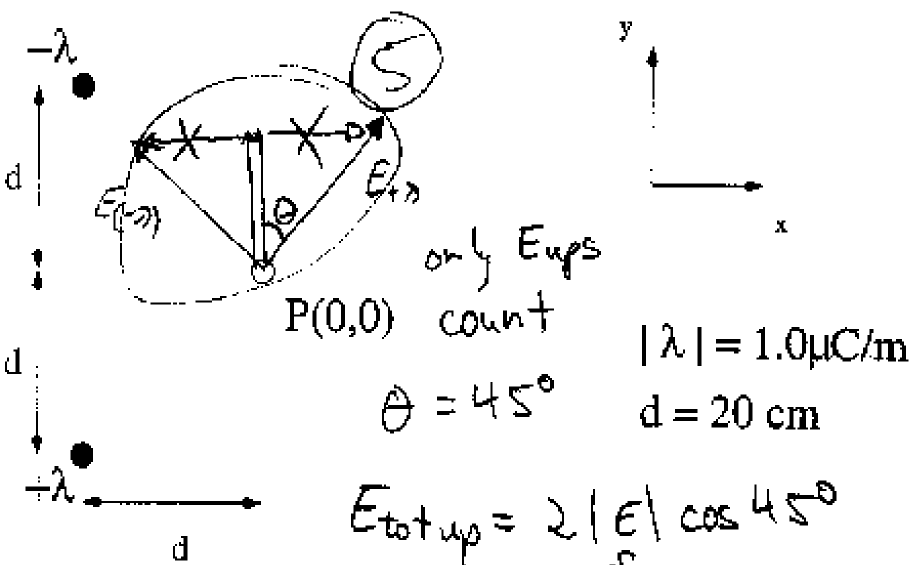
outer shell has $q_{net} = +q$

[$Charge = -4q$] (2 pts)

$+q = 5q_{\text{inner surface}} + Q$ $Q = -4q$

4) Electric Fields (Vectors)(show your work)

Two infinite lines of charge, one with charge/length, $+\lambda$, of $1.0\mu C/m$ and the with a charge/length, $-\lambda$, of $-1.0\mu C/m$ are placed as shown: the negative line of charge at (-d,d) and the positive line of charge at (-d,-d). (assume d = 20 cm and that all lines of charge are absolutely parallel)



$E_{tot\,up} = 2|E|_{\text{of one}} \cos 45°$

a) What is the magnitude and direction of the electric field at the point P (0,0)?

(5 pts. each)

$|E| = \dfrac{(1.0\mu C/m)}{\varepsilon_0 (2\pi r)}$ (2)

$r = \sqrt{.20m^2 + .20m^2} = 0.2828\,m$

$E_X = 0$

$E_Y = 90k\frac{N}{C}$

$2|E|\cos\theta = 2\left(\dfrac{1.0\times10^{-6}\,C/m}{\left(8.85\times10^{-12}\frac{C^2}{m^2N}\right)(2\pi)(.2828m)}\right)\cos 45°$ (2)

$= 89{,}931.6\ N/C$ (1)

4) Electric Fields (cont'd)

b) Where should a third positive line of charge with charge/length, λ, be placed so that the total electric field vanishes (equals zero) at point P? (5 pts. each)

want to cancel up E need to place positive directly above point P

$\left[X = 0 \quad Y = 0.20m\right]$ (5)

$|E| = \frac{\lambda}{2\pi r_{new} \varepsilon_0}$

$\left(90 \frac{kN}{C}\right) = \frac{(1.0 \mu C/m)}{\varepsilon_0 2\pi r_{new}}$ (2)

(2) $\frac{1}{r_{new}} = \frac{\left(8.85 \times 10^{-12} \frac{m^2 N}{C^2}\right)(2\pi)\left(84{,}931.6 \frac{N}{C}\right)}{(1 \times 10^{-6} C)}$

$\frac{1}{r_{new}} = \frac{1}{5.0}$

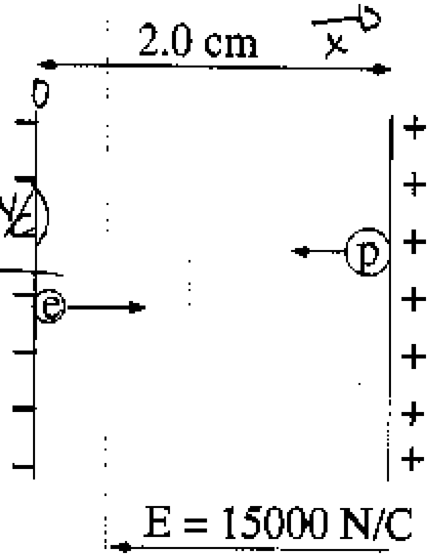
$r_{new} = 0.20m$ (1)

(pretty close to where we started.

The idea for this problem came from you guys!

5) Electric Fields (Position, Velocity, Acceleration) (show some work please)

Two large parallel copper plates are 2.0 cm apart and have a uniform electric field of magnitude 1.5×10^4 N/C between them as depicted below. An electron is released from the negative plate at the same time that a proton is released from the positive plate. Neglect the force of the particles on each other and gravity.



Will pass @ same time & place

$a_e = \frac{F_e}{m_e} = \frac{q_e E_e}{m_e} = \frac{(1.6\times10^{-19} C)(1.5\times10^4 N/C)}{9.11\times10^{-31} kg} = 2.63\times10^{15} m/s^2$

$a_p = \frac{F_p}{m} = \frac{q_p E_p}{m_p} = \frac{(1.6\times10^{-19} C)(1.5\times10^4 N/C)}{1.67\times10^{-27} kg} = 1.437\times10^{12} m/s^2 = -1.437\times10^{12} m/s^2 (-\hat{x})$

$a_p/a_e = \frac{-1.437\times10^{12}}{2.63\times10^{15}} = 5.4\times10^{-4}$

a) Find the distance from the positive plate when they pass each other. (10 pts.)

$x_e = \frac{1}{2}a_e t^2$

$x_p = 2.0cm + \frac{1}{2}a_p t^2$

$x_e = x_p$

$t^2 = \frac{2x_e}{a_e}$

$\frac{1}{2}a_e t^2 = 2.0cm + \frac{1}{2}a_p t^2$

$x_e = 2.0cm + \frac{1}{2}a_p\left(\frac{2x_e}{a_e}\right)$

$x_e - \left(\frac{a_p}{a_e}\right)x_e = 2.0cm$

$x_e = \frac{2.0cm}{\left(1-\frac{a_p}{a_e}\right)} = 1.9989125 cm$ or $1.09\times10^{-5} m$ from positive plate

[Distance = $1.1\times10^{-5} m$]

b) How fast is the electron moving when they pass each other. (5 pts.)

[Velocity = $1.0\times10^7 m/s$]

$V_e = a_e t = a_e\left(\frac{2x_e}{a_e}\right)^{1/2}$

$= \sqrt{2x_e a_e} = \sqrt{2(0.01999m)(2.63\times10^{15} m/s^2)} = 1.0254\times10^7 m/s$

c) How much work did it take to accelerate the electron to this velocity? (5 pts.)

$\frac{1}{2}mv^2 = \text{Work} = \frac{1}{2}(9.11\times10^{-31})kg \times (1.0254\times10^7 m/s)^2 = 4.79\times10^{-17} J$

[Work = $4.8\times10^{-17} J$]

check $W = Fd = (1.6\times10^{-19}C)(1.5\times10^4 N/C)(0.01999 m) = 4.798\times10^{-17} J$ close!