Last time I introduced you to the concepts of the wave function

what you're used to seeing

$E(x,t) = E_0 \sin(kx - \omega t) + E_0 \sin(kx - \omega t + \phi)$ @ a point on the screen

$I(x,t) \propto E^2(x,t)$

$I_{av} = E^2{}_{av}/c\mu_0$

what we said about the wavefunction

$\psi(x,t) = \underbrace{A\sin(kx-\omega t) + A}_{\text{represent equal probabilities}}\sin(kx - \omega t + \phi)$

& $\psi^2(x,t)$ represents some observable amplitude in that ψ^2_{av} is just like the Intensity in that it represents where a quanta will likely end up. (a probability map you try to make $\int \psi^2_{av}\,dx = 1$)

we also introduced the uncertainty principle

$\Delta x \Delta p_x \geq \frac{h}{2\pi} \Leftarrow$ used single slit to show this behavior

$\Delta E \Delta t \geq \frac{h}{2\pi} \Leftarrow$ said for a big Δt (like an average over time) ΔE is small & we can talk about the energy of a state)

Another example: We also said the Bohr model could describe the energy levels in hydrogen pretty well. What is actually going on is that there is a wave function that ~~represents~~ each energy level of hydrogen, the lowest lying 4 looks like a ball of ~~fuzz~~.

Now, if you confine an electron in a small space, this fuzz, theres going to be some minimum momentum allowable from the uncertainty principle

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$

So, suppose we try to find out the minimum Δx for hydrogen.

Recall, $E = \frac{p^2}{2m} - \frac{k|q_e q_p|}{r}$ let $p \Rightarrow p_x$, $r \Rightarrow x$ & $x p_x = \frac{h}{2\pi}$

$$E = \left(\frac{h}{2\pi}\right)^2 \frac{1}{2m} \frac{1}{x^2} - \frac{k|q_e q_p|}{x}$$

@min $\frac{dE}{dx} = 0 = \left(\frac{h}{2\pi}\right)^2 \frac{1}{2m} \frac{-2}{x^3} - k|q_e q_p|\left(\frac{-1}{x^2}\right)$

or $\left(\frac{h}{2\pi}\right)^2 \frac{1}{m} \frac{1}{x^3} = \frac{k|q_e q_p|}{x^2}$

$$\left(\frac{h}{2\pi}\right)^2 \frac{1}{k|q_e q_p| m} = x = \left(\frac{hc}{2\pi}\right)^2 \frac{1}{k|q_e q_p| mc^2}$$

$$= \left(\frac{1240\,eVnm}{2\pi}\right)^2 \frac{1}{\left(9\times10^9 \frac{Vm}{C}\right) e\left(1.6\times10^{-19}\,C\right)} \frac{1}{511,}$$

$$= 5.29\times10^7 (nm)^2/m \cdot \frac{1\,m}{10^9\,nm}$$

$= 0.0529\,nm$

$0.529\,Å$ what we got before!

We'll talk more about hydrogen & wave functions on Monday.

Now, we said ψ was special because it is supposed to contain all sorts of information about a particle. In fact, we said

$$\underbrace{-\left(\frac{h}{2\pi}\right)^2\frac{1}{2m}\frac{\partial^2}{\partial x^2}\psi(x)}_{KE} + \underbrace{U(x)}_{PE} = \underbrace{E}_{\text{total } E}$$

2 ways to think about this

1) $\frac{\partial^2}{\partial x^2}\sin kx$ gives a $-k^2 = -\frac{(2\pi)^2}{\lambda^2}$ & $KE = \frac{p^2}{2m}$

2) $\frac{\partial^2}{\partial x^2}\psi$ tells us information about the curvature of ψ which is essentially like λ

Bonus

In order to be useful, ψ has to be a finite (enough) quantity so we can say the total probability of finding a particle $\psi^2_{av} = 1$

