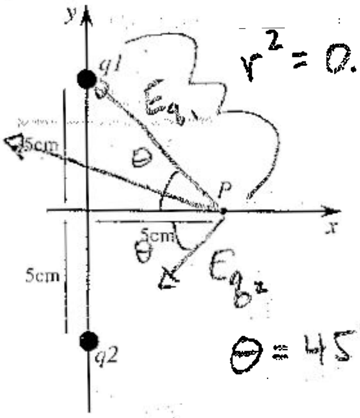
Point Charges (20 points total)

7. Two charges are placed at opposite locations on the y-axis as shown on the right. If $q_1 = -10\text{nC}$ and $q_2 = -5\text{nC}$, answer the following questions:



$$\vec{E}_1 = (18{,}000\ N/C)\left(-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$+\ \vec{E}_2 = (9{,}000\ N/C)\left(-\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$-\frac{27{,}000}{\sqrt{2}}\frac{N}{C}\hat{i} + \frac{9000}{\sqrt{2}} N/C\ \hat{j}$$

Enet

E_{q_1}

E_{q_2}

$r^2 = 0.05m^2 + 0.05m^2$

$\theta = 45°$

(a) Draw a vector on the graph at point P which shows the approximate direction of the electric field at that point. (2)

(b) What is the electric field $\vec{E}$ at point P?

For q_1
$$\vec{E}_1 = \frac{(9\times10^9 \frac{Nm^2}{C^2})(10\times10^{-9}C)}{(0.05m)^2 + (0.05m)^2}(-\cos 45°\hat{i} + \sin 45°\hat{j})$$

$$\vec{E}_2 = \frac{(9\times10^9 \frac{Nm^2}{C^2})(5\times10^{-9}C)}{(0.05m)^2 + (0.05m)^2}(-\cos 45°\hat{i} - \sin 45°\hat{j})$$

$E_x = -19092\ N/C$ (3)

$E_y = 6364\ N/C$ (3)

(c) What is the electric potential V at point P?

$$V = \frac{Kq_1}{r} + \frac{Kq_2}{r} = \frac{(9\times10^9 N\frac{m^2}{C^2})}{\sqrt{0.05m^2 + 0.05m^2}}(-10\times10^{-9}C - 5\times10^{-9}C) = -1909\left(\frac{Nm}{C}\right)$$

$V = 1909\ V$ (4)

(d) A third charge $q_3 = 8\text{nC}$ is brought to from far away to point P. How much work was done on q_3 to bring it to point P?

$q_3 > 0$, wants to get closer, work <0

$$\Delta U = q\Delta V = (8nC)(-1909V) = -15{,}272nJ$$

$W = 1.53\times10^{-5}J$ (4)

(e) When q_3 is at P, what is the total electric potential energy of the three charges?

$$U_{tot} = \frac{Kq_1q_2}{r_{12}} + \underbrace{\frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}}}_{-1.53\times10^{-5}J}$$

$$\frac{(9\times10^9\frac{Nm^2}{C^2})(-10\times10^{-9}C)(-5\times10^{-9}C)}{0.1m}$$

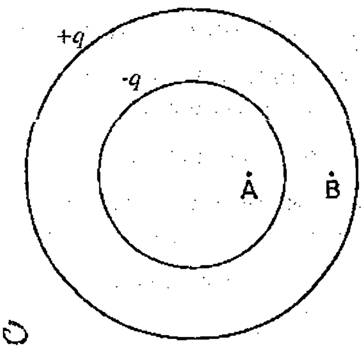
$$= 4.5\times10^{-6}J - 1.53\times10^{-5}J$$
$$= -1.08\times10^{-5}J$$

$U_{tot} = -1.08\times10^{-5}J$ (4)

Multiple Choice (8 points total)

1. Which of the following statements about electric field lines due to a charge arrangement is **correct**?
   (a) They can never cross.
   (b) The less densly the lines are spaced, the stronger the electric field.
   (c) If the electric force on an electron (which has a negative charge) points upward at a point, the direction of the electric field is upward at that point.
   (d) They point toward positive charges.
   (e) Charges must move along these lines.

