

Equations

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \quad (\text{Point Charge}) \quad \vec{E} = \vec{F}/q_{\text{test}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r_{12}^2} \hat{r}_{12}$$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint \vec{E} \cdot \hat{n} dA = \frac{q_{\text{enclosed}}}{\epsilon_0}, \quad \Phi = E A \quad (\text{special cases})$$

$$(\text{Sphere}) A = 4\pi r^2, (\text{Cylinder}) A = 2\pi r L, (\text{Sheet}) A = L^2 + L^2 (\text{two sides})$$

$$\vec{F} = m\vec{a}, \quad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2, \quad v_x = v_{0x} + a_x t$$

$$-\Delta U = W = \int \vec{F} \cdot d\vec{s}, \quad \text{Kinetic Energy} = \frac{1}{2}mv^2$$

$$\vec{F}_{\text{total}} = \sum_i \vec{F}_i \quad \vec{E}_{\text{total}} = \sum_i \vec{E}_i \quad V_{\text{total}} = \sum_i V_i$$

$$Q = CV, \quad U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}, \quad C = \kappa C_0, \quad E = \frac{E_0}{\kappa}, \quad \epsilon = \kappa\epsilon_0$$

$$(V_F - V_I) = - \int_I^F \vec{E} \cdot d\vec{s}, \quad E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z},$$

$$C = \frac{\epsilon_0 A}{d} \quad \text{Parallel Plate}$$

$$C = \frac{2\pi\epsilon_0 L}{\ln(b/a)} \quad \text{Cylindrical}$$

$$C = 4\pi\epsilon_0 \frac{ab}{a-b} \quad \text{Spherical}$$

$$(\text{series}) 1/C_{\text{equiv}} = 1/C_1 + 1/C_2 + \dots, \quad (\text{parallel}) C_{\text{equiv}} = C_1 + C_2 + \dots$$

Constants

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

$$e = 1.60 \times 10^{-19} C$$

$$M_{\text{electron}} = 9.11 \times 10^{-31} kg$$

$$M_{\text{proton}} = 1.67 \times 10^{-27} kg$$

$$k = \frac{1}{4\pi\epsilon_0}$$

Name: Key

I.D.#: _____

Please circle your section:

Section 1

Section 2

Section 3

Section 4

Instructions

This is a one hour, closed book examination. Put answers in the boxes provided, or circle the best answer. If numerical answers are needed, you must include units. If required, any work needed to justify the answer must be shown in the space provided. A correct answer without the necessary justifying work may not receive any credit. You may use the formula sheet on the back of the exam.

Total point scores for each problem will appear in the table below and in () beside each problem number. Do what is easiest first. AVOID glancing at anyone else's paper during the exam! The honor code is in effect.

You must select 4 problems from the choice of 5 given. I will try to grade all 5 if you do them, and give you the best 4 of 5, but I may run out of steam. There are no guarantees! I may just grade the first 4 you did and calculate your score from them.

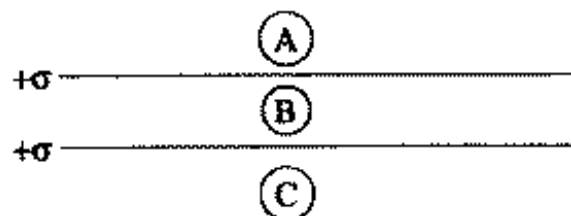
(Grade based on the 4 problems you chose.)

Point Tallies for the Exam Problems

Problem	Description	Max Score	Actual Score
1	Short Answer	25	
2	Various Subjects	25	
3	Electric Fields	25	
4	Capacitors	25	
5	Electric Potential	25	
Total	4 out of the 5	100	

1. Short Answer (25 points total)

- a) (4 points) Describe, briefly, the electric field at the points indicated on the figure to the right. Each horizontal line represents an infinite sheet of charge with identical positive charge density.