& we made some general observations

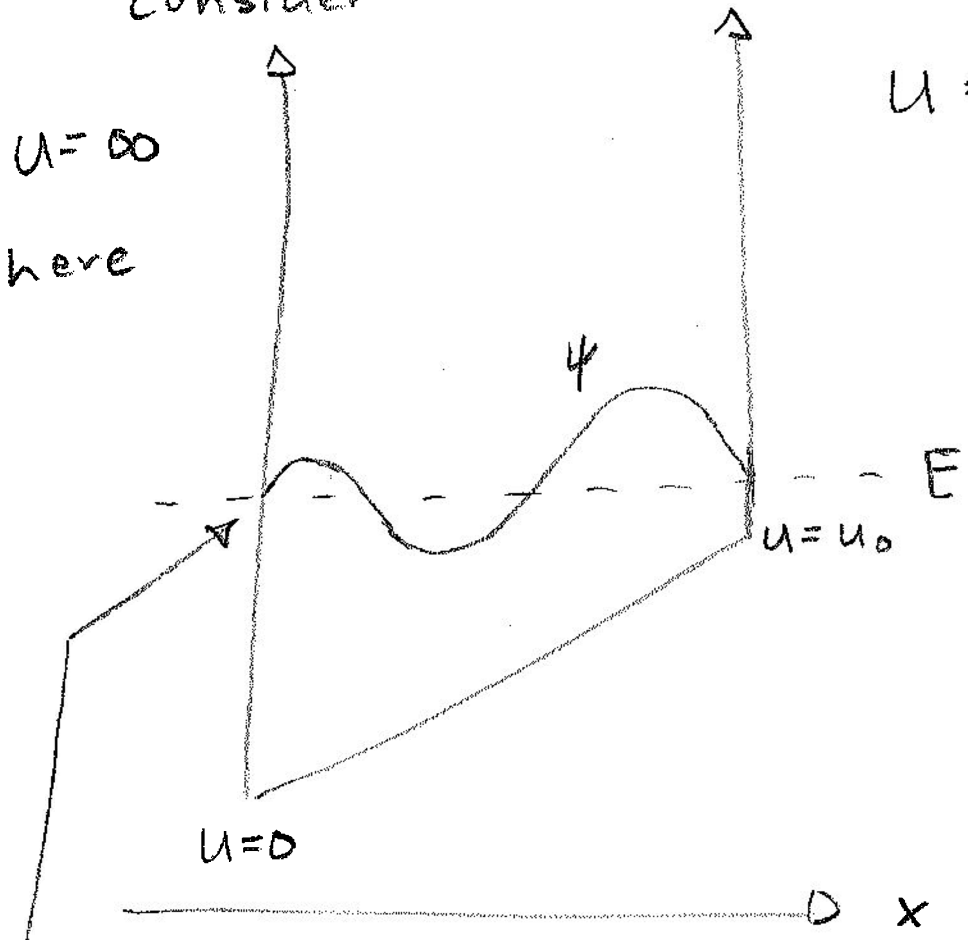
A) if $E > U$ $\quad\psi$ behaves like some $\sin kx$ or $\cos kx$

B) if $U > E$ $\quad\psi$ " e^{ax} e^{-ax}

If $|U-E|$ is large, curvature is large, KE is large

$\Rightarrow$ said where $|U-E|$ is large ψ is small compared to where $|U-E|$ is small)

Consider



This is a digram representi[ng] where E_{tot} is in relation to U so we can guess what ψ will look like

as we increase x, KE is decreasing
- particle is moving slower
- has to spend more time where $E-U$ is small

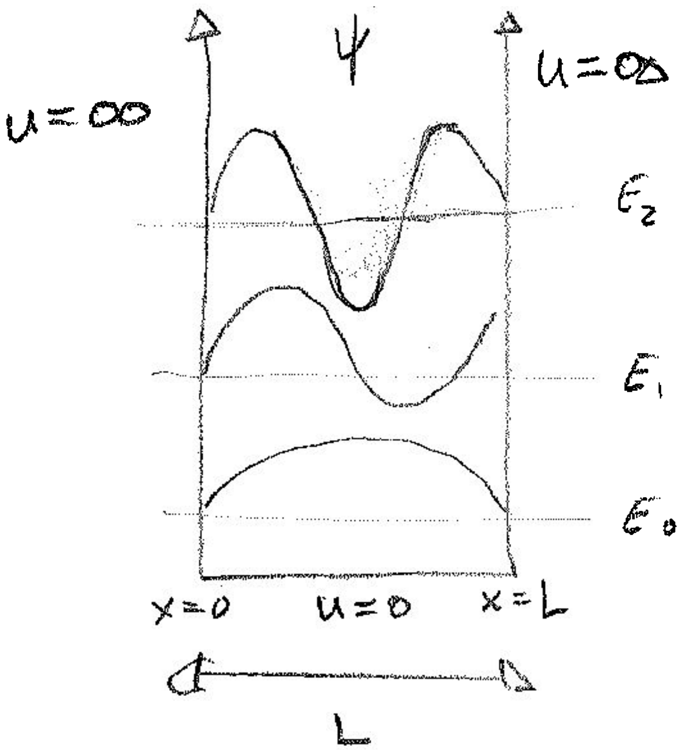
so, ψ^2 should be bigger there

- λ is getting bigger too

(particle can be anywhere here, ψ gives likely spots when we measure

couple of ways to think of this
1) curvature is ∞ goes to the E line right away
2) beyond wall, particle is forbidden, KE is huge right @ wall & particle spends 0 time there

