

1) Short Answer (50 points)(Show Your Work!)

- a) If a 0.09 kg particle with charge 1.4 C is placed in an electric field, $E=2.3$ (N/C) and then released, how fast is the particle moving after a time $t = 2.1$ sec? (at $t=0$, the particle is at rest)

$$F = qE = ma$$

$$v = at$$

$$= \frac{qE}{m} t = \left(\frac{(1.4 \text{ C})(2.3 \text{ N/C})}{0.09 \text{ kg}} \right) 2.1 \text{ s} = 75.13 \text{ m/s}$$

$v = 75 \text{ m/s}$

(5)

- b) What is the resistance of a copper wire of length 2.1 m, diameter 1.1 mm, and resistivity of 1.69×10^{-8} Ohm-meters. If this is the resistivity at 20 degrees C, what is the resistance at 42 degrees C?

$$R = \rho \frac{L}{A} = \frac{(1.69 \times 10^{-8} \Omega \text{ m})(2.1 \text{ m})}{\pi (0.0011 \text{ m}/2)^2}$$

$R_{20} = 0.037 \Omega$
 $R_{42} = 0.041 \Omega$

(5)
(5)

$$= 0.0373 \Omega$$

$$\frac{R}{A} (\rho - \rho_0) = (\rho_0 \alpha (T - T_0)) \frac{L}{A}$$

$$R - R_0 = R_0 \alpha (T - T_0)$$

$$R = R_0 + R_0 \alpha (T - T_0) = 0.0373 \Omega + 0.0373 \Omega (4.3 \times 10^{-3}) (22^\circ \text{C}) = 0.0408 \Omega$$

- c) A circuit contains an unknown resistor and a charged 15.0 mF capacitor. If the voltage across the 15.0 mF capacitor decreases from 5.5 V at $t = 0.0$ sec to 4.5 V at 1.5 sec when a switch is closed at $t = 0.0$ sec, how much current is flowing through the resistor at $t = 1.5$ sec? Discharging

$I =$

(5)

$$Q = Q_0 e^{-t/RC}$$

$$\frac{Q}{C} = \frac{Q_0}{C} e^{-t/RC}$$

$$V = V_0 e^{-t/RC}$$

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}$$

$$R = -\frac{t}{C \ln\left(\frac{V}{V_0}\right)} = -\frac{1.5 \text{ s}}{0.015 \text{ F} \ln\left(\frac{4.5}{5.5}\right)}$$

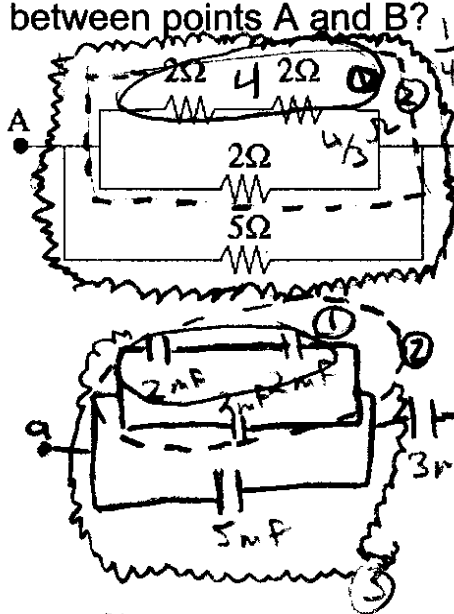
$$= 498.3 \Omega$$

$$I = V/R = 4.5 \text{ V} / 498.3 \Omega$$

$$= 9.03 \times 10^{-3} \text{ A}$$

1) Short Answer (Show Your Work!)

d) Calculate the resistance between points A and B. Now, if I replace each resistor with a capacitor such that 1 Ohm = 1 mF (For example, a 3 Ohm resistor becomes a 3mF capacitor), what is the capacitance between points A and B?



Resistor network analysis:

$$\frac{1}{4\Omega} + \frac{1}{2\Omega} = \frac{3}{4\Omega} \quad R_{\text{eq}} = \frac{4}{3}\Omega$$

$$\frac{1}{5\Omega} + \frac{3}{4\Omega} = \frac{19}{20\Omega}$$

$$R_{\text{eq}} = \frac{20}{19}\Omega \quad R_{AB} = \frac{20}{19}\Omega + 3\Omega = 4.05\Omega$$

