Physics 116b

First Partial Examination

September 20, 2001

Name: Key

Please circle your section:

Section 1 Section 2 Section 3 Section 4

Instructions

This is a one hour, closed book examination. Put answers in the boxes provided, or circle the best answer. If numerical answers are needed, you must include units. If required, any work needed to justify the answer must be shown in the space provided. A correct answer without the necessary justifying work may not receive any credit. You may use the formula sheet on the back of the exam.

Total point scores for each problem will appear in the table below and in ( ) beside each problem number. Do what is easiest first. AVOID glancing at anyone else's paper during the exam! **The honor code is in effect.**

You must select 4 problems from the choice of 5 given. I will try to grade all 5 if you do them, and give you the best 4 of 5, but I may run out of steam. There are no guarantees! I may just grade the first 4 you did and calculate your score from them.

(Grade based on the 4 problems you chose.)

Point Tallies for the Exam Problems

Problem	Description	Max Score	Actual Score
1	Short Answer	25	
2	Various Subjects	25	
3	Electric Fields	25	
4	Capacitors	25	
5	Gauss's Law	25	
Total	4 out of the 5	100	

1. Short Answer (25 points total)

a) (5 points) A constant electric field of 10 N/C points along the x direction. An electron, initially at rest, moves over a distance of 1.0 m. How fast is the electron moving at the end of its 1.0 m journey?

do B first $\frac{1}{2}mv^2 = qV$ $V = Ed$

$$v = \sqrt{\frac{2qEd}{m}} = \sqrt{\frac{2(1.6\times10^{-19}C)(10\frac{N}{C})(1.0m)}{9.11\times10^{-31}kg}}$$

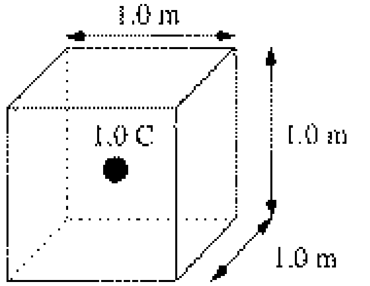
$$= 1.87\times10^{6} m/s$$

b) (5 points) Through what potential difference (V) has the electron moved over its 1.0 m journey in part a) ? (Vfinal - Vinitial)

$$\Delta V = Ed = (10\frac{N}{C})(1.0m)$$

$$= 10 \text{ Volts}$$

c) (5 points) A +1.0 C charge is located in the center of a 1m x 1m x 1m box. What is the Electric Flux going through each face of the box? (hint: Total flux = ?)



$\Phi_{total} = \frac{q_{enc}}{\epsilon_0}$

each face has same area, charge is in center each face has $\frac{\Phi_{tot}}{6}$

$$= \left(\frac{\frac{1.0C}{8.85\times10^{-12}\frac{C^2}{Nm^2}}}{6}\right) = 1.88\times10^{10}\frac{N}{C}m^2$$

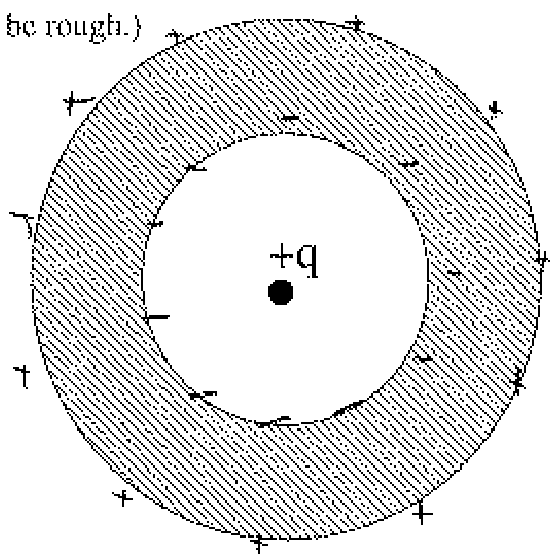
d) (5 points) At what distance is the force between two electrons equal to 1.0 pN?