2. A small conducting spherical shell with charge q is placed inside a larger spherical conducting shell with charge $-q$. The electric field is measured at three points as shown. Which of the following statements about the magnitudes of those measurements is **true**?

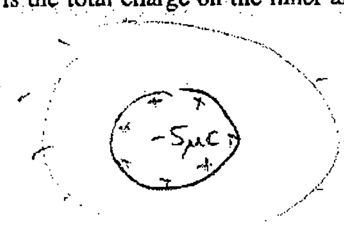


   (a) $E_A > E_B$
   (b) $E_A = E_C$
   (c) $E_A > E_C$
   (d) $E_C > E_A$
   (e) $E_C > E_B$

$q_{enc} = 0$
for C & A

$\Rightarrow$ Spherical symmetry $E = 0$

3. An uncharged conductor has a hollow cavity inside of it. Within this cavity there is a charge of -5 μC that does not touch the conductor. There are no other charges in the vicinity. What is the total charge on the inner and outer surfaces of the conductor?



   (a) $Q_{\text{inner}} = -5\,\mu\text{C}$, $Q_{\text{outer}} = 5\,\mu\text{C}$
   (b) $Q_{\text{inner}} = 5\,\mu\text{C}$, $Q_{\text{outer}} = 0$
   (c) $Q_{\text{inner}} = 0\,\mu\text{C}$, $Q_{\text{outer}} = -5\,\mu\text{C}$
   (d) $Q_{\text{inner}} = 5\,\mu\text{C}$, $Q_{\text{outer}} = -5\,\mu\text{C}$
   (e) $Q_{\text{inner}} = 0\,\mu\text{C}$, $Q_{\text{outer}} = 0\,\mu\text{C}$

4. A positive test charge q is released from rest in a uniform electric field E, at a point where the electric potential is V_a. Only the electric field acts on the particle. At some later time, the charge has moved a distance r to a new point where the electric potential is V_b. How much kinetic energy has the particle gained?
   (a) $q(V_b - V_a)$
   (b) $q(V_a - V_b)$
   (c) q/r
   (d) qE
   (e) none

$KE > 0$

$U = qV$ $\quad q > 0$

want part to go from high U to low U

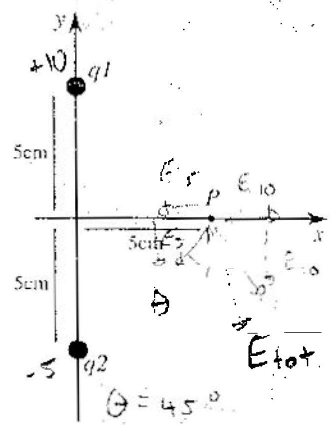
for $q > 0$ means high V to low V
start (a) finish (b)

Point Charges (20 points total)

7. Two charges are placed at opposite locations on the y-axis as shown on the right. If $q_1 = +10\text{nC}$ and $q_2 = -5\text{nC}$, answer the following questions:

$r = \sqrt{.05m^2 + .05m^2}$

$= 7.071 \times 10^{-2} m$



(a) Draw a vector on the graph at point P which shows the approximate direction of the electric field at that point. (2)

(b) What is the electric field $\vec{\mathbf{E}}$ at point P?

$|E_{10}| = \dfrac{(9\times10^9 N\frac{m^2}{C^2})(10nC)}{(7.071\times10^{-2}m)^2} = 18000\ N/C$

$\boxed{E_x = 6364 N/C}$ (3)

$|E_5| = 9000 N/C$

$E_x = 18000 N/C \cos 45° - 9000 N/C \cos 45°$

$\boxed{E_y = 19{,}092 N/C}$ (3)

$E_y = (-18000 \sin 45° - 9000 \sin 45°) N/C$

(c) What is the electric potential V at point P?

$V = \sum \frac{kq}{r} = \left(\dfrac{9\times10^9 N\frac{m^2}{C^2}}{7.071\times10^{-2}}\right)(10nC - 5nC) =$

$\boxed{V = 636V}$ (4)

(d) A third charge $q_3 = 8\text{nC}$ is brought to from far away to point P. How much work was done on q_3 to bring it to point P?

