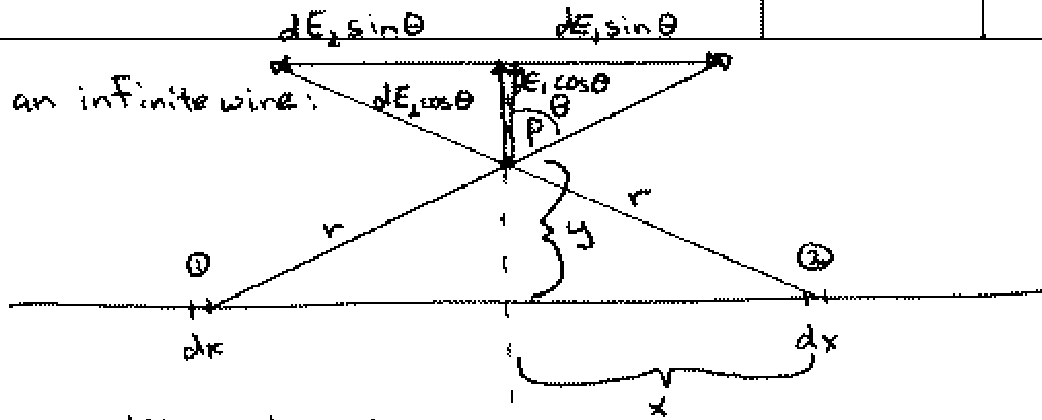
FERMILAB
ENGINEERING NOTE

SECTION	PROJECT	SERIAL-CATEGORY	PAGE

SUBJECT: Sheet of charge, using "wires" side-by-side

NAME	
DATE	REVISION DATE

Recall an infinite wire:



The charge in a little slice dx is $\lambda\, dx$

λ = charge / length

The x components cancel & we're left with

$$dE_{tot} = 2 dE_1 \cos\theta$$
$$= 2\left(\frac{k\lambda\, dx}{r^2}\right)\cos\theta$$

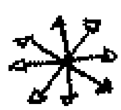
$$r^2 = x^2 + y^2$$
$$= y^2\tan^2\theta + y^2$$
$$= y^2\left(\frac{\sin^2\theta}{\cos^2\theta}\right) + y^2\left(\frac{\cos^2\theta}{\cos^2\theta}\right) = \frac{y^2}{\cos^2\theta}(\sin^2\theta + \cos^2\theta)$$
$$= \frac{y^2}{\cos^2\theta} \quad \text{or} \quad \cos\theta = \frac{y}{r} \quad r^2 = \left(\frac{y}{\cos\theta}\right)^2$$

$$\frac{x}{y} = \tan\theta$$
$$x = y\tan\theta$$
$$dx = \frac{y\, d\theta}{\cos^2\theta}$$
$$k = \frac{1}{4\pi\varepsilon_0}$$

so $$dE_{tot} = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\lambda \frac{y\,d\theta}{\cos^2\theta}}{y^2/\cos^2\theta}\cos\theta = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\cos\theta\, d\theta$$

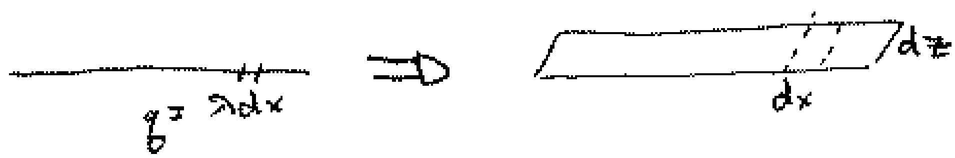
$$E_{tot} = \int_0^{\pi/2} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\cos\theta\, d\theta$$
$$= \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}$$

looks like



from the end

now, suppose instead of a line, we have a ribbon



$q = \sigma\, dx\, dz$

we're essentially replacing λ with σdz

now, we're going to place a lot of these ribbons on end

FERMILAB
ENGINEERING NOTE

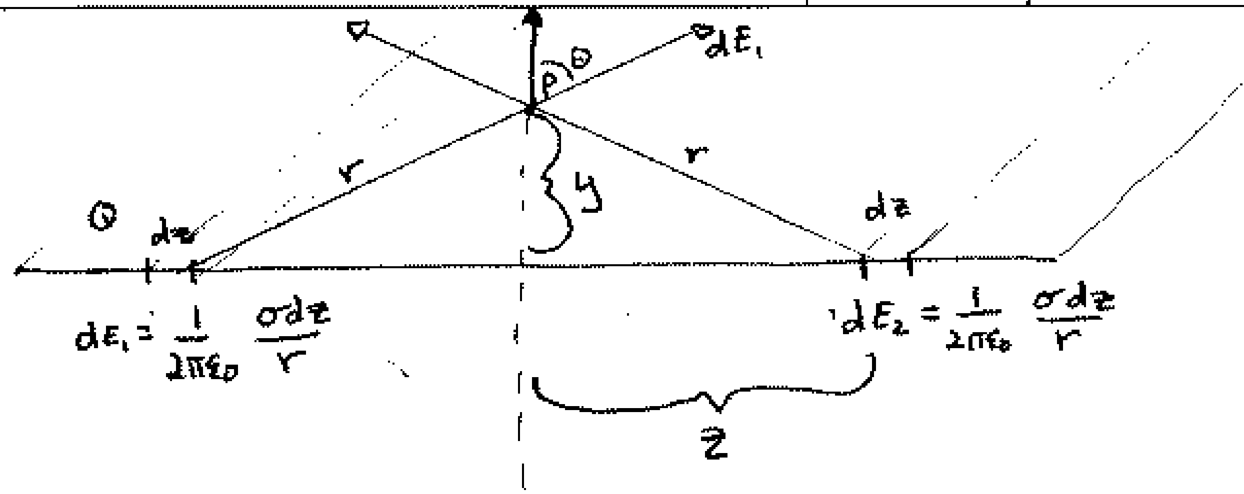
SECTION	PROJECT	SERIAL-CATEGORY	PAGE

SUBJECT

NAME

DATE

REVISION DATE



$$dE_1 = \frac{1}{2\pi\varepsilon_0}\frac{\sigma dz}{r} \qquad dE_2 = \frac{1}{2\pi\varepsilon_0}\frac{\sigma dz}{r}$$

so, were actually only changing our earlier calculation a little bit.

$$dE_{tot} = 2\left(\frac{1}{2\pi\varepsilon_0}\frac{\sigma dz}{r}\right)\cos\theta$$

$$r = \frac{y}{\cos\theta} \qquad dz = \frac{y\,d\theta}{\cos^2\theta}$$

$$dE_{tot} = \left(\frac{1}{\pi\varepsilon_0}\right)\sigma\frac{\left(\frac{\cancel{y}d\theta}{\cancel{\cos^2\theta}}\right)}{\left(\frac{\cancel{y}}{\cancel{\cos\theta}}\right)}\cancel{\cos\theta} = \frac{1}{\pi\varepsilon_0}\sigma d\theta$$

$$E_{tot} = \int_0^{\pi/2}\frac{\sigma}{\pi\varepsilon_0}d\theta = \frac{\sigma}{2\varepsilon_0}$$

which is what we got for Gauss' Law!