

As we did for capacitors, we will do for Resistors ①

Capacitors $Q = CV$

Resistors $IR = V$

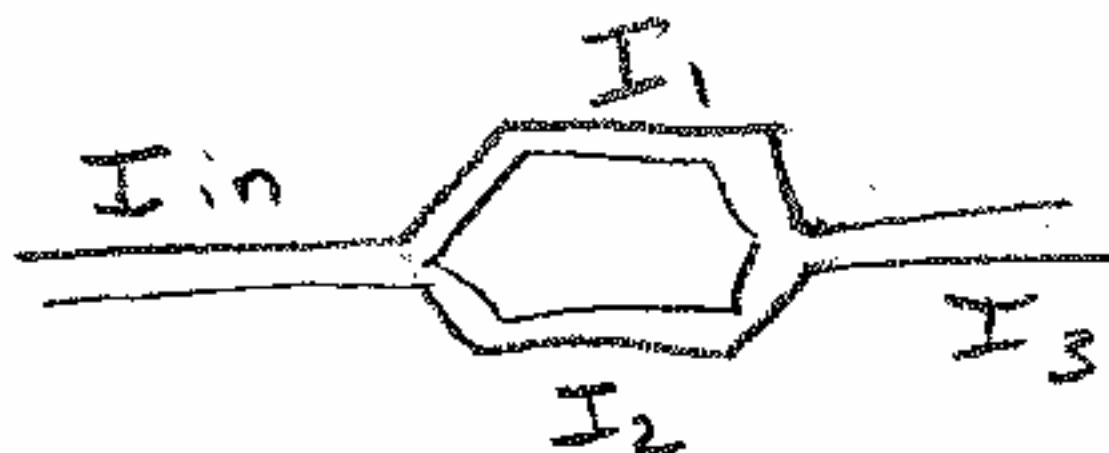
or

$$I = \left(\frac{1}{R}\right)V$$

Expect $\frac{1}{R}$ to behave like C ?

Consider

current conservation = no build up of charge in steady state flow. We've seen charge build up on capacitors & we'll get to dealing with those in a circuit too in a bit.



