

$$\frac{3}{2d} = \frac{1}{R} \quad R = d \frac{2}{3}$$

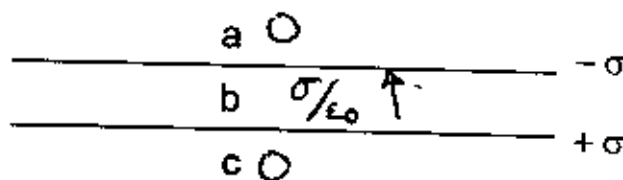
### 1) Short Answer (25 points)

a) At what distance is the force between 2 electrons equal to 5.0 nN? (5pts)

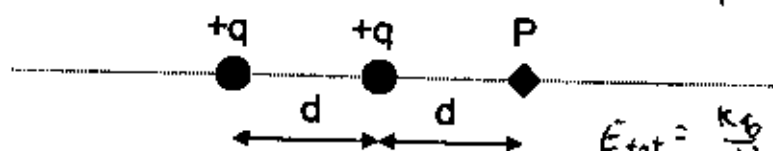
$$\frac{k_e q_1 q_2}{r^2} = 5 \times 10^{-9}$$

$$r = \sqrt{\frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{5 \times 10^{-9}}} = 2.15 \times 10^{-10} \text{ m}$$

b) Describe, briefly, the electric field at the points indicated on the figure to the right. Each horizontal line represents an infinite sheet of charge with equal but opposite sign charge densities. (5pts)



c) Suppose there are 2 positive charges with charge of +q, separated by a distance d as shown below. Where should a third positive charge of +q be placed so that the electric field vanishes at the point P? (5pts)



$$\frac{1}{d^2} + \frac{1}{(2d)^2} = \frac{1}{d^2} \left(1 + \frac{1}{4}\right) = \frac{5}{4d^2}$$

$$E_{tot} = \frac{k_e q}{d^2} + \frac{k_e q}{(2d)^2} = \frac{k_e q}{r^2}$$

$$r = \sqrt{\frac{1}{\frac{5}{4d^2}}} \quad \text{or } \sqrt{\frac{4}{5}} d \quad \text{or } \frac{2d}{\sqrt{5}}$$

d) Estimate the capacitance of the earth? (5 pts) (hint: Assume the earth is a spherical conductor, put some charge on it and calculate the electric potential.) The radius of the earth is 6.4E6 meters.

$$V = \frac{k_e q}{r} = \frac{(9 \times 10^9) Q}{6.4 \times 10^6} = 1406.25 \frac{Q}{\text{m}} \quad C = \frac{1}{1406} = 7 \times 10^{-4} \text{ F} \quad \left(\frac{\text{C}}{\text{Vm}}\right)$$

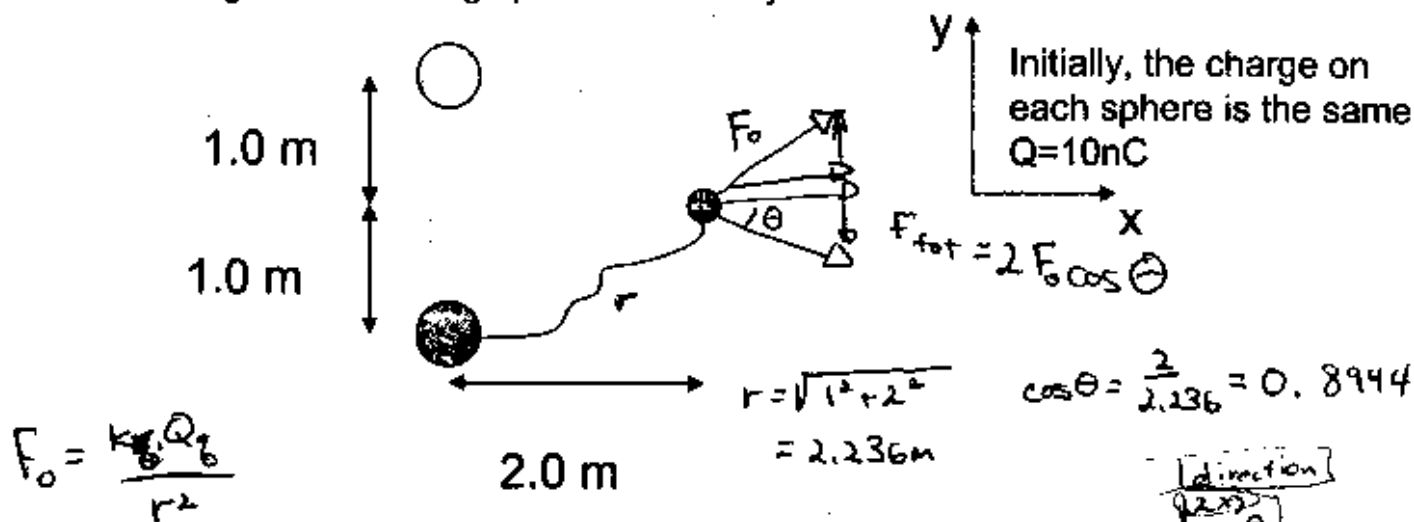
e) At what points in space is the electric field due to the potential:  $V = a(x-1)^2 + b(y+2)^2$  Equal to zero?

