recall parallel plate $V = Ed = \frac{\sigma}{\varepsilon_0} d = \frac{(q/A)}{\varepsilon_0} d$ (1)

notice $V = q\left(\frac{d}{\varepsilon_0 A}\right)$ or $\left(\frac{\varepsilon_0 A}{d}\right) V = q$

The amount of charge is determined by the physical layout of the conductors & the applied potential diff

cylindrical $V = 2k\lambda \ln(b/a) = \frac{2}{4\pi\varepsilon_0}(q/\ell) \ln(b/a)$

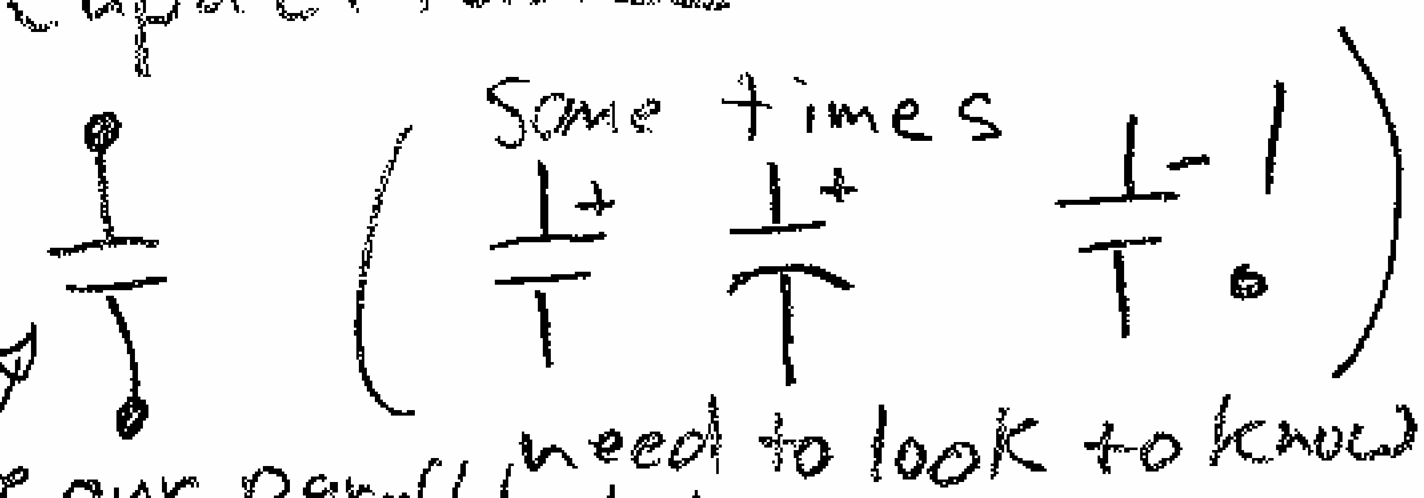
$$\ell\left(\frac{2\pi\varepsilon_0}{\ln(b/a)}\right) V = q$$

There is a short hand used to describe this situation that lets us get away with a lot less work, define

$$CV = q$$

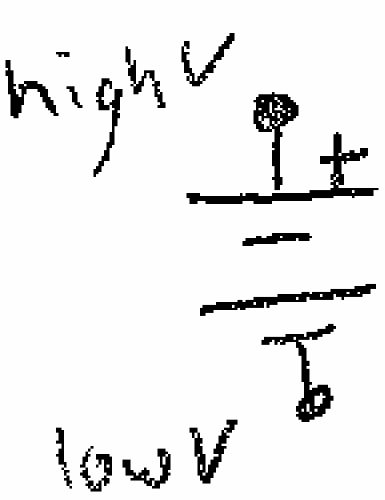
called capacitance ← more = more q for same V