Capacitor network analysis (1 Ohm = 1 mF):

$$C_{\text{eq}} = 1\text{mF} \text{ (from } (\frac{1}{2} + \frac{1}{2})^{-1} \text{)}$$

$$C_{\text{eq}} = 3\text{mF} \text{ (1mF + 2mF)}$$

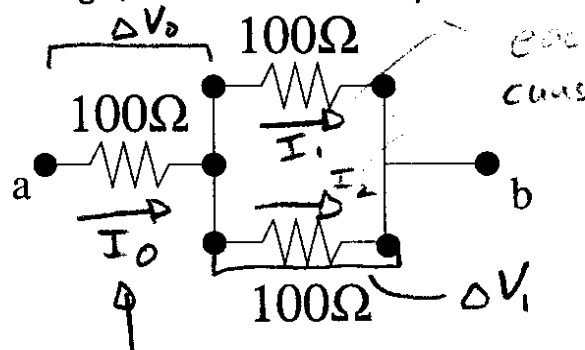
$$C_{\text{eq}} = 3\text{mF} + 5\text{mF} = 8\text{mF}$$

$$\frac{1}{C_{AB}} = \frac{1}{8\text{mF}} + \frac{1}{3\text{mF}} = \frac{3}{24\text{mF}} + \frac{8}{24\text{mF}} = \frac{11}{24\text{mF}}$$

$$C_{AB} = 2.18\text{mF}$$

$R_{AB} = 4\Omega$
 $C_{AB} = 2.18\text{mF}$

e) Three 100 Ohm resistors are connected as shown below. If the maximum power any one of the resistors can dissipate is 25.0W, what is the maximum voltage that can be applied between the terminals a and b? At this maximum voltage, what is the total power dissipated by all three resistors? (10pts)



each $\frac{1}{2} I_0$ cause R's the same

$V_{ab} = 75.0\text{V}$
 $P_{\text{tot}} = 37.5\text{W}$

most current through $P = I^2 R = 25.0\text{W}$

check V^2/R

$$I_0 = \sqrt{\frac{25.0\text{W}}{100\Omega}} = 0.5\text{A}$$

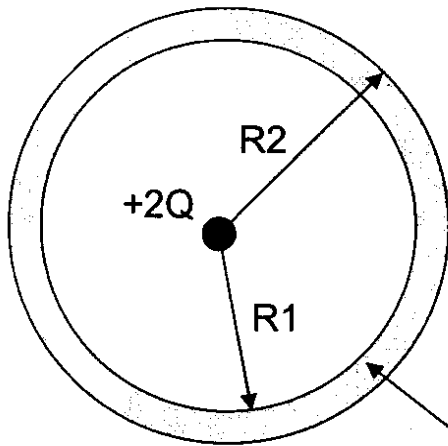
$$\Delta V_0 = (0.5\text{A})(100\Omega) = 50\text{V}$$

$$\Delta V_1 = \left(\frac{0.5\text{A}}{2}\right) 100\Omega = 25\text{V}$$

$$P_{\text{due to } V_1} = \frac{V_1^2}{R} + \frac{V_1^2}{R} = 2 \frac{(25\text{V})^2}{100\Omega} = 12.5\text{W}$$

1) Short Answer (Show Your Work! Explain your reasoning!)

- f) A conducting spherical shell of inner radius R_1 and outer radius R_2 has a charge of $+2Q$ placed at its center and a net charge of $-3Q$ placed on it (that is, the sum of the inner surface charge and the outer surface charge is $-3Q$). What is the electric field inside the shell ($r < R_1$), inside the conductor ($R_1 < r < R_2$), and outside the shell ($r > R_2$)? Also, what is the Electric potential inside the shell ($r < R_1$)? (Hint: Remember what E is inside a conductor, that E is a result of total enclosed charge, and that the Electric Potential is 0 at $r = \text{infinity}$.)



$Q_{\text{ins}} = +2Q$
 symmetry is spherical for whole problem
 at electrostatic equilibrium, must be true
 $-3Q$ net charge on the conductor

$$E(\text{inside}) = \frac{k(2Q)}{r} \quad (2)$$

$$E(\text{conductor}) = 0 \quad (2)$$

$$E(\text{outside}) = \frac{k(-Q)}{r} \quad (3)$$

$$V(\text{inside}) = \frac{2kQ}{r} - \frac{2kQ}{R_1} - \frac{kQ}{R_2} \quad (3)$$