$V = 0 @ \infty$

$\Delta V = 636V$

+ charge so + ΔU

$W = q\Delta V = 8\times10^{-9} C\ 636V = 5.09\times10^{-6}$

$\boxed{W = 5.1 \mu J}$ (4)

(e) When q_3 is at P, what is the total electric potential energy of the three charges?

$U_{tot} = \dfrac{kq_1q_3}{r_{13}} + \dfrac{kq_1q_2}{r_{12}} + \dfrac{kq_2q_3}{r_{23}}$

$\boxed{U_{tot} = 0.59 \mu J}$ (4)

$= 5.09\mu J + \dfrac{9\times10^9 N\frac{m^2}{C^2}}{0.10m}(10nC)(-5nC) = 0.59\mu J$

Multiple Choice (8 points total)

1. Which of the following statements about electric field lines due to a charge arrangement is **incorrect**?

   (a) They can never cross.

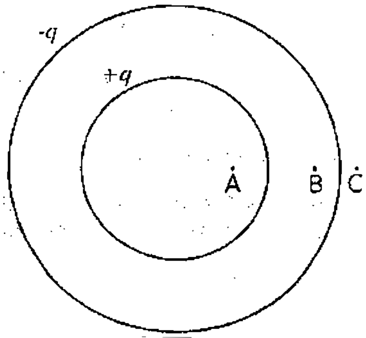
   (b) The more closely the lines are spaced, the stronger the electric field.

   (c) If the electric force on an electron (which has a negative charge) points upward at a point, the direction of the electric field is downward at that point.

   (d) They point toward negative charges.

   (e) Charges must move along these lines.

2. A small conducting spherical shell with charge q is placed inside a larger spherical conducting shell with charge $-q$. The electric field is measured at three points as shown. Which of the following statements about the magnitudes of those measurements is **true**?



   (a) $E_A > E_B$

   (b) $E_A > E_C$

   (c) $E_C > E_A$

   (d) $E_C > E_B$

   (e) $E_A = E_C$

3. An uncharged conductor has a hollow cavity inside of it. Within this cavity there is a charge of $+10\,\mu C$ that does not touch the conductor. There are no other charges in the vicinity. What is the total charge on the inner and outer surfaces of the conductor?

   (a) $Q_{inner} = -5\,\mu C, \quad Q_{outer} = -5\,\mu C$

   (b) $Q_{inner} = -10\,\mu C, \quad Q_{outer} = 0$

   (c) $Q_{inner} = -10\,\mu C, \quad Q_{outer} = +10\,\mu C$

   (d) $Q_{inner} = 0\,\mu C, \quad Q_{outer} = -10\,\mu C$

   (e) $Q_{inner} = 0\,\mu C, \quad Q_{outer} = 0\,\mu C$

4. A positive test charge q is released from rest in a uniform electric field E, at a point where the electric potential is V_a. Only the electric field acts on the particle. At some later time, the charge has moved a distance r to a new point where the electric potential is V_b. How much kinetic energy has the particle gained?

   (a) q/r

   (b) $q(V_b - V_a)$

   (c) $q(V_a - V_b)$

   (d) qE

   (e) none

Short Answer (8 points total)

5. A charge of -1 nC is placed at the origin of a coordinate system. A second charge of -4 nC is placed on the x-axis at the point $x = 1$ cm. Find the location on the x-axis where the electric field is zero.

$\boxed{x = \frac{1}{3}\text{ cm}}$ (4)

$-1nC$ $\vec{E}$ $\quad$ $-4nC$ $\vec{E}$

$-1nC \qquad x \qquad -4nC$

$$\vec{E}_{-1nC} = \frac{k(-1nC)}{x^2}\hat{x}$$

$$\vec{E}_{-4nC} = \frac{k(-4nC)}{(1cm - x)^2}\,-\hat{x}$$

$$\vec{E}_{-1nC} + \vec{E}_{-4nC} = 0 \quad \text{or} \quad \frac{k\,1nC}{x^2} = \frac{k(4nC)}{(1cm - x)^2} \quad \text{or} \quad \frac{1}{x^2} = \frac{4}{(1cm - x)^2}$$