$F = \frac{kq_1q_2}{r^2}$ $r = \sqrt{\frac{kq_1q_2}{F}} = \sqrt{\frac{9\times10^{9}N\frac{m^2}{C^2}(1.6\times10^{-19}C)^2}{1.0\times10^{-12}N}}$

$$= 1.52\times10^{-8} m$$

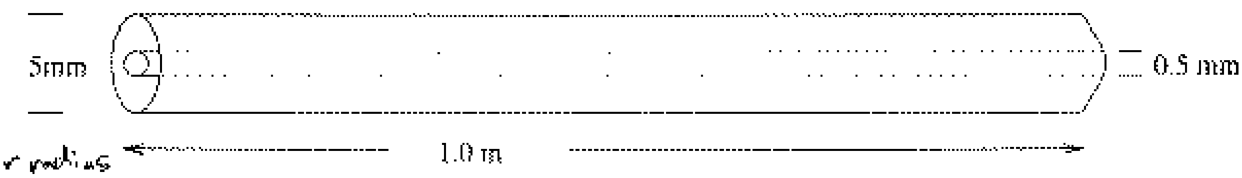
e) (5 points) A positive charge is placed inside an uncharged conductor. Sketch the distribution of the charges on the conducting sphere due to the influence of the charge in the center.

(note: q is large, and your sketch can be rough.)



2. Short Answer (25 points total)

a) (5 points) A coaxial cable is basically a cylindrical capacitor. Please estimate the capacitance in a cable that might be used to connect a TV to the cable outlet in the wall. Please use the digram below for dimensions.



b = outer radius
a = inner

$$C = \frac{2\pi \varepsilon_0 L}{\ln(b/a)} = \frac{2\pi \; 8.85\times10^{-12}\frac{F}{m}\; 1.0\,m}{\ln(10)} = 24.1\,pF$$

b) (5 points)Since the cable has plastic in it, the answer for part a) needs to be modified. If the dielectric constant of the plastic used = 2.5, what is the value for capcitance now? (It's ok to guess part a) to do part b))

$$C_{new} = \kappa C_{old} = 2.5\,(24.1\,pF) = 60.3\,pF$$

c) (5 points) Now, instead of hooking up your television, you decide to hook up a 9.0V battery to your cable. How much stored energy is there in the cable after you charge it up with the 9.0 V battery?

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(60.3\,pF)(9V)^2 = 2.4\,nJ$$

d) (5 points) Suppose in a certain region of space, the electric potential is described by V = -30 xyz. What is the electric field at a point x=1.0 m, y=1.0 m, z=1.0 m?

$$E_x = -\frac{\partial V}{\partial x} = +30yz \text{ @ } 1,1,1 = 30$$

$$E_y = +30\,xz = 30 \qquad \vec{E} = 30\hat{x} + 30\hat{y} + 30\hat{z} \;\; N/C$$

$$E_z = +30yx = 30$$

e) (5 points) In class, we determined that the electric field above a charged conducting sheet was σ/ε_0 (σ is the charge/area). Show that the electric field just above the surface of a conducting sphere carrying charge Q, is described by the same formula.

$$E = \frac{kQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$ outside charged sphere.

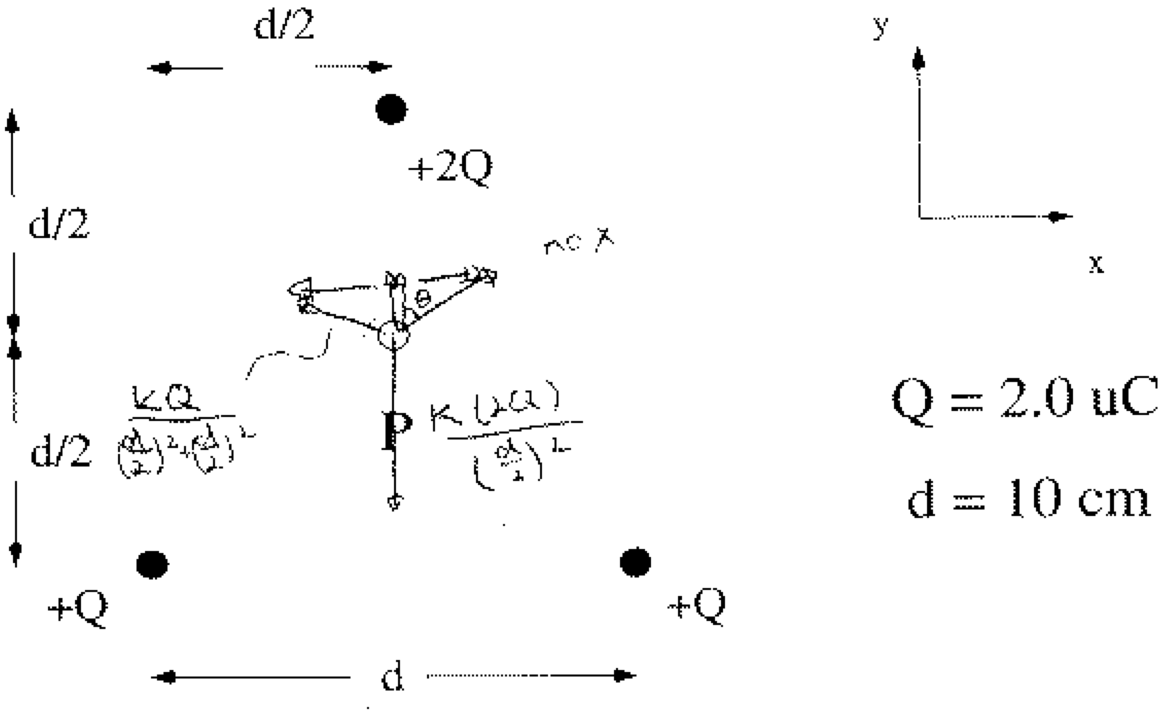
@surface R $$E = \left(\frac{Q}{4\pi R^2}\right)\frac{1}{\varepsilon_0}$$