$\rightarrow Q_{\text{inside Gaussian Surf}} = -3Q + 2Q = -Q$

$\rightarrow \text{Know } V_{\infty} = 0$

$V_{\text{in cond}} = \text{constant}$

$V_{\text{outside due to } E_{\text{outside}}} = -\frac{kQ}{r}$

@ cond Surf $V = -\frac{kQ}{R_2}$ & inside cond

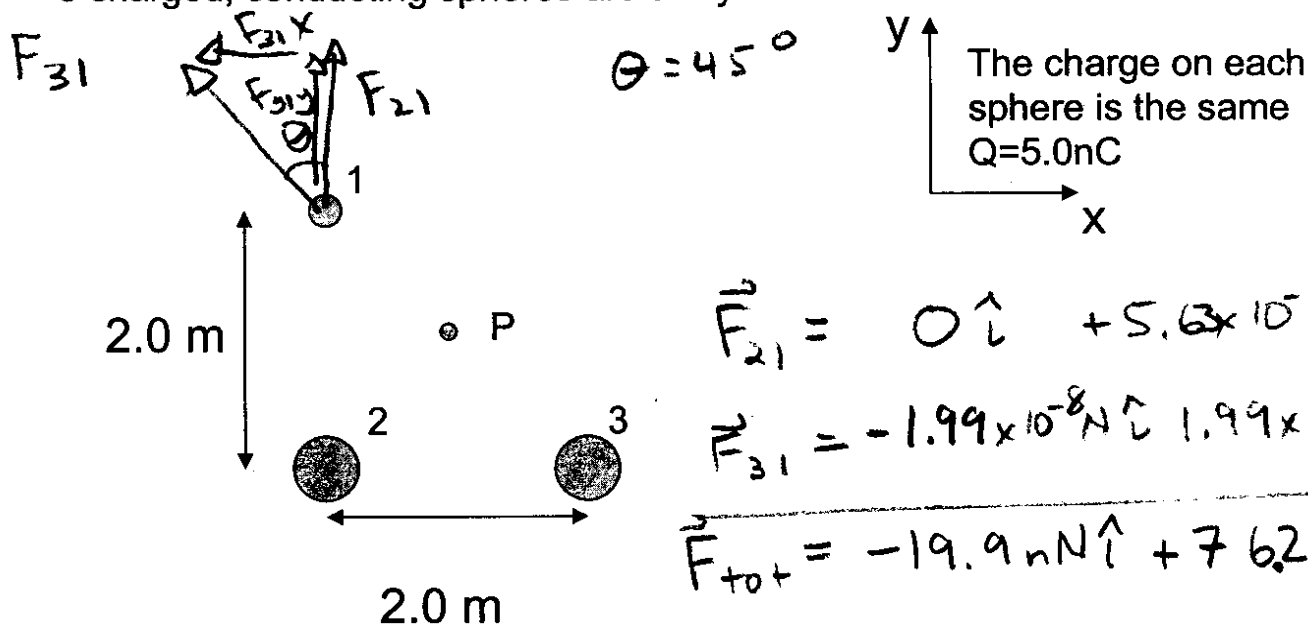
Inside has $\frac{k(2Q)}{r}$ plus something to give $V = -\frac{kQ}{R_2}$ @ $r = R_1$

or $\frac{k(2Q)}{R_1} + V_{\text{need}} = -\frac{kQ}{R_2}$

$V_{\text{need}} = -\frac{kQ}{R_2} - \frac{k(2Q)}{R_1}$

2) Force and Potential (20 points Show Work!)

3 charged, conducting spheres are arrayed as shown below:



$$\vec{F}_{21} = 0 \hat{i} + 5.6 \times 10^{-8} \text{ N} \hat{j}$$

$$\vec{F}_{31} = -1.99 \times 10^{-8} \text{ N} \hat{i} + 1.99 \times 10^{-8} \text{ N} \hat{j}$$

$$\vec{F}_{\text{tot}} = -19.9 \text{ nN} \hat{i} + 7.62 \text{ nN} \hat{j}$$

What is the force on the little sphere (1), due to the 2 big spheres (2 & 3)?

$$\vec{F}_{21} = 0 \hat{i} + \frac{(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(5.0 \times 10^{-9} \text{ C})^2}{(2.0 \text{ m})^2} \hat{j}$$

$$F_x = -19.9 \text{ nN} \quad (5)$$

$$F_y = 7.62 \text{ nN} \quad (5)$$

$$\vec{F}_{31} = \left[\frac{(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(5.0 \times 10^{-9} \text{ C})^2}{((2 \text{ m})^2 + (2 \text{ m})^2)^{3/2}} \right] (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

What is the Electric potential at a point P, located midway between sphere (1) and sphere (3)? How much work would have to be done to bring in a charge of 5.0 nC from infinity to point P?

V is just a sum of the 3
all have equal distance

$$V \text{ at P} = 95.5 \text{ Nm/C} \quad (5)$$

$$\text{Work} = 477 \text{ nJ} \quad (5)$$

$$V = 3 \frac{kQ}{\sqrt{(1 \text{ m})^2 + (1 \text{ m})^2}} = 3 \frac{(9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(5 \times 10^{-9} \text{ C})}{\sqrt{2} \text{ m}} = 95.46 \text{ N/C}$$

Bring in another positive charge

- charge repelled by others, need to do positive work on charge

$$U_{\infty} = 0 \quad U_P = qV = (5.0 \times 10^{-9} \text{ C}) 95.46 \frac{\text{Nm}}{\text{C}} = 4.77 \times 10^{-7} \text{ Nm}$$