$$I_3 = I_{in}$$

$$I_{in} = I_1 + I_2$$

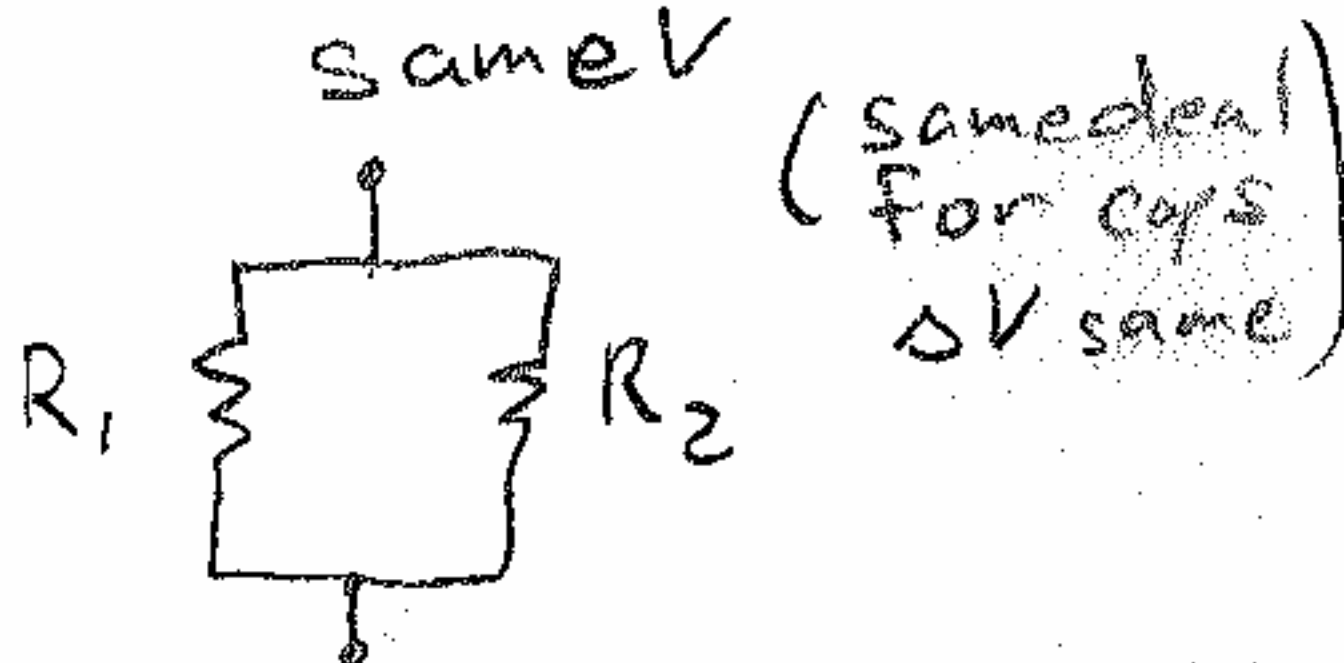
$$I_3 = I_1 + I_2$$



Resistors in series
same I

[capacitors in series]
same Q !

Resistors in Parallel
same V



We've just chosen a different convention

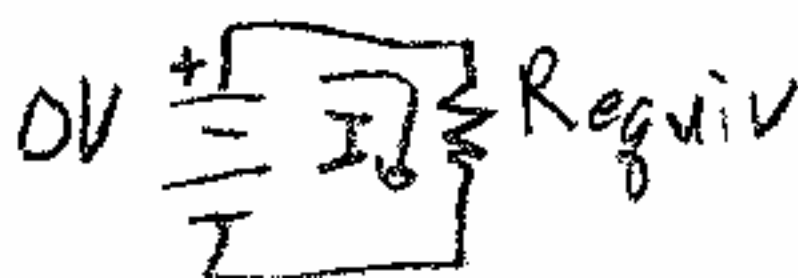
series



$$IR_1 + IR_2 + IR_3 = \Delta V$$

$$I(R_1 + R_2 + R_3) = \Delta V$$

R_{equiv}



parallel



$$\Delta V = I_1 R_1$$

$$\Delta V = I_2 R_2$$

$$I_0 = I_1 + I_2$$

$$I_0 = \frac{\Delta V}{R_{equiv}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$



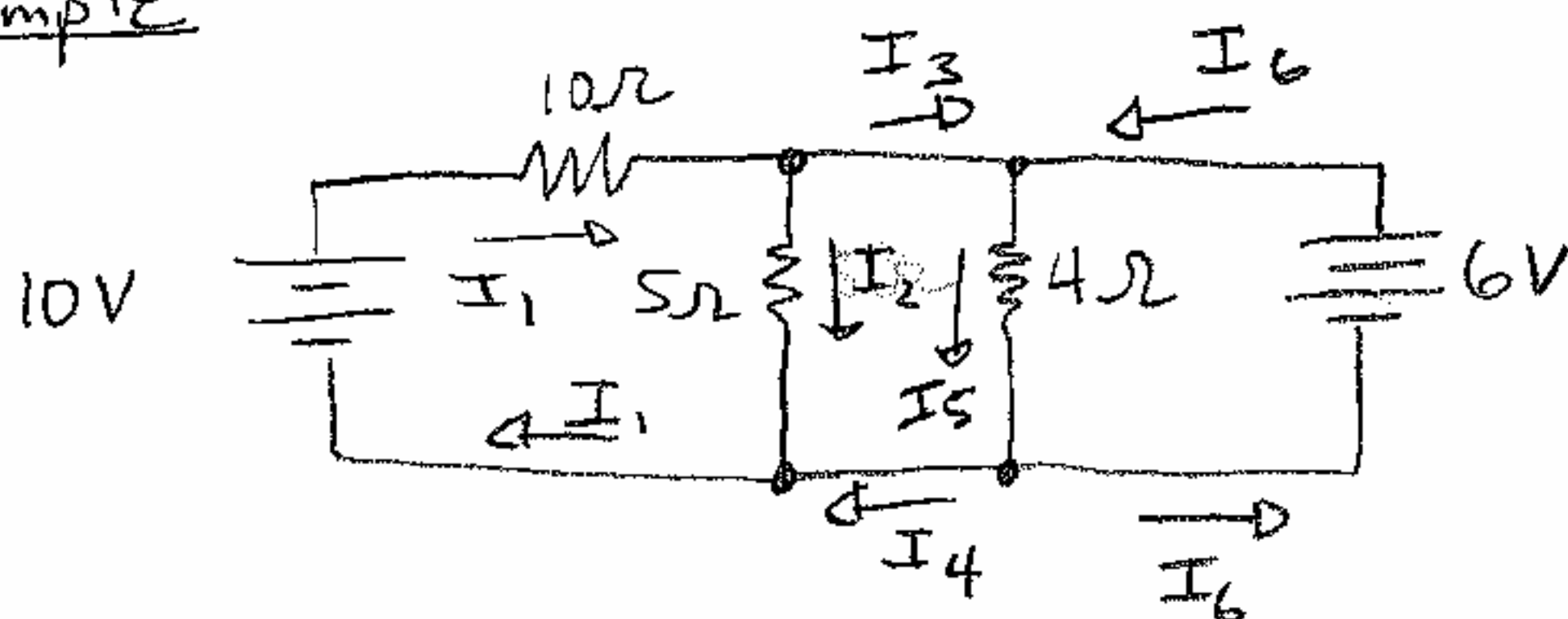
$$\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2}$$