$\Rightarrow$ know $x > 0$ so $\sqrt{\text{both sides}}$ $\quad \frac{1}{x} = \frac{2}{1cm - x} \quad (1cm - x) = 2x$

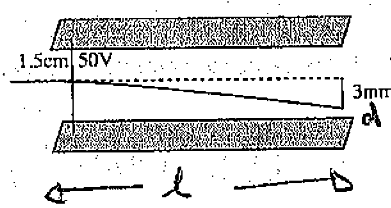
$\Rightarrow 1cm = 3x$

$x = \frac{1}{3}$ cm

check $\frac{1}{\left(\frac{1}{3}\right)^2} = 9$

$\frac{4}{\left(\frac{2}{3}\right)^2} = 4\left(\frac{9}{4}\right) = 9!$

6. An electron is accelerated from rest through a potential difference of 2000 V. The electron then enters a pair of parallel plates separated by a distance of 1.50 cm. The electron velocity is parallel to the plates when it enters the plates. If the potential difference between the plates is 50.0 V, and the electron is deflected by 3.00 mm at the end of the plates, what is the magnitude of the final velocity of the electron?



$E = \frac{50V}{1.5cm}$

$\boxed{v = 2.66 \times 10^7 \frac{m}{s}}$ (4)

Total energy of particle at end

$= qV_{accel} + qV_{deflect}$ $\qquad V_{deflect} = E(3mm) = \frac{50V}{1.5cm}\,0.3cm = 10$

$= \frac{1}{2}mv^2 \qquad \sqrt{v^2} = \sqrt{2(qV_{accel} + qV_{deflect})/m}$

$$= \sqrt{2\,\frac{(1.60\times10^{-19}C)(2000V + 10.0V)}{9.11\times10^{-31}\,kg}} = 2.66\times10^{7}\,m/s$$

check $\frac{1}{2}m\frac{v^2}{c^2}c^2 = 2010eV$

$\left(\sqrt{2\,\frac{2010eV}{(511{,}000eV)}}\right)3e8\frac{m}{s} = 2.66\times10^{7}\,m/s$

alternate

$V_0 = \sqrt{2\frac{qV_0}{m}} = 2.65\times10^{7}\,m/s$

$V_f = \sqrt{V_0^2 + V^2}$

$d = \frac{1}{2}at^2,\ t^2 = \frac{2d}{a}\quad t = \sqrt{\frac{2d}{a}}$

$v = at = \sqrt{2ad} \qquad a = \frac{qE}{m} = \frac{q\,50V/0.015m}{m}$

$v = 1.874\times10^{6}\,m/s$

Short Answer (8 points total)

5. A charge of +3 nC is placed at the origin of a coordinate system. A second charge of +1 nC is placed on the x-axis at the point $x = 1$ cm. Find the location on the x-axis where the electric field is zero.

+3nC +1nC

$\rightarrow E \leftarrow$

$$\frac{k(3nC)}{x^2} = \frac{k(1nC)}{(1-x)^2}$$

$$3(1-x)^2 = x^2 \quad x>0$$

$$\sqrt{3} - \sqrt{3}x = x$$

$$\sqrt{3} = (1+\sqrt{3})x$$

$$\frac{\sqrt{3}}{1+\sqrt{3}} = x = 0.634 \text{ cm}$$

$\boxed{x = 0.634\text{cm}}$ (4)

6. An electron is accelerated from rest through a potential difference of 1000 V. The electron then enters a pair of parallel plates separated by a distance of 1.00 cm. The electron velocity is parallel to the plates when it enters the plates. If the potential difference between the plates is 25.0 V, and the electron is deflected by 2.00 mm at the end of the plates, what is the magnitude of the final velocity of the electron?

$$E = \frac{25.0V}{1.0cm}$$

$$\Delta V = \frac{25.0V}{1.0cm} 0.2cm = 5V$$

$$E_{tot} = e1000V + e5V = e1005V = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(1.60\times10^{-19}C)(1005V)}{9.11\times10^{-31}kg}} = 1.88\times10^{7} m/s$$

$\boxed{v = 1.88\times10^{7} m/s}$ (4)