(A) ↑ 2 × 1 sheet

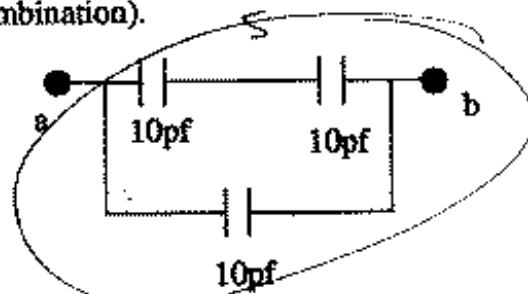
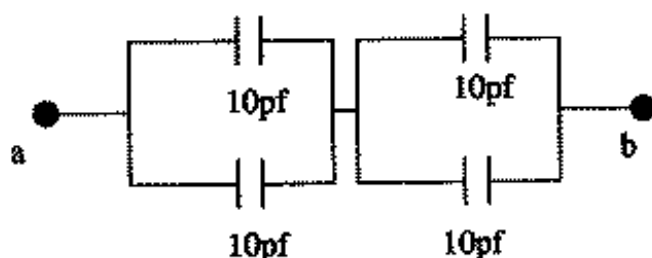
(B) 0

(C) ↓ 2 × 1 sheet

- b) (2 points) A constant Electric field is pointing in the direction indicated in the figure. Please draw an arrow to indicate the direction of INCREASING electric potential (V).



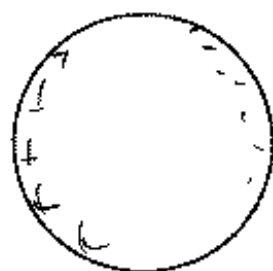
- c) (2 points) Circle the combination of capacitors with the largest capacitance between the two ends (labeled a & b for each combination).



$$\frac{20}{11} + \frac{20}{11} = 10$$

$$5 + 10 = 15$$

- d) (3 points) If I held a positively charged plastic wand next to a soda can that is lying on its side on a flat table, and the can is uncharged, would the soda can be attracted to, or repelled by, the soda can? Also, indicate on the figure, the arrangement of the free charge in the soda can after the charged wand is brought near.



Attracted

the free charge in the soda can after the charged wand is brought near.

(hint: we did this in class)

1. Short Answer (Continued)

- e) (2 points) At Electrostatic Equilibrium (when charges stop moving around), the electric field inside a conductor is: _____

SMALL

LARGE

- f) (2 points) If I lift my pen up, I'm _____ the gravitational potential energy of the pen.

INCREASING

DECREASING

- g) (2 points) The Electric flux through a closed (or complete) surface (sometimes called a closed, Gaussian surface) is proportional to the charge _____ the closed surface.

INSIDE

OUTSIDE

- h) (2 points) The Electric Potential (V) inside a conductor at Electrostatic Equilibrium is best described as:

ZERO

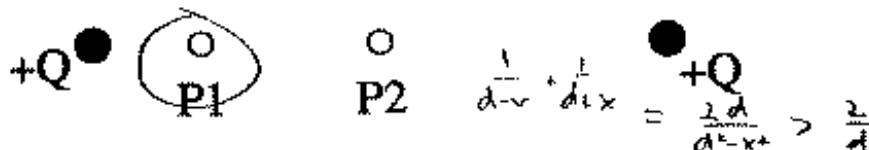
CONSTANT

- i) (2 points) If I insert a dielectric material of dielectric constant $K > 1$ between the plates of a capacitor that had only air between the plates before (i.e. $K = 1$ before, $K > 1$ after), the capacitance of the capacitor _____

INCREASES

DECREASES

- j) (2 points) Two charges, each of value $+Q$ are arranged as shown in the figure below. At which location, P1 or P2, is the electric potential the largest? (See figure below)



- k) (2 points) If 2 conducting spheres are connected by a wire so that they are at the same potential, which of the spheres would you expect to have the greatest electric field at its surface? (assume there is some charge on the combination)

THE SMALLER SPHERE

THE BIGGER SPHERE

2) Various Subjects (5 points each)

(Circle the Correct Response, Show work for Partial Credit):

1) A $+4.0 \text{ C}$ (positive) charge and a -2.0 C (negative) charge are located 0.50 m apart from each other. What is the force on the $+4.0 \text{ C}$ (positive) charge due to the -2.0 C (negative) charge? (5 pts.)

- I) $7.2 \times 10^{10} \text{ N}$ and repulsive
- II) $7.2 \times 10^{10} \text{ N}$ and attractive
- III) $1.4 \times 10^{11} \text{ N}$ and repulsive
- IV) $2.9 \times 10^{11} \text{ N}$ and attractive ✓
- V) $2.9 \times 10^{11} \text{ N}$ and repulsive

$$\frac{9 \times 10^9 (8)}{(0.5)^2}$$

2) Two widely separated spheres, one with a radius of 1.0 cm and charge on its outer surface $+q$ ($Q_{\text{small}} = +q$), and one sphere with a radius of 10.0 cm and no charge on its outer surface ($Q_{\text{big}} = 0$), are connected by a thin wire, i.e. the wire forces them to be at the same potential. The charge on the big ($r = 10.0 \text{ cm}$) sphere after the spheres are connected by a thin wire is: (5 pts.)