The methods we've been using to solve these problems are sometimes called kirchoff's rules ②

$\sum_{\text{into}} I = 0$ sum of currents into a junction = 0
(or battery!)

$\sum_{\text{loop}} V = 0$ sum of voltages around a loop = 0

Example



Typically with these problems, you get a set of equations & unknowns. (We could make this simpler but we'll do it the hard way (st!))

3 loops

$$\begin{aligned} \textcircled{a} \quad 10V - I_1(10\Omega) - I_2 5\Omega &= 0 \\ \textcircled{b} \quad I_2 5\Omega - I_5 4\Omega &= 0 \\ \textcircled{c} \quad 6V - I_1 10\Omega - 10V &= 0 \end{aligned}$$

make sure you get all the R's & V's

$$\begin{aligned} \textcircled{d} \quad I_6 + I_3 - I_5 &= 0 \\ I_5 - I_6 - I_4 &= 0 \quad (\text{these imply } I_5 = I_4) \\ \textcircled{e} \quad I_1 - I_2 - I_3 &= 0 \\ I_2 + I_4 - I_6 &= 0 \quad (\text{these too!}) \end{aligned}$$

need at least 6 equations to find all the currents

① is easiest $I_1 10\Omega = (10V - 6V) \Rightarrow I_1 = 0.4A$

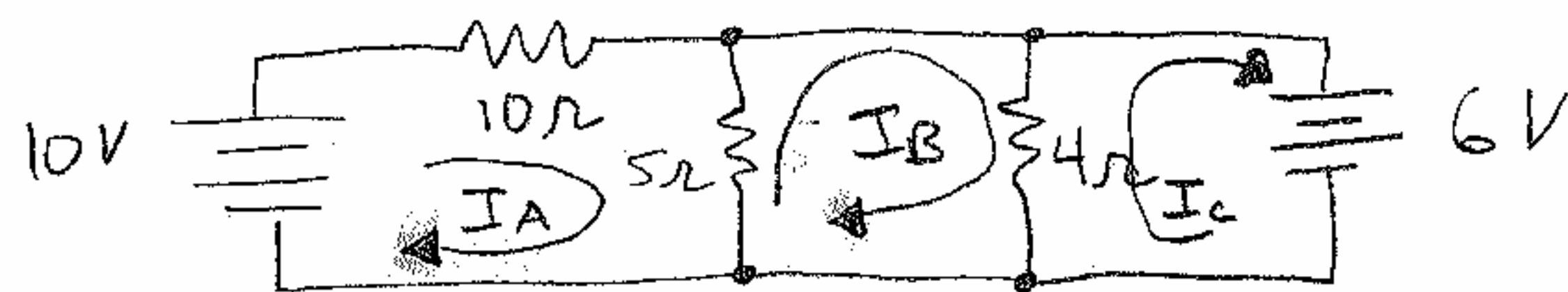
② $10V - 0.4A(10\Omega) - I_2 5\Omega = 0 \Rightarrow 6V = I_2 5\Omega \Rightarrow I_2 = \frac{6}{5}A$

