1) Short Answer (16 points)(Show Your Work!)

a) What is the slit width if a second order minima is found at an angle of 44.5 degrees when light is incident normally on a single slit and a diffraction pattern forms. Assume the wavelength of the light used is 632.8nm. (4 pts)

Second order $m=2$

Single slit $a\sin\theta = m\lambda$

$$a = \frac{m\lambda}{\sin\theta}$$

$$= \frac{2(632.8\,\text{nm})}{\sin 44.5^\circ} = 1806\,\text{nm}$$

Width = 1806nm

b) A light source emits 2 wavelengths of light. The manufacturer claims the 2 wavelengths are 500nm and 501nm. You have a diffraction grating that has a grating spacing (d) of 9000nm. How large a beam must you use (or how much of the diffraction grating must you illuminate) in order to resolve these 2 wavelengths with the diffraction grating you have if you look at the 2nd order maxima?(4 pts)

Resolving Power $R = mN = \frac{\lambda}{\Delta\lambda}$

$m=2$

Beam Size = 2.25 mm

N = # slits to illuminate to resolve the two wavelengths

$$N = \frac{1}{2}\frac{(500+501)/2}{501-500} = \frac{500.5}{2} = 250.25$$

beam size $\underset{\substack{\uparrow \\ \text{grating sep}}}{9000\,\text{nm}} \times \underset{\substack{\uparrow \\ \text{\# gratings}}}{250.25} = 2.25\times10^{6}\,\text{nm}$

1) Short Answer cont'd (Show Your Work!)

c) Since it tends to be very difficult to make an X-ray microscope, electron microscopes are often used to see very small objects. What potential difference must an electron be accelerated from rest with, to produce a DeBroglie wavelength of 100.0 pm for an electron? (4 pts)

De Broglie

$$\lambda = \frac{h}{p} = \frac{hc}{pc}$$

$$KE = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2}$$

$$= \frac{(hc/\lambda)^2}{2m_ec^2} = \frac{(1239.8\,eVnm / 0.100nm)^2}{2\,(511{,}000eV)}$$

$$= 150.4eV$$

V = 150.4 V

d) As you watch in the laboratory (i.e. you are at rest!), a particle moving at a speed of 0.999c along the x axis is created at x = 0.0 and explodes (or dies) a short time later at x = 1.0 cm as measured by you in the laboratory. If this particle were created at rest instead of a frame of reference moving at 0.999c with respect to your laboratory, how long would it have lived? (4 pts)

Time= 1.49 ps

Length in Fixed frame = 1.0cm

time in frame where particle moving (same as above) $= \frac{0.01m}{0.999(2.9979\times10^8 m/s)}$

$= 3.339\times10^{-11}s$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.999^2}}$$

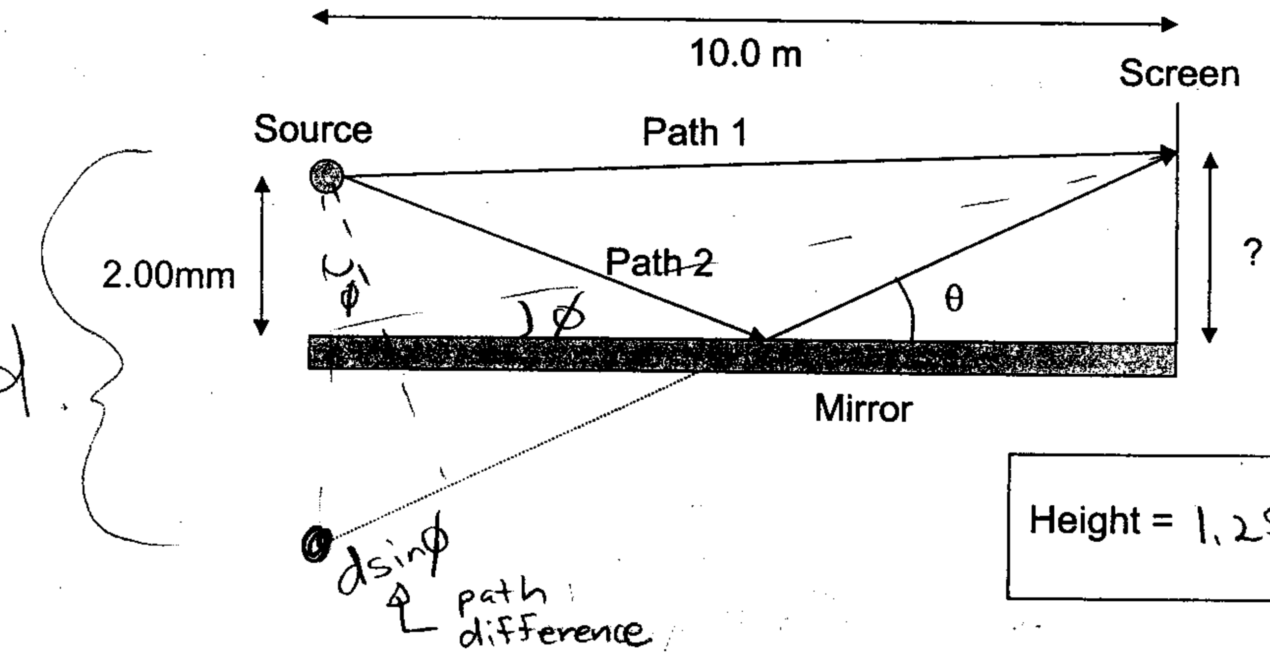
$= 22.366$

time in particle (rest) frame $= \frac{3.339\times10^{-11}s}{22.366}$

$= 1.49\times10^{-12}s$

2) Interference (show your work)