charge/volume

$$= \sigma/\varepsilon_0 \; !$$

3) Electric Field Vectors (25 points total)

Consider the charges arranged at the corners of a triangle as shown in the figure below.



a) Calculate the x and y components of the electric field at the point indicated by **P**.

$$E_y = -2\left(\frac{KQ}{(\frac{d}{2})^2}\right) + 2\left(\frac{KQ}{(\frac{d}{2})^2+(\frac{d}{2})^2}\cos\theta\right)$$

$$= -2\left(\frac{KQ}{(\frac{d}{2})^2}\right) + \left(\frac{KQ}{(\frac{d}{2})^2}\right)\cos\theta$$

$$\boxed{E_x = 0} \quad (5)$$

$$\boxed{E_y = -9.3\times10^6 \frac{N}{C}} \quad (5)$$

$$= (-2+\cos\theta)\left(\frac{9\times10^9\,N\frac{m^2}{C^2}\;2\times10^{-6}C}{(.05m)^2}\right)$$

$$= 9.3\ M\ \frac{N}{C}$$

10^6

3) Electric Field Vectors (25 points total contd.)

b) To what position must the +2Q charge be moved in order to make the electric field be zero ($|E| = 0$) at point P? (let P be the point (0,0))

$\cos 45° = 0.707$

want $\left|\frac{2KQ}{y^2}\right| = \frac{KQ}{(\frac{d}{2})^2}\cos\Theta$

$2\left(\frac{d}{2}\right)^2 \frac{1}{\cos\Theta} = y^2$

$y = \sqrt{\frac{(.05m)^2 \times 2}{0.707}}$

$= 0.084m$

$[x = 0]$ (5)

$[y = 0.084m]$ (5)

c) Calculate the Electric Potential (assume $V = 0$ at r = infinity) at point P for the configuration of charges in part b).

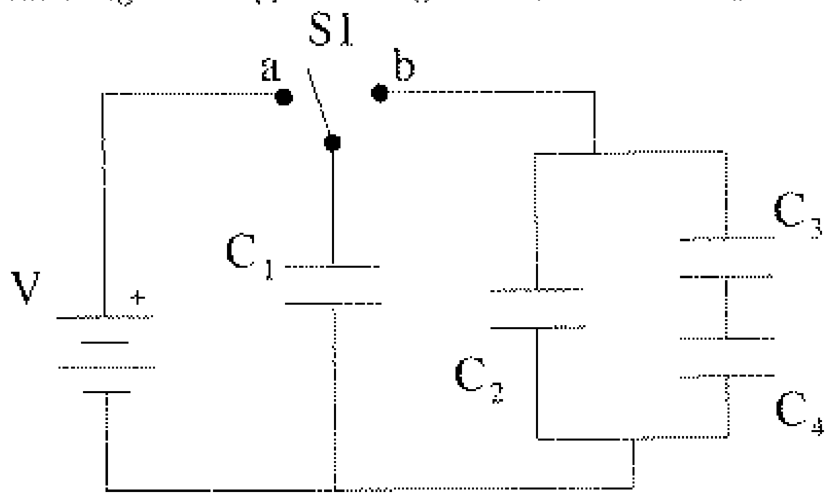
$V_{tot} = 2\left(\frac{KQ}{\sqrt{(\frac{d}{2})^2 + (\frac{d}{2})^2}}\right) + \frac{2KQ}{.084m}$