③ $(\frac{6}{5}A) 5\Omega - I_5 4\Omega = 0 \Rightarrow (\frac{6}{5}A) 5\Omega = I_5 4\Omega \Rightarrow I_5 = \frac{6}{4}A = \frac{3}{2}A$

$= 0.4A - 1.2A = -0.8A = I_4$

$= I_5 - I_3 = \frac{3}{2}A + 0.8A = 2.3A$

You can also use a hybrid technique that ignores nodes 3



note: current through 5Ω is $I_A - I_B$
 4Ω is $I_B - I_C$ etc.

I was trained to do the drops around each loop

$$10V - I_A 10\Omega - (I_A - I_B) 5\Omega = 0$$

$$(I_A - I_B) 5\Omega - (I_B - I_C) 4\Omega = 0$$

$$6V + (I_B - I_C) 4\Omega = 0 \Rightarrow (I_B - I_C) = +\frac{3}{2}A \text{ (} I_5 \text{ from previous!)}$$

$$(I_A - I_B) 5\Omega - \left(\frac{3}{2}A\right) 4\Omega = 0$$

$$(I_A - I_B) = -\frac{6}{5}A \text{ (} I_2 \text{ from previous!)}$$

$$10V - I_A 10\Omega - \frac{6}{5}A(5\Omega) = 0 \Rightarrow 4V - I_A 10\Omega = 0, I_A$$

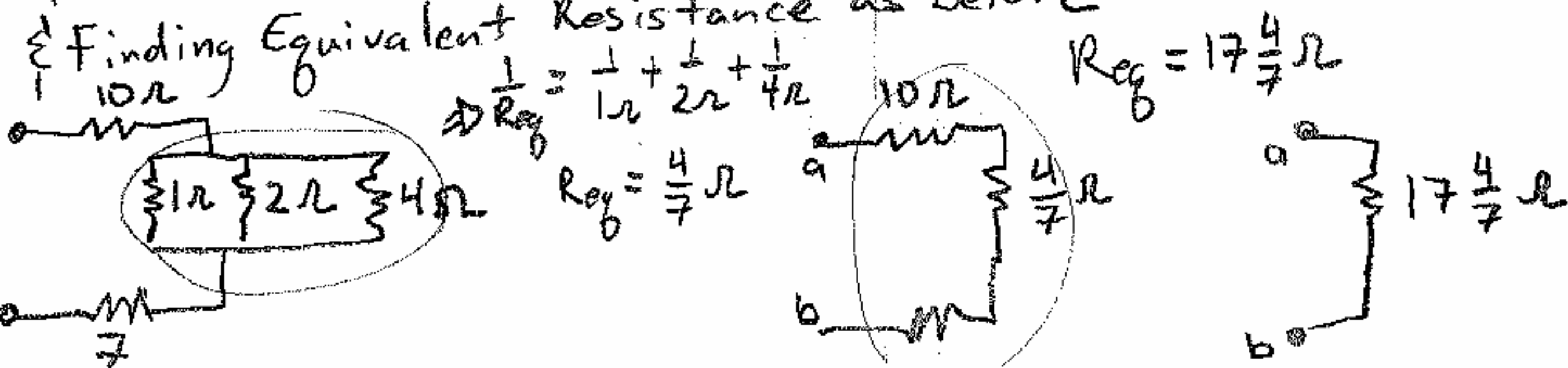
$$0.4A - I_B = 1.2A \quad I_B = -0.8A \text{ (} I_3, I_4 \text{ from before)}$$

$$-0.8A - I_C = 1.5A \quad I_C = -0.8A - 1.5A = -2.3A$$

($-I_6$ from before)