$$E_x = -2a(x-1) = 0 @ x = 1 \quad (1, -2)$$

$$E_y = -2b(y+2) = 0 @ y = -2$$

## 2) Force and Potential (25 points)

3 charged, conducting spheres are arrayed as shown below:



What is the force on the little sphere, due to the 2 big spheres?

$$F_0 = \frac{k_e Q_b Q_s}{r^2} = \frac{(9 \times 10^9 \text{ N m}^2/\text{C}^2)(10 \times 10^{-9} \text{ C})^2}{(2.236 \text{ m})^2} = 1.8 \times 10^{-7} \text{ N}$$

$$\begin{aligned} F_x &= 3.22 \times 10^{-7} \text{ N} & (5) \\ F_y &= 0 & (5) \end{aligned}$$

$$F_x = 2(1.8 \times 10^{-7} \text{ N})(0.8944) = 3.22 \times 10^{-7} \text{ N}$$

Next, all three spheres are connected together with a thin wire and charge is redistributed. If the little sphere has  $\frac{1}{2}$  the radius of the big spheres, what is the force now? And what is the charge on the little sphere?

$$\begin{aligned} \sqrt{\frac{Q_{\text{big}}}{r_{\text{big}}}} &= \frac{Q_{\text{small}}}{r_{\text{small}}} \\ Q_{\text{small}} &= \frac{r_{\text{small}}}{r_{\text{big}}} Q_{\text{big}} = \frac{Q_{\text{big}}}{2} \end{aligned}$$

$$\begin{aligned} F_x &= \cancel{3.22 \times 10^{-7} \text{ N}} & (5) \\ F_y &= 0 & (5) \\ Q_{\text{small}} &= 6 \text{ nC} & (5) \end{aligned}$$

$$\begin{aligned} \sqrt{Q_{\text{tot}}} &= 30 \text{ nC} = Q_{\text{small}} + Q_{\text{big}} + Q_{\text{big}} \\ &= \frac{Q_{\text{big}}}{2} + Q_{\text{big}} + Q_{\text{big}} = \frac{5}{2} Q_{\text{big}} \end{aligned}$$

$$Q_{\text{big}} = \frac{2}{5} 30 \text{ nC} = 12 \text{ nC}$$

$$Q_{\text{small}} = 6 \text{ nC}$$

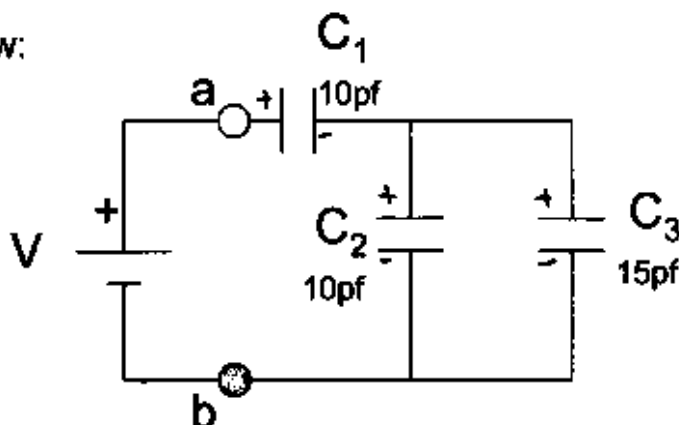
$$F_0^{\text{new}} = F_0^{\text{old}} \frac{(12 \text{ nC})(6 \text{ nC})}{(10 \text{ nC})(10 \text{ nC})} = 0.72 F_0^{\text{old}}$$

$$F_x^{\text{new}} = 0.72 F_x^{\text{old}}$$

#### 4) Capacitors (25 points)

Consider the circuit shown below:

(Switch S is open and the circuit is at electrostatic equilibrium)



a) For each capacitor, mark the plate that has positive charge with a "+" sign, and the plate that has a negative charge with a "-" sign. (5)

b) If the charge on  $C_1$  is 5.0 pC, what is  $V$ ?

$$V = 0.70 \text{ V} \quad (5)$$

$Q_1 = \text{charge on } C_2 C_3 \text{ combo to } 0$   
 $= \text{charge on } C_1 \text{ \& } (C_2 C_3) \text{ since all in series}$

$$V = \frac{5.0 \text{ pC}}{7.14 \text{ pF}}$$

$$\frac{1}{C_{ab}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{1}{10} + \frac{1}{25} = \frac{7}{50} \quad C_{ab} = 7.14 \text{ pF}$$

c) What is the charge on  $C_2$ ?

$$Q_2 = 2 \text{ pC} \quad (5)$$

$$V_{\text{across } C_1} = \frac{5 \text{ pC}}{10 \text{ pF}} = 0.5 \text{ V}$$

$$\text{So } V_{\text{across } C_2} = 0.7 \text{ V} - 0.5 \text{ V} = 0.2 \text{ V}$$

$$Q_2 = 2 \text{ pC}$$

d) What is the equivalent capacitance between points a and b?

$$C = 7.14 \text{ pF} \quad (5)$$