Symbol for capacitor is

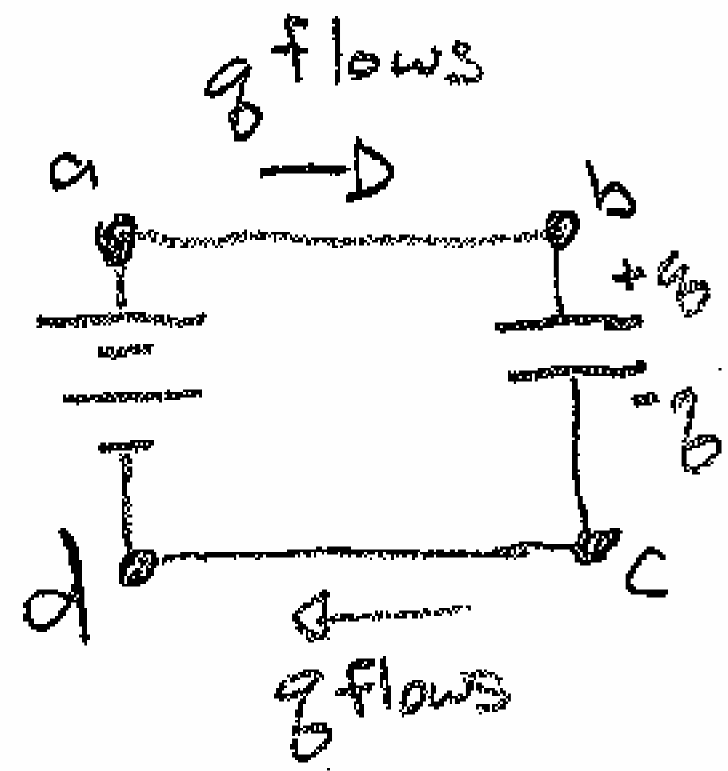


Remember, to create that E field we need a source of potential difference, a battery

An ideal battery looks like



& when we hook up a capacitor



$V_a = V_b$

$V_d = V_c$

$V_a - V_d = V_{batt}$

Unit of Capacitance is called the Farad ②

(After Micheal Faraday - spent lots of time in basements used little math)

$1F = (C/V)$

last friday we had a 30cm long tube with inner radius 15×10^{-6}m & outer radius $5.0\ 10^{-3}$

$$C = \frac{2\pi\,(8.85\times10^{-12}\,F/m)}{\ln\left(5\times10^{-3}/15\times10^{-6}\right)}\,0.30\,m$$

$$= 2.9\times10^{-12}\,F = 2.9\,pF$$

& if we put 2000V across it

$$Q = (2.9\times10^{-12}F)(2000V) = 5.8\times10^{-9}C$$

Notice for a parallel plate capacitor

$C = \frac{\varepsilon_0 A}{d}$ to get lots of C we want a lot of A very close together (d)

if we want to hold lots of Q, we will want ΔV high

Typically, some sort of insulating material, or non-conducting oxide is used to get plates close together, & sometimes lot of area is achieved with many layers or by rolling things

p. Consequences 1) Electric Field is altered by material

- ε_0 replaced by $\kappa\varepsilon_0$ for C, or $\varepsilon_0 \to \varepsilon_0/\kappa$ without dielectric
  dielectric constant

weakens the field so can put more Q for same ΔV

2) maximum E may change too before a "lightning strike"

so $Q = CV$

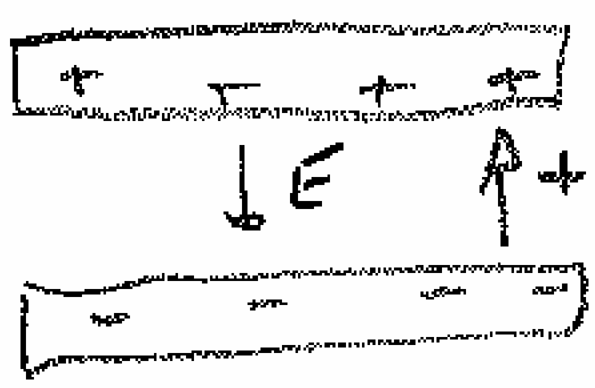
will have some max V etc.

