

1) Short Answer (Show Your Work!)

- a) An inductor is in series with a 12V battery and a 300 Ohm resistor. At $t=0$ no current flows through the circuit and then a switch is closed and current begins to flow. At $t = 0.01$ ms, the current flowing through the circuit is 2.4 mA. What is the inductance of the inductor? How much energy has been stored in the inductor at $t = 1.3$ ms?

$$I_{max} = 12V / 300\Omega = 40mA$$

$$I(t) = I_{max} (1 - e^{-t/\tau}) \quad \tau = L/R$$

$$2.4mA = 40mA (1 - e^{-t/\tau}) \quad = \frac{0.0485s}{300\Omega}$$

$$\ln \left(1 - \frac{2.4}{40} \right) = -t/\tau = -\frac{tR}{L} \quad @ t = 0.0013s$$

$$L = 1 \times 10^{-5} s (300\Omega) / \ln(0.94) = 0.0485s$$

Inductance=

$$48.5mH$$

(5)

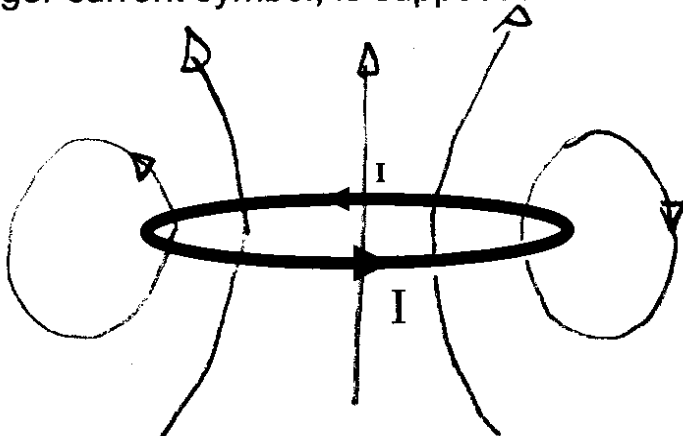
$$Energy = \frac{1}{2} L I^2$$

$$= \frac{1}{2} (0.0485s) (0.04A)^2$$

$$= 38.8 \mu J$$

(5)

- b) Sketch the magnetic field for a single coil of current. (5) (Your point of view is slightly above the coil. This means the lower part, with the bigger arrow and the bigger current symbol, is supposed to be closer to you.)



- c) A particle, with a net charge equal to the charge on an electron, moves in a circular orbit in a 1.6 T magnet field. If the particle makes 122,000 revolutions/s, what is the mass of the particle?

$$\frac{mv^2}{r} = qvB$$

$$m \left(\frac{v}{r} \right) = qB$$

$$\frac{v}{r} = \frac{qB}{m} = 2\pi f \Rightarrow m = \frac{qB}{2\pi f}$$

$$\omega = \frac{v}{r} = 2\pi f$$

$$Mass = 3.34 \times 10^{-25} kg$$

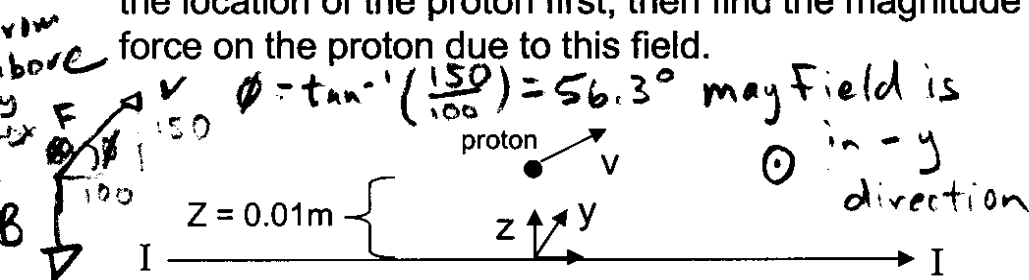
(5)

$$= \frac{(1.6 \times 10^{-19} C) 1.6 T}{2\pi (122000)}$$

$$= 3.34 \times 10^{-25} kg$$

1) Short Answer cont'd (Show Your Work!)

d) What is the magnitude and direction of the force on a proton located 1.0 cm above the origin of the x y axis, i.e. at the position (0.0, 0.0, 0.01m), and moving at a velocity of 100 m/s in the x direction and 150 m/s in the y direction, if there is a wire located along the x axis, with a current of 10A? Calculate the magnitude and direction of Magnetic Field due to the wire at the location of the proton first, then find the magnitude and direction of the force on the proton due to this field.