A piece of glass (n>1) acts like a very good mirror for light reflected at shallow angles. If we place a source of light a little above the surface of a very large sheet of glass and look at a screen far away from the source, an interference pattern can be seen. If a source of 500nm light is placed 10.0m from the screen and 2.00 mm above the mirror, find the height to the first dark band above the surface of mirror. (Hint: Think about the image the source makes in the "mirror", and make use of some of the machinery you have developed to describe 2 slit interference. Careful! Does the glass cause a phase shift?) (8 pts)



Height = 1.25mm

so, one gets a path difference & a phase difference from the reflection

$$(d\sin\phi)\frac{2\pi}{\lambda} + \pi = \pi, 3\pi, 5\pi$$

for destructive

or

$$d\sin\phi\,\frac{2\pi}{\lambda} = 0, 2\pi, 4\pi \ldots$$

$\phi = 0$ is at surface, use 2π

$$\frac{d\sin\phi}{\lambda} = 1 \qquad \sin\phi \approx \phi = \frac{\lambda}{d} = \frac{500\times10^{-9}}{4\times10^{-3}} = 1.25\times10^{-4}$$

$$h = (10.0\,m)\overset{\approx\tan\phi}{\phi} = (10.0\,m)(1.25\times10^{-4})$$

3) Photoelectric Effect (Show your work!)

For this problem you will determine un unknown wavelength, and the work function, ϕ, of the material under test. (Please use hc=1239.8 eVnm from the equation sheet.) You determine that the stopping potential of your test material for light with wavelength of 250nm is 3.1V and the stopping potential for light with unknown wavelength is 0.72V. From your data, determine the unknown wavelength and the work function of the material you are testing. (8 pts)

use $eV_{stop} = \frac{hc}{\lambda} - \phi$

$_1 = 3.1V$

$_2 = 0.72V$

$_1 = 250nm$

$eV_1 = \frac{hc}{\lambda_1} - \phi$

$eV_2 = \frac{hc}{\lambda_2} - \phi$

$\lambda = 480.7nm$
$\phi = 1.86eV$

find λ_2

$$eV_2 - eV_1 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$

$$\left(eV_2 - eV_1 + \frac{hc}{\lambda_1}\right) = \frac{hc}{\lambda_2}$$

$$\lambda_2 = \frac{hc}{eV_2 - eV_1 + \frac{hc}{\lambda_1}}$$

$$= \frac{1239.8eVnm}{0.72eV - 3.1eV + \frac{1239.8eVnm}{250nm}}$$

$$= 480.7nm$$

$$\phi = \frac{hc}{\lambda_1} - eV_1 = \frac{1239.8eV}{250nm} - 3.1eV$$

$$= 1.86eV$$

4) Compton Effect (Show your work!)

At what photon scattering angle is the ***electron kinetic energy*** a **maximum**? (8 pts. A correct answer with no supporting work here will receive only 2 pts. Hint: If you are stuck, just try some angles and look at the trend to predict the maximum.) At this angle, what is the **energy, E'**, of the ***scattered*** **photon** in terms of the initial photon energy, E_0?(4 pts.) Note: An energy of zero is not acceptable for the scattered photon or the electron!

$\theta = \pi, \quad 180°$

$E' = 1/\left(\frac{1}{E_0} + \frac{2}{m_e c^2}\right)$

Compton Scattering

$$\lambda' - \lambda_0 = \frac{hc}{m_e c^2}(1 - \cos\phi)$$

$$KE_{electron} = E_0 - E'$$

Want to minimize E' or maximize λ'

occurs @ $\phi = 180°$ $(\cos\phi = -1)$ $\lambda' - \lambda$ is max

or

$$\frac{1}{E'} - \lambda_0 = \frac{1}{m_e c^2} - \frac{\cos\phi}{m_e c^2}$$

differentiate $-\frac{dE}{E'^2} = \frac{\sin\phi}{m_e c^2} d\phi$

$$\frac{dE}{d\phi} = \frac{-E'^2}{m_e c^2}\sin\phi$$

not possible!

& $E'^2 = 0$

extreme points @ $\phi = 0, \pi, 2\pi$

max

minima

@ $\phi = \pi$ $\quad 1 - \cos\phi = 2$

$$\lambda' = \overset{\frac{hc}{E_0}}{\lambda_0} + 2\frac{h}{m_e c} = \frac{hc}{E'}$$

$$E' = \frac{hc}{\frac{hc}{E_0} + 2\frac{hc}{m_e c^2}} = \frac{1}{\frac{1}{E_0} + \frac{2}{m_e c^2}}$$