Can change things with dielectric material

$C \Rightarrow \kappa C_0$ ← without κ

which will have a new value for where the max ΔV we can put on it occurs (called dielectric strength)

It takes energy to "charge up" a capacitor



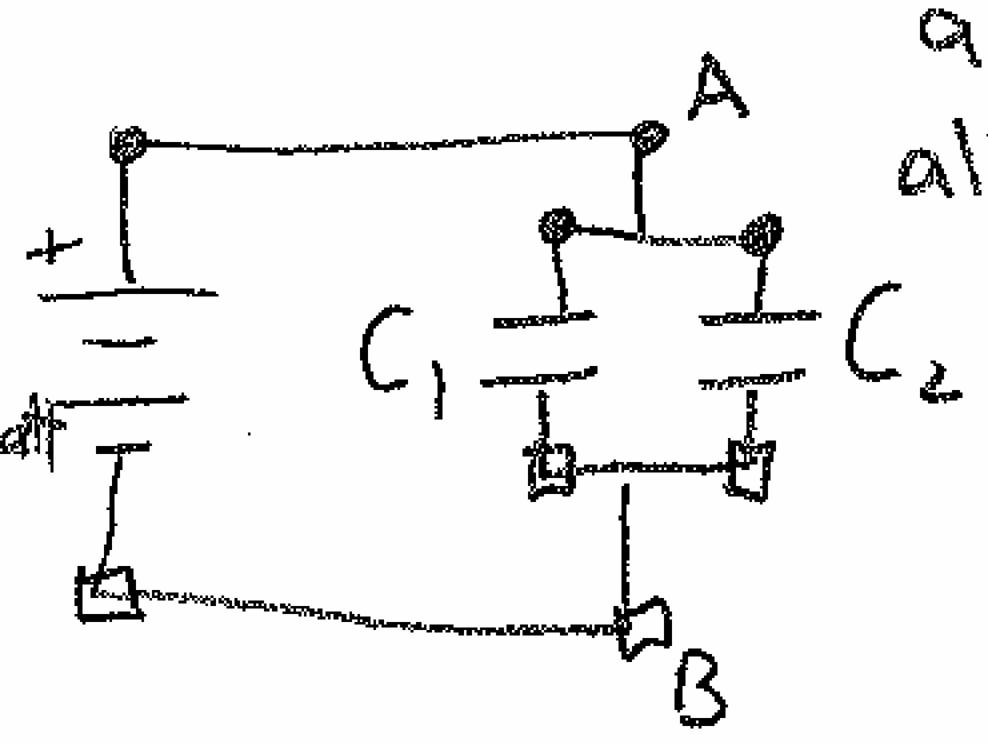
charging up equivalent to moving one charge between plates

$\Delta U = (Q_{moved})\,\Delta V_{plates}$

or $\int dU = \int \left(\frac{Q_{on\,plates}}{C}\right) \underbrace{dQ}_{\text{little bit moves}}$

$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$

Finally, one can use combinations of capacitors to get different values of capacitance. Consider:



all ● @ same V so V_1 across C_1 is
all □ @ same V the same as V_2 across C_2 is the same as V_{batt}

$q_1 = C_1 V_1 = C_1 V_{batt}$
$q_2 = C_2 V_2 = C_2 V_{batt}$

total charge that flowed from batt

$q_1 + q_2 = Q_{batt}$

(4)

so, if I wanted to replace the 2 caps w/ one between A & B I'd need $C_{one} V_{batt} = q_1 + q_2$

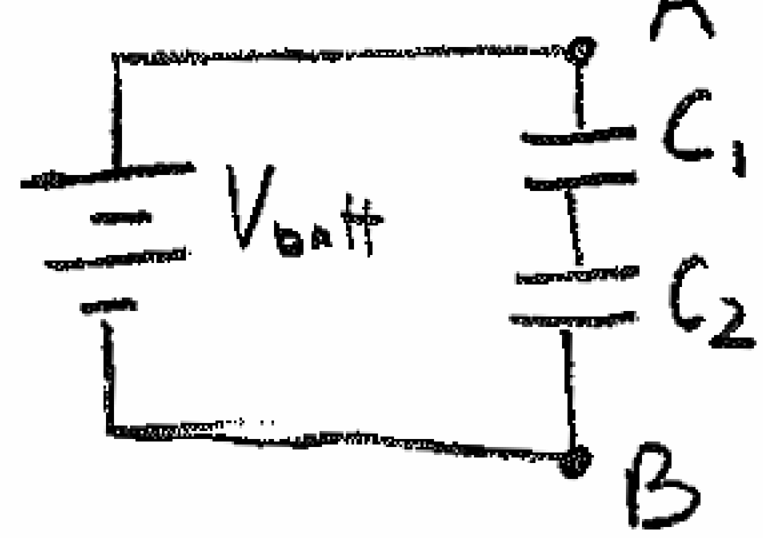
$= C_1 V_{batt} + C_2 V_{batt}$

$= (C_1 + C_2) V_{batt}$

Parallel Combination (always more)