$$\begin{aligned} |\text{Magnetic Field}| &= 2.0 \times 10^{-4} \text{ T} \\ \text{Direction} &= -\hat{y} \end{aligned} \quad (5)$$

only x component of \vec{v} matters for force direc.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}})(10 \text{ A})}{(2\pi)(0.01 \text{ m})} = 2 \times 10^{-4} \text{ T}$$

$$\begin{aligned} |\text{Force}| &= 3.2 \times 10^{-21} \text{ N} \\ \text{Direction} &= -\hat{z} \end{aligned} \quad (5)$$

$$\vec{F} = q\vec{v} \times \vec{B} \quad (\text{only } x \text{ comp. } v = 100 \text{ m/s})$$

$$\begin{aligned} |F| &= qvB \sin \theta = (1.6 \times 10^{-19} \text{ C})(100 \text{ m/s})(2.0 \times 10^{-4} \text{ T}) \sin(146.3^\circ) \\ &= 3.2 \times 10^{-21} \text{ N} \end{aligned}$$

e) What is the self-inductance of a solenoid of length 2.5 m, ~~radius 3.0 cm~~ ^{diameter 6.0 cm}, and 2500 turns? If 4.9 A of current flows through this solenoid, what is the magnitude of the magnetic field in its center?

$$L = \mu_0 n^2 A l \quad n = \frac{N}{l}$$

$$= \mu_0 \frac{N^2}{l} A$$

$$= (4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}) \frac{(2500)^2}{2.5 \text{ m}} \pi \left(\frac{0.06 \text{ m}}{2}\right)^2$$

$$= 8.83 \text{ mH}$$

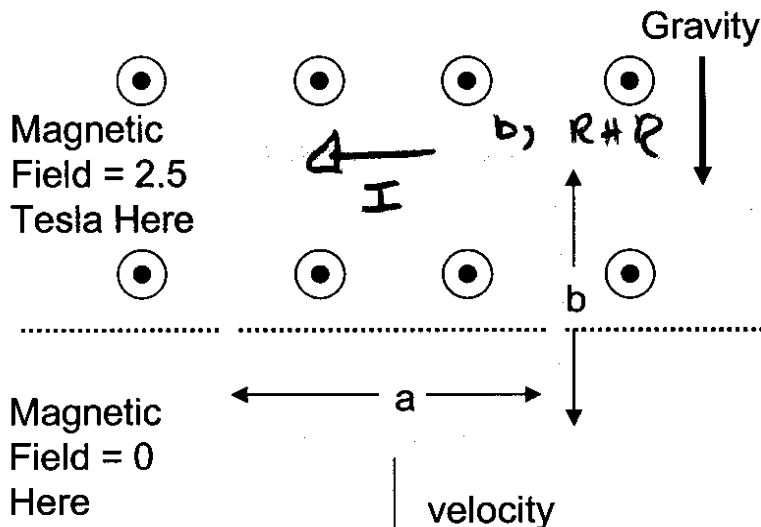
$$\begin{aligned} \text{Self-inductance} &= 8.83 \text{ mH} \end{aligned} \quad (5)$$

$$\begin{aligned} \text{Magnetic Field} &= 6.16 \text{ mT} \end{aligned} \quad (5)$$

$$\begin{aligned} B &= \mu_0 n I = (4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}) \left(\frac{2500}{2.5 \text{ m}}\right) (4.9 \text{ A}) \\ &= 6.16 \text{ mT} \end{aligned}$$

2) Induced EMF (20 points) (Show Your Work!)

In the figure below, the conducting loop has fallen a short distance and is now falling at a constant velocity, v . The loop has sides of length $a = 0.50$ m and $b = 0.45$ m, has a mass of 14.0 grams, and has a resistance of 1.20 Ohms. If the magnetic field out of the page is 2.5 Tesla above the dashed line and zero below the dashed line, answer the following questions:



What force is pulling down on the loop?

$$\text{Force} = 0.1372 \text{ N} \quad (2)$$

Since the loop is moving at a constant velocity, there must be an equal force acting to hold the loop up. What current do you need to flow in the loop to produce this force? Please indicate the direction of this current on the figure as well.

$$\text{Current} = 0.1098 \text{ A} \quad (5)$$

(drawing direction of current on figure) (3)

Since there is a resistance in this loop, you can calculate the induced EMF. This EMF is produced by the changing flux. By setting these equal to each other, calculate the constant velocity.

$$\text{Velocity} = 0.105 \text{ m/s} \quad (5)$$

If you had trouble solving for velocity using the flux, calculate how much power is supplied by gravity. This should equal the power generated by the current and the EMF in the loop. So, how much power is supplied by gravity?

$$\text{Power} = 0.0144 \text{ W} \quad (5)$$

$$F = mg = (0.014 \text{ kg})(9.8 \text{ m/s}^2) = 0.1372 \text{ N}$$

$$F = I L B \quad I = \frac{F}{L B} = \frac{0.1372}{(0.50 \text{ m})(2.5 \text{ T})} = 0.1098 \text{ A}$$

$$\text{changing Flux} = B(a v)$$

$$\mathcal{E} = (0.1098 \text{ A}) 1.2 \Omega = 0.132 \text{ V}$$

$$v = \frac{\mathcal{E}}{B a} = \frac{0.132 \text{ V}}{(2.5 \text{ T})(0.5 \text{ m})} = 0.105 \text{ m/s}$$

$$P = I V = (0.1098 \text{ A})(0.132 \text{ V}) = 0.0145 \text{ W}$$

$$\text{check } mgv = (0.1372)(0.105 \text{ m/s}) = 0.0144 \text{ W}$$

pretty good