Consider a simple case



KE is increasing

ψ looks like

$A \sin(kx)$

λ has allowed values

$E_0 \hookrightarrow \lambda = 2L$

$E_1 \hookrightarrow \lambda = L$ or $\lambda = \frac{2L}{n}$

$E_3 \hookrightarrow \lambda = \frac{2}{3}L$

so, since $u=0$ $\quad E\psi(x) = -\left(\frac{h}{2\pi}\right)^2 \frac{1}{2m}\frac{\partial^2}{\partial x^2}\psi(x)$

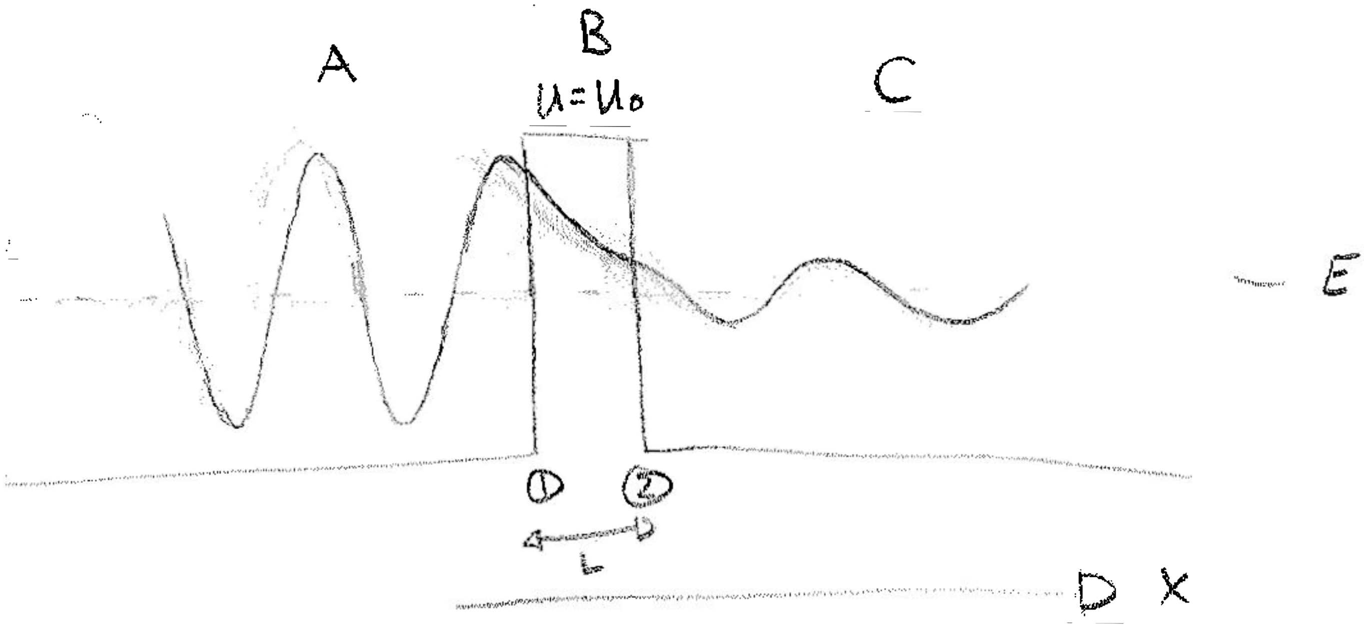
$$E = KE = \left(\frac{n}{2L}\right)^2 \frac{h^2}{2m} \quad n = 1, 2, 3 \text{ etc}$$

For $L = 0.5\,nm$

$E_0 = 1.5\,eV$ or so $= \left(\frac{1}{2(0.5nm)}\right)^2 \frac{(1240\,eV\,nm)^2}{2(511{,}000\,eV)} = 1.50\,eV$

book calls this E_∞

And we'll finish with our tunneling case



We can't do this exactly, it will involve matching ψ_A ψ_B & ψ_C at each boundary ψ'_A ψ'_B & ψ'_C

but we can note that probability that particle in A or C should have a term like $\left(\psi_C/\psi_A\right)^2$ should go like $\left(e^{-a(L)}/e^{-a(0)}\right)$

where a is like k in $\sin(kx)$

and $-\left(\frac{h}{2\pi}\right)^2 \frac{1}{2m} \frac{\partial^2}{\partial x^2} e^{-ax} = (E - U_0)$

or $a^2 = \left(\frac{2\pi}{h}\right)^2 2m(U_0 - E)$

& your book gives the probability of transmission as $T = G e^{-2kL}$

$G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)$

$k = \left(\frac{2\pi}{h}\right)\sqrt{2m(U_0 - E)}$