- I) $Q_{\text{big}} = 0.50 q$
- II) $Q_{\text{big}} = 0.01 q$
- III) $Q_{\text{big}} = 1.0 q$
- IV) $Q_{\text{big}} = 0.91 q$ ✓
- V) $Q_{\text{big}} = 1.1 q$

$$\frac{Q_b}{r_b} = \frac{Q_s}{r_s} \quad Q_b + Q_s = +q$$

$$Q_b \left(\frac{1}{r_b} + \frac{1}{r_s} \right) = \frac{q}{r_s} \quad Q_b = \frac{q}{1.0} \left(\frac{1}{1} + \frac{1}{10} \right) = .91 q$$

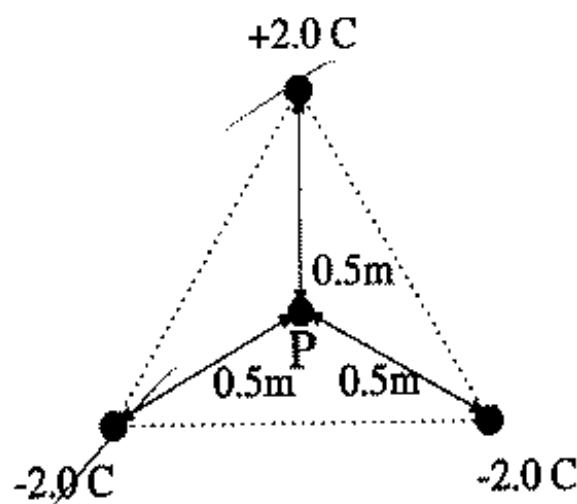
3) If a 10.0 kg particle with charge 3.0 C is placed in an electric field, $\vec{E} = 2.0 \text{ (N/C)} \hat{x}$, and then released, how fast is the particle moving after 5.0 s ? (at $t = 0$ the particle is at rest) (5 pts.)

- I) $v = 3.0 \text{ m/s}$ ✓
- II) $v = 9.0 \text{ m/s}$
- III) $v = 15 \text{ m/s}$
- IV) $v = 33 \text{ m/s}$
- V) $v = 0.33 \text{ m/s}$

$$a = \frac{F}{m} = \frac{(3.0 \text{ C})(2.0 \text{ N/C})}{10 \text{ kg}} = .6 \text{ m/s}^2$$

$$v = at = .6 \text{ m/s}^2 \times 5 \text{ s} = 3 \text{ m/s}$$

2) Various Subjects (Continued)



4) What is the electric potential at point P for the three charges in the figure, each of which is placed at one point of an equilateral triangle? (5 pts.)

- I) $V = 0.0 \text{ Volts}$
- II) $V = -3.6 \times 10^{10} \text{ Volts}$
- III) $V = -7.2 \times 10^{10} \text{ Volts}$
- IV) $V = -1.1 \times 10^{11} \text{ Volts}$
- V) $V = -1.8 \times 10^{10} \text{ Volts}$

$$\frac{-k(2C)}{.5m} = \frac{(9 \times 10^9)(2)}{.5} = 3.6 \times 10^{10} V$$

5) What is the gap between the plates of a parallel plate capacitor in air if the Area of the gap is 0.5 m^2 and the capacitance is $2.5 \times 10^{-10} \text{ F}$? (5 pts.)

- I) $1.1 \times 10^{-23} \text{ cm}$
- II) 0.58 cm
- III) 0.14 cm
- IV) 1.8 cm
- V) 7.1 cm