$= \frac{2(9\times10^9 \frac{Nm^2}{C^2} 2\times10^{-6}C)}{(.0707m)} + \frac{2(9\times10^9 N\frac{m^2}{C^2})(2\times10^{-6}C)}{.084m}$

$[V_P = 9.38\times10^5 V]$ (5)

4) Combinations of Capacitors **Show Your Work!** (25 points total)

All capacitors are initially uncharged. The switch, S1, is open. (I.e. not connected to either position a or position b) (Electrostatic equilibrium means after all the charge has stopped moving around, or after a long time.)



$C_1 = 15$ pf

$C_2 = 20$ pf

$C_3 = 35$ pf

$C_4 = 30$ pf

$V = 6.0$ Volts

I) Switch S1 is moved to position a. What is the charge on C_1 at electrostatic equilibrium?

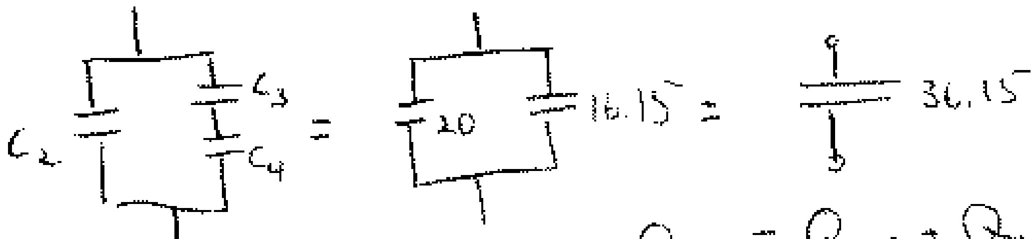
$$q = CV$$
$$= (15pF)(6.0V)$$
$$= 90pF$$

[$Q_1 =$ 90pF] (5)

II) Switch S1 is now moved to position b. What is the charge on C_1 at electrostatic equilibrium?

replace combo with equivalent

[$Q_1 =$ 26.4pC] (10)



$$Q_{tot} = Q_{on\,C_1} + Q_{on\,C_{new}} \Rightarrow Q_{on\,C_1}\left[1 + \frac{C_{new}}{C_1}\right]$$

$$V_{on\,C_1} = V_{on\,C_{new}}$$

$$Q_{on\,C_{new}} = \frac{Q_{on\,C_1}}{C_1} C_{new}$$

$$Q_{on\,C_1} = \frac{90pF}{\left[1 + \frac{36.15}{15}\right]} = 26pC$$

$$\frac{1}{C_{new}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{35} + \frac{1}{30}$$

$$C_{new} = 16.15pF$$

4) Combinations of Capacitors **Show Your Work!**(contd.)

III) How much electrostatic energy is stored by all the capacitors in part II)?

so 63.6pC on 36.15pF cap $\left[U_{tot} = 79.2\,pJ\right]$ (5)

$$U_{tot} = \frac{1}{2}\frac{(26.4pC)^2}{15pF} + \frac{1}{2}\frac{(63.6pC)^2}{36.15pF}$$

$$= 79.18\,pJ$$

could also calculate ΔV & C_{equiv}

IV) Switch S1 is now moved back to position a, and the circuit is allowed to come to electrostatic equilibrium. Then Switch S1 is moved again to position b, and the circuit is allowed to come to electrostatic equilibrium. How much electrostatic energy is stored by all the capacitors now?

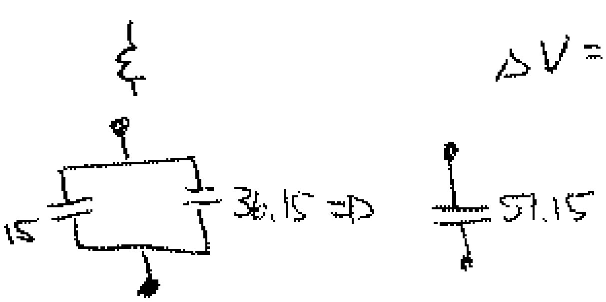
$\left[U_{tot} = 232\,pJ\right]$ (5)

means Q tot changed !

$$Q_{tot} = Q_{old\ on\ 36.15} + \underset{\text{total charged up C1}}{90pF} = 153.6\,pC$$

same ratios apply $\quad Q^{new}_{c_1} = \frac{153.6pC}{1+\frac{36.15}{15}} = 45pC$