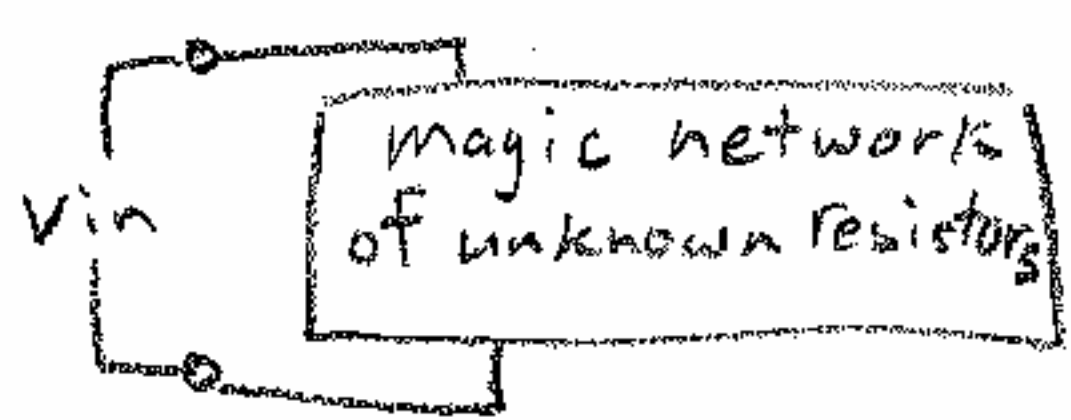
Both techniques are a valid way to approach the problem. Notice how we could have surmised from the start that 6V must be across both the 5Ω & 4Ω .

Finding Equivalent Resistance as before



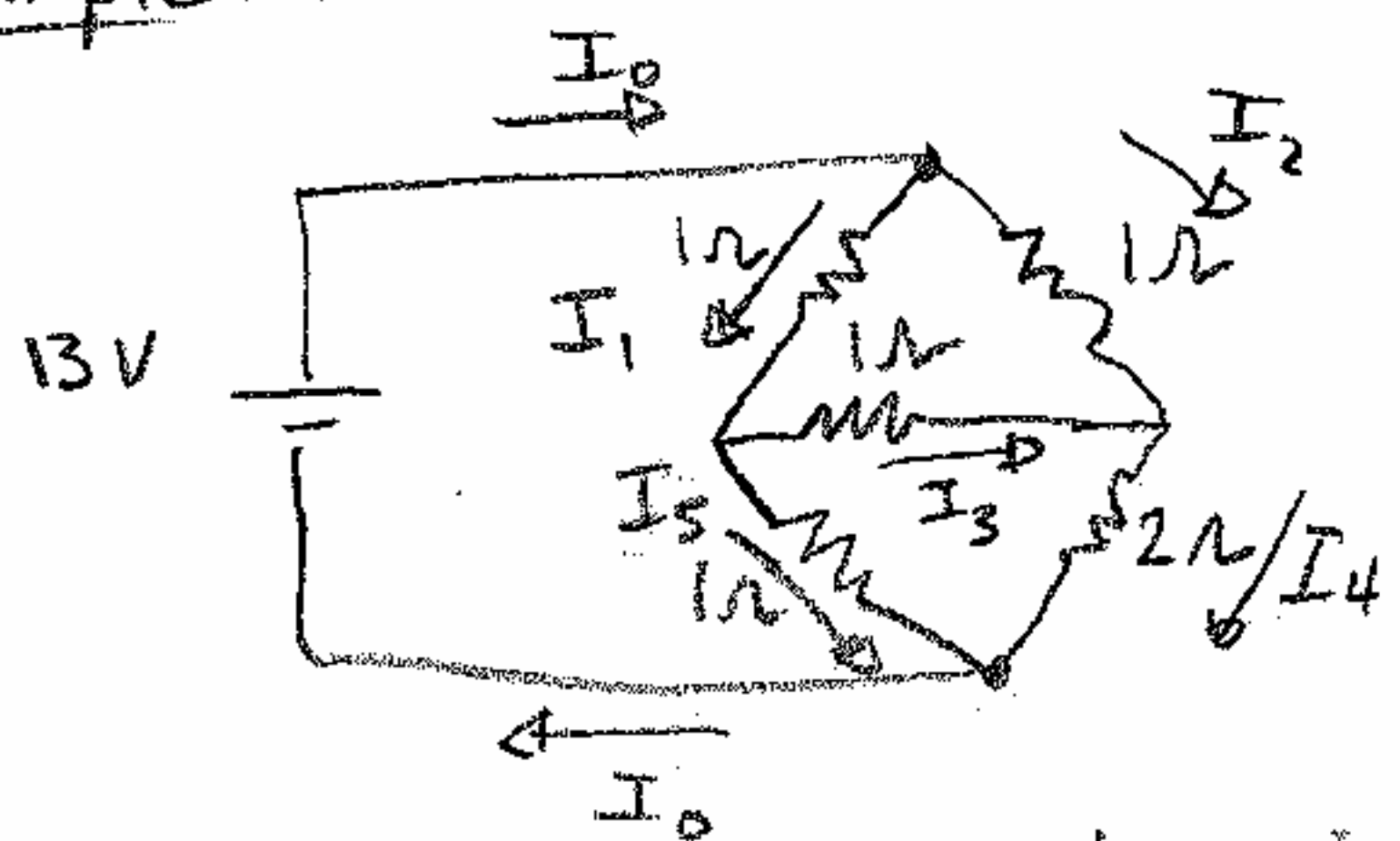
You can also define an equivalent Resistance