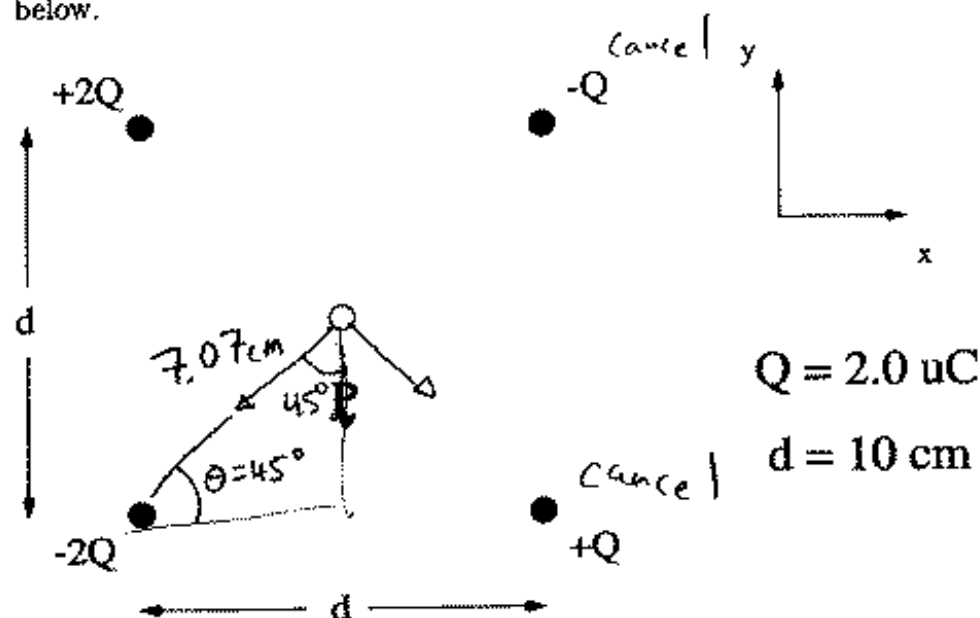
$$C = \frac{\epsilon_0 A}{d}$$

$$d = \frac{\epsilon_0 A}{C}$$

$$= \frac{(8.85 \times 10^{-12} \text{ F/m})(.5 \text{ m}^2)}{2.5 \times 10^{-10} \text{ F}} = 1.78 \times 10^{-2} \text{ m}$$

3) Electric Field Vectors (25 points total)

Consider the charges arranged at the corners of a square as shown in the figure below.



a) Calculate the x and y components of the electric field at the point indicated by P.

$$2 \times \text{mag from one in } Y$$

$$|E| = \frac{k(2.0 \mu\text{C})}{((d/2)^2 + (d/2)^2)} \frac{1}{\sqrt{2}}$$

$$\begin{cases} E_x = 0 \text{ (From Symmetry)} & (10) \\ E_y = 5.1 \times 10^6 \text{ N/C} & (10) \end{cases}$$

$$(E_{\text{tot}}) = \frac{(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2})(2.0 \times 10^{-6} \text{ C})}{(.0707 \text{ m})^2} (2) \cos 45^\circ = 5.09 \times 10^6 \text{ N/C}$$

b) Suppose we have a cube that encloses all 4 of the charges. (I.e. we put the charges in a box.) What is the total electric flux through the surface of the entire cube?

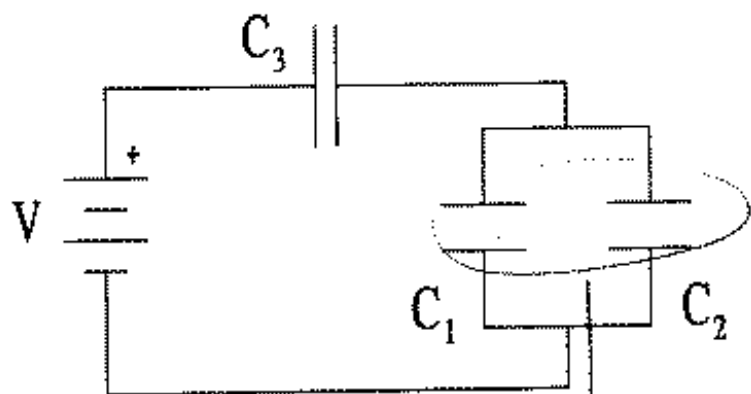
$$q_{\text{inside}} = 0$$

$$[\Phi_{\text{tot}} = 0] (5)$$

4) Capacitors in a circuit (25 points total)

Consider the circuit shown below:

The battery has been hooked up for a long time. All the charge has stopped moving around. The circuit has reached its equilibrium state. Please answer the following questions:



$$C_1 = 10 \text{ pf}$$

$$C_2 = 20 \text{ pf}$$

$$C_3 = 30 \text{ pf}$$

$$V = 6.0 \text{ Volts}$$

For all 3 $\frac{1}{C_{\text{equiv}}} = \frac{1}{30 \text{ pf}} + \frac{1}{30 \text{ pf}}$ $C_{12} = 30 \text{ pf}$
 $C_{\text{equiv}} = 15 \text{ pf}$

a) What is the charge on C_3 ?