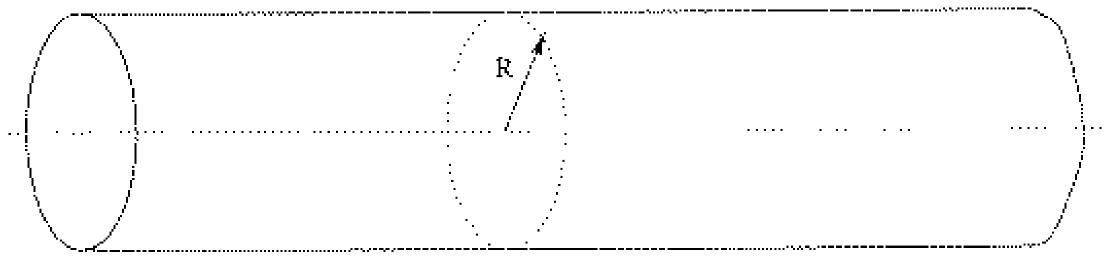
$\Delta V = (45pC)/(15pF) = 3V$



$\frac{1}{2}CV^2 = \frac{1}{2}(51.5pF)(3V)^2 = 231.75$

5) Gauss's Law (25 points total)

A very long (you can assume it is infinitely long) non-conducting cylinder of radius R has charge uniformly distributed throughout its volume. I.e. the density of charge, or charge/unit volume, $\rho = Q/(\pi R^2 \ell)$.



a) Calculate the Electric Field as a function of the radius both inside and outside the cylinder. (hint: Find Q enclosed and use Gauss's Law)

Inside $q_{enc} = \left(\frac{Q}{\cancel{\pi} R^2 \cancel{\ell}}\right)\cancel{\pi} r^2 \cancel{\ell}$

$= \frac{Q r^2}{R^2}$

Gauss' Law $EA = q_{enc}/\varepsilon_0$

so $E(2\pi r \ell) = \frac{Q r^2}{R^2}\frac{1}{\varepsilon_0}$

$E = \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{\ell}\right)\frac{r}{R^2}$

check @ $r = R$, get old answer $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$

Inside the Cylinder: $r < R$

$\left[E(r) = \frac{1}{2\pi\varepsilon_0}\frac{Q}{\ell}\frac{r}{R^2}\right]$ (10)

Outside the Cylinder: $r > R$

$\left[E(r) = \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{\ell}\right)\frac{1}{r}\right]$ (5) all charge contained

b) Calculate the Electric Potential as a function of the radius both inside and outside the cylinder. (hint: Use your answer in a), assume V at ~~infinity~~ $r = R$ is zero.)

$\underbrace{V_F}_{0} - V_I = \int_r^R \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{\ell}\right)\frac{r}{R^2}dr$

$0 - V_I = \frac{1}{2\pi\varepsilon_0}\left(\frac{Q}{\ell}\right)\frac{1}{R^2}\left(\frac{r^2}{2}\Big|_r^R\right)$

$= \left(\frac{1}{2\pi\varepsilon_0}\right)\left(\frac{Q}{\ell}\right)\frac{1}{R^2}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$

$V = K\left(\frac{Q}{\ell}\right)\left(\frac{r^2}{R^2} - 1\right)$

(increases as r gets bigger

Inside the Cylinder: $r < R$

$\left[V(r) = K\frac{Q}{\ell}\left(\frac{r^2}{R^2} - 1\right)\right]$ (5)

$\int \frac{1}{r}dr = \ln(r)$

Outside the Cylinder: $r > R$

$\left[V(r) = \frac{1}{2\pi\varepsilon_0}\ln\left(\frac{r}{R}\right)\right]$ (5)

outside

$V_F - \underbrace{\cancel{V_I}}_{0} = \int_R^r \frac{1}{2\pi\varepsilon_0}\frac{Q}{\ell}\frac{1}{r}dr = \frac{1}{2\pi\varepsilon_0}\frac{Q}{\ell}\left(\ln(r)\Big|_R^r\right)$

$= \frac{1}{2\pi\varepsilon_0}\frac{Q}{\ell}\left(\ln(r) - \ln(R)\right) = \frac{1}{2\pi\varepsilon_0}\frac{Q}{\ell}\ln\left(\frac{r}{R}\right)$