$$C_{equiv} = C_1 + C_2 + \ldots$$

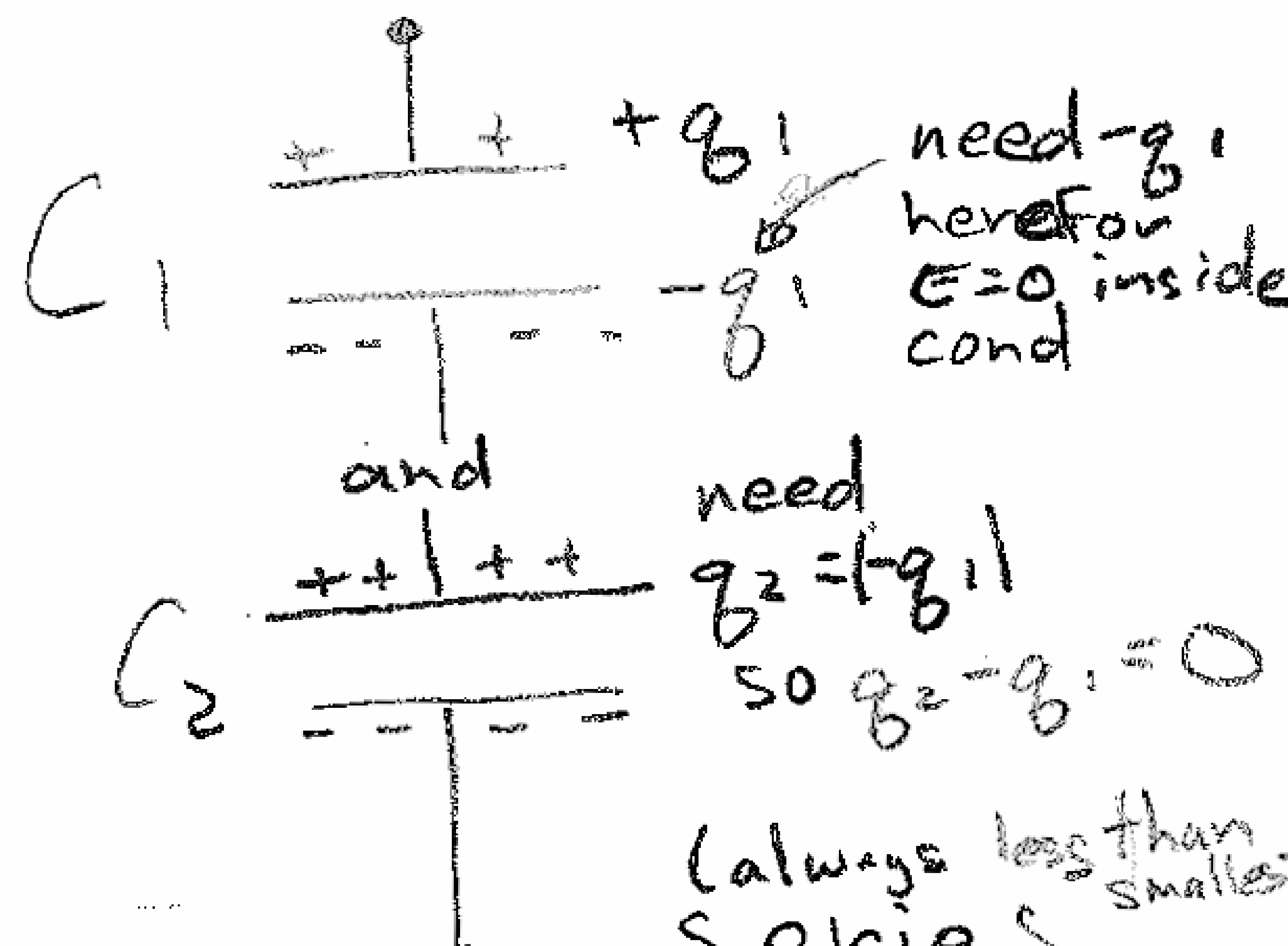
Consider:



Before we attach the battery, C_1 & C_2 are uncharged (example when not on web)

This combo can be dangerous!

when q flows from battery onto C_1



So, in this combination, V_1 & V_2 can be different, but q_1 & q_2 are the same

$$Q_{Batt} = q_1 = q_2 !$$

$$V_{batt} = V_1 + V_2 = \frac{q_1}{C_1} + \frac{q_2}{C_2}$$

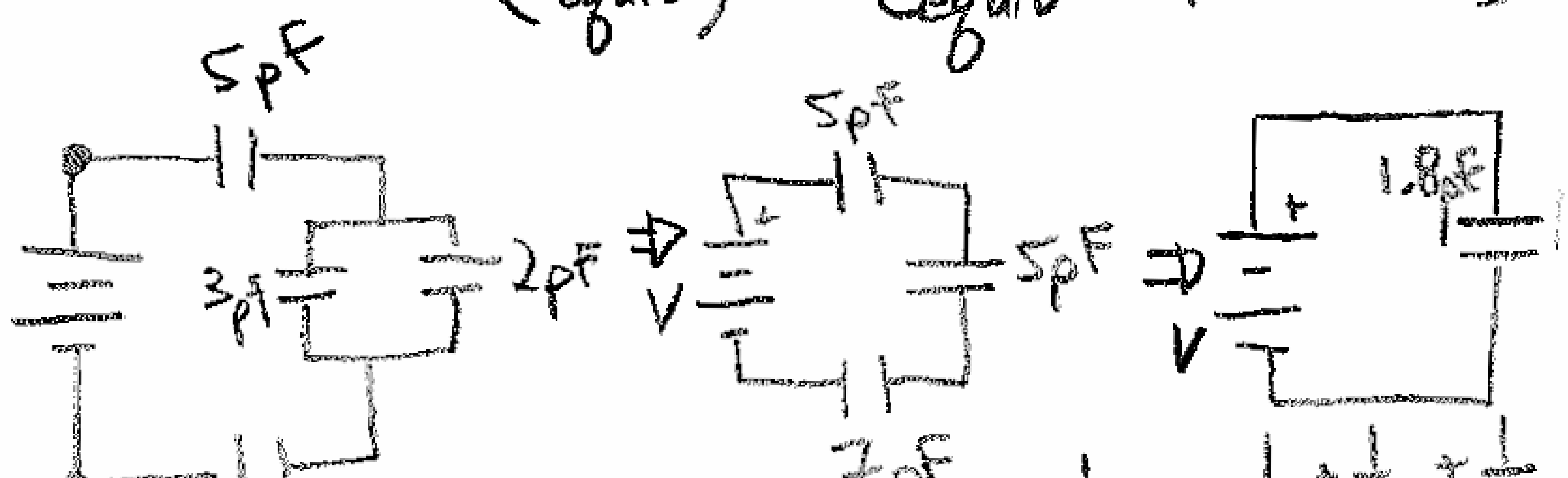
$$= Q_{batt}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = Q_{Batt}\left(\frac{1}{C_{equiv}}\right)$$

(always less than smallest!)

Series

or $\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$

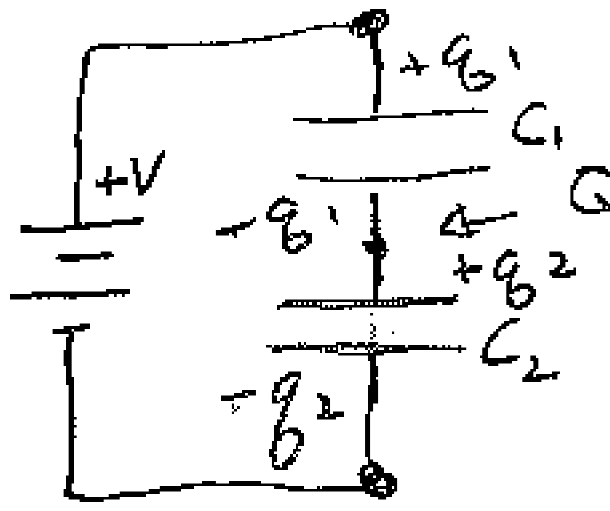
more complicated networks are a of divide conquer