Same as charge on C_{equiv} (series) $[Q = 90 \text{ pC}]$ (8)

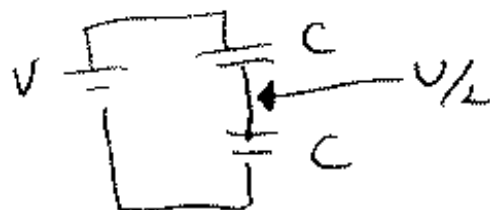
$$Q = (15 \text{ pF})(6.0 \text{ V})$$

$$= 90 \text{ pC}$$

b) What is the potential difference across C_1 ?

$\frac{1}{2}$ voltage, both caps in series 30 pf $[V = 3.0 \text{ V}]$ (4)

check $\frac{90 \text{ pC}}{30 \text{ pF}} = 3 \text{ V}$



c) What is the charge on C_2 ?

3V across C_2

$$[Q = 60 \mu C] (3)$$

20 pF

$$Q = 3V (20 \mu F) = 60 \mu C$$

d) What is the total energy stored in the capacitors?

$$\frac{1}{2} C_{equiv} V^2$$

$$[U = 270 \mu J] (5)$$

$$= \frac{1}{2} 15 \mu F (6V)^2 = 270 \mu J$$

e) A material of dielectric constant K ($K > 1$) is inserted between the plates of C_2 . Circle the correct answer below (there may be more than 1). The potential difference across each capacitor behaves as follows. (5 pts.)

- I) C_1 goes up III) C_2 goes up V) C_3 goes up
 II) C_1 goes down IV) C_2 goes down VI) C_3 goes down

C_2 increases, so C_{12} increases

$$C_{12} > C_3$$

q on each same

$$V_{12} = \frac{q}{C_{12}} \quad V_3 = \frac{q}{C_3}$$

$$V_{12} C_{12} = V_3 C_3$$

$$V_{12} = \frac{C_3}{C_{12}} V_3$$

V_1, V_2 go down

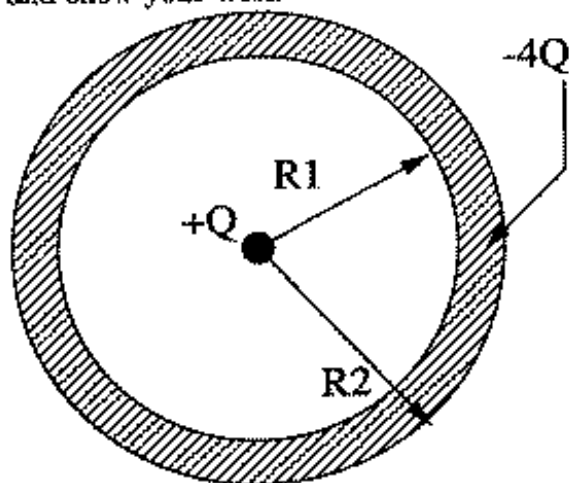
V_3 goes up

$$V_{12} < V_3$$

$$V_{12} + V_3 = 6V \text{ still}$$

5) Electric Potential (25 points total)

A conducting spherical shell of inner radius R_1 and outer radius R_2 has a charge of $+Q$ placed at its center and a charge $-4Q$ placed on it (this means that the sum of the charge on the inner surface of the conductor and the outer surface of the conductor is equal to $-4Q$). (see the figure below). Briefly explain your reasoning and show your work.



The electric potential V is zero at radius of $r = \infty$. Please find the algebraic expression for the electric potential in all regions of space.

consider work needed to move charge from R_1 to ∞

Inside the Shell: $r < R_1$

$$\left[V = \frac{kq}{r} - \frac{kq}{R_1} - \frac{3kq}{R_2} \right] (10)$$

$$W = \int_r^{R_1} \frac{kq}{r^2} dr + \int_{R_1}^{R_2} 0 \cdot dr + \int_{R_2}^{\infty} \frac{k(-3Q)}{r^2} dr$$

$$= \left(\frac{kq}{r} - \frac{kq}{R_1} \right) 0 + \left(\frac{k(-3Q)}{R_2} - \frac{k(-3Q)}{\infty} \right)$$

Inside the Conductor: $R_1 < r < R_2$

$$\left[V = -\frac{3Q}{R_2} \right] (10)$$

$$= \frac{kq}{r} - \frac{kq}{R_1} + \frac{k(-3Q)}{R_2}$$

constant @ surface $V = \frac{kq}{a}$

when $r = R_1$

$$V = -\frac{3Q}{R_2}$$

Outside the Shell: $r > R_2$

$$\left[V = -\frac{3kQ}{r} \right] (5)$$

treat as pt chg