$\frac{V_{in}}{I_0}$ consider



& you'll see problems where the "divide & conquer" method fails. (purely resistive networks don't have the same "hidden charge" issue)

Example in book



$R_{eq} = ?$

$$\begin{aligned} I_0 &= I_1 + I_2 \\ I_1 &= I_3 + I_5 \\ I_2 &= I_4 - I_3 \\ I_5 + I_4 &= I_0 \end{aligned}$$

choose enough paths for each resistor

- a) $13V - I_1(1\Omega) - I_5(1\Omega) = 0 = 13V - I_1(1\Omega) - (I_1 - I_2)(1\Omega) = 0$
- b) $13V - I_2(1\Omega) - I_4(2\Omega) = 0 = 13V - I_2(1\Omega) - (I_2 + I_3)2\Omega = 0$
- c) $-I_1(1\Omega) - I_3(1\Omega) + I_2(1\Omega) = 0$

$$13V - I_1(2\Omega) + I_3(1\Omega) = 0 \Rightarrow 13V + I_3(1\Omega) = I_1(2\Omega) \Rightarrow I_1 = 6.5A + \frac{1}{2}I_3$$

$$13V - I_2(3\Omega) - I_3(2\Omega) = 0 \Rightarrow I_2(3\Omega) = 13V - I_3(2\Omega) \Rightarrow I_2 = \frac{13}{3}A - \frac{2}{3}I_3$$

$$-(6.5A + \frac{1}{2}I_3)1\Omega - I_3(1\Omega) + (\frac{13}{3}A - \frac{2}{3}I_3)1\Omega = 0 \Rightarrow -2.17A + 2.17I_3 = 0$$

$$I_3 = -1A, I_1 = 6A, I_2 = \frac{15}{3}A = 5A \quad I_0 = 11A \quad R_{eq} = \frac{13V}{11A} = 1.18\Omega$$

note: If $I_3 = 0$



$$\begin{aligned} I_1 &= I_5 & I_1 R_1 &= I_2 R_3 \\ I_2 &= I_4 & I_1 R_2 &= I_2 R_4 \end{aligned}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{Bridge}$$