$$\frac{1}{1.8\,pF} = \frac{1}{5pF} + \frac{1}{5pF} + \frac{1}{7pF}$$

non-zero q on series caps

suppose



Q_{tot} here $= (q^2 - q^1)$

know $V = \frac{q'}{C_1} + \frac{q^2}{C_2}$

$$Q_{tot} = q^2 - q^1$$
$$Q_{tot} + q^1 = q^2$$

$$V = \frac{q'}{C_1} + \frac{(Q_{tot} + q_1)}{C_2}$$

$$\left(V - \frac{Q_{tot}}{C_2}\right) = \frac{q'}{C_1} + \frac{q'}{C_2} = q'\left(\frac{C_2 + C_1}{C_2 C_1}\right)$$

$$\frac{C_2 C_1}{C_1 + C_2}\left(V - \frac{Q_{tot}}{C_2}\right) = q'$$

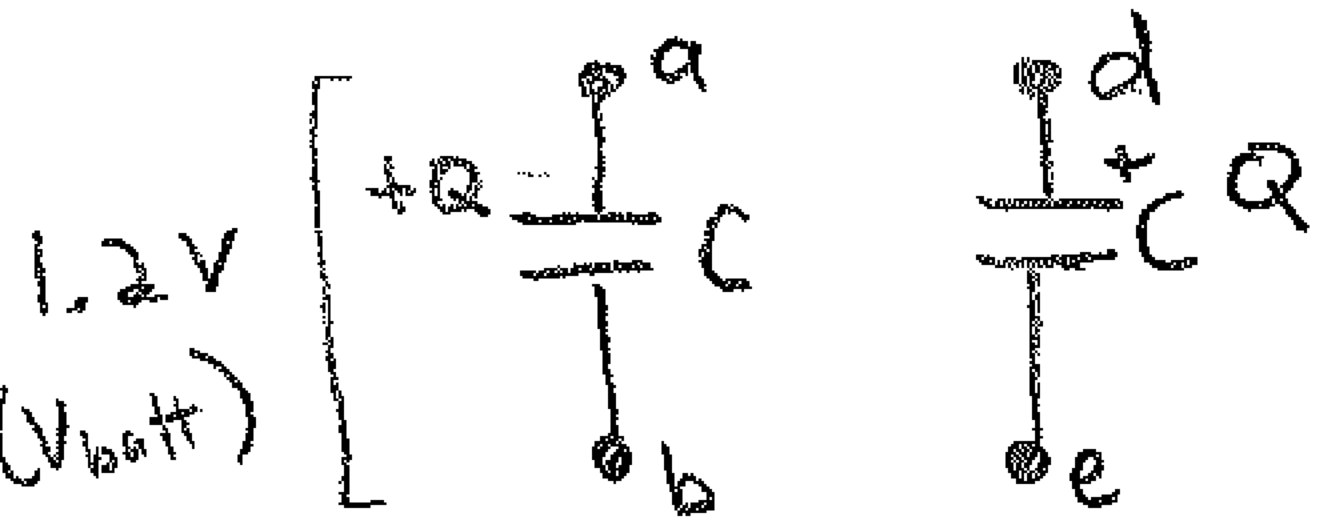
if $Q_{tot} = 0$ (get back C_{eq})

if not .. now $V_1 = q'/C_1 = \frac{C_2}{C_1 + C_2}\left(V - \frac{Q_{tot}}{C_2}\right)$

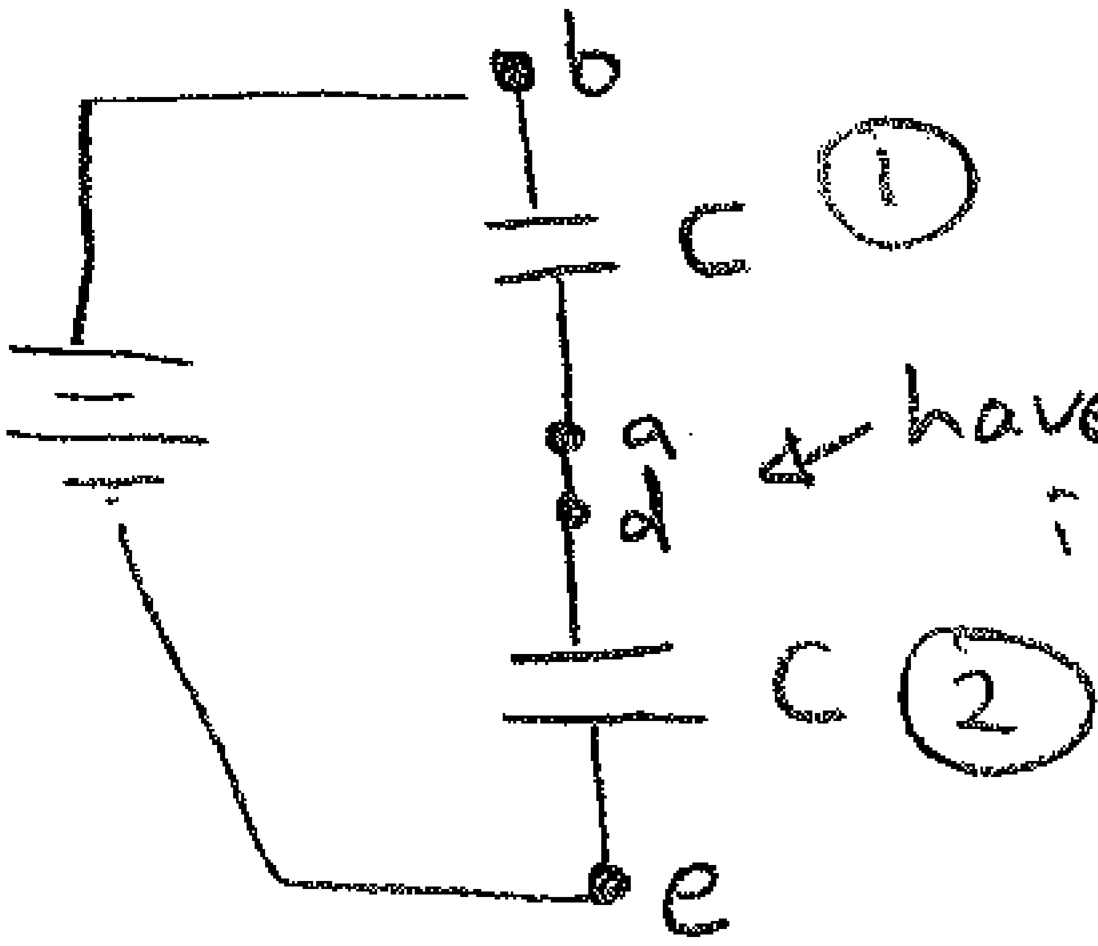
$$V_2 = q^2/C_2 = \frac{1}{C_2}\left(Q_{tot} + \frac{C_2 C_1}{C_1 + C_2}\left(V - \frac{Q_{tot}}{C_2}\right)\right)$$
$$= \frac{Q_{tot}}{C_2} + \left(\frac{C_1}{C_1 + C_2} V\right) - \frac{C_1}{C_1 + C_2}\left(\frac{Q_{tot}}{C_2}\right)$$

if $Q_{tot} < 0$, easy to get more ΔV across C_1 than you expect (& hence less across C_2)

Take the example from class. Consider 2 equal capacitors, both charged up to 1.2V;



now hook them up



& hook 1.2V up battery

← have +2Q stuck in there

$$V_1 = \frac{C_2}{C_1+C_2}\left(V_{batt} - \frac{Q_{tot}}{C_2}\right)$$

$Q_{tot} = 2Q$

$Q = C V_{batt}$

$C_1 = C_2 = C$

$$V_1 = \frac{C}{C+C}\left(V_{batt} - \frac{2CV_{batt}}{C}\right)$$

$$= \frac{1}{2}\left(V_{batt} - 2V_{batt}\right) = -V_{batt}/2 \quad \left(V_2 = \frac{3}{2}V_{bat}\right.$$

If we had connected e to b $Q_{tot} = -2Q$

& $V_1 = \frac{3}{2}V_{batt}$ $\left(V_2 = -\frac{1}{2}V_{batt}\right.$

& we might have just exceeded the dielectric strength!