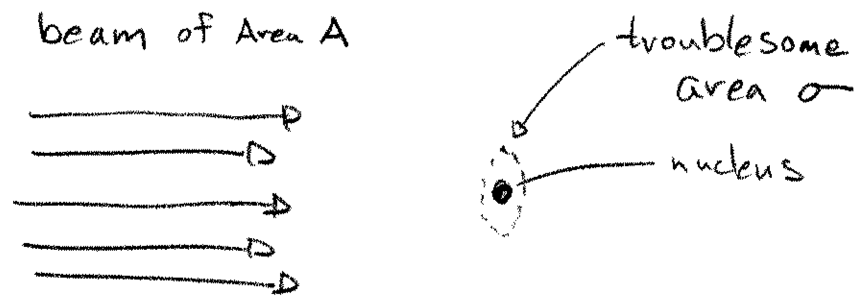
More Fission

What's Wrong with ^{238}U (take too high energy neutron to get it to scatter. These leave the reactor quickly, while slower neutrons have a better chance of getting picked up by another nucleus)

To understand this quantitatively, we need to put the concept of a cross section in our toolbox.

Imagine an incoming beam of particles incident on a big ole nucleus. We can characterize the nucleus as a troublesome disk, that causes beam particles to scatter when they come within a certain radii (on average of course!)



The probability that one of the particles in the beam hits this disk is going to be related to 1) the area of the beam (particle can be anywhere in here)
2) the area of the disk (one spot)

$$P_{scattered} = \left(\frac{\sigma}{A}\right)$$

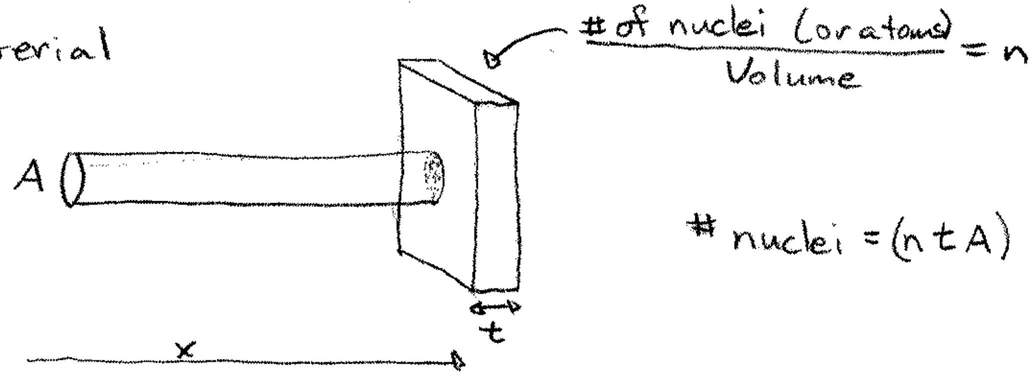
This increases if we add more nuclei

$$P_{new} = (\#\,\text{nuclei})\left(\frac{\sigma}{A}\right)$$

and the rate of scattering looks like

$$\frac{\#\,\text{incident particles}}{s} P_{new} = \frac{\#\,\text{scattered particles}}{s}$$

Now, consider a beam incident on a slab of material



$\#$ nuclei $= (ntA)$

$$\frac{\#\text{ scattered particles}}{s} = (ntA)\frac{\sigma}{A}\,\frac{\#\text{ of incident particles}}{s}$$

$$= nt\sigma\,\frac{\#\text{ of incident particles}}{s}$$

now for fun, consider a very small thickness Δt

$\#$ particles at surface $+ dt$ $= \#$ incident $- \#$ scattered

in terms of intensity as a function of depth (let $\Delta x = \Delta t$)

$$I(x+dx) = I(x) - I(x)\,\sigma n\,dx$$

& our usual trick

$$\frac{I(x+dx) - I(x)}{\Delta x} = -I(x)\,\sigma n = \frac{dI(x)}{dx}$$

end up with $I(x) = I_0 e^{-\sigma n x}$

in terms of thickness

$$I(t) = I_0\, e^{-\sigma n t}$$

this little σ is called a cross section, and it is a very useful quantity for describing what happens in a nuclear reation

For cross sections of nuclear reactions, we use units that are a hold over from the Manhattan Project. They were figuring out what happens when neutrons hit Uranium nuclei "as big as a barn". And they very well could not discuss numbers of uranium nuclei!

$$\text{Area of uranium disk} = \pi \left(r_0 A^{1/3}\right)^2$$
$$= \pi\left(1.2\times10^{-15}\,\text{m}\,(235)^{1/3}\right)^2$$
$$= 1.72\times10^{-28}\,\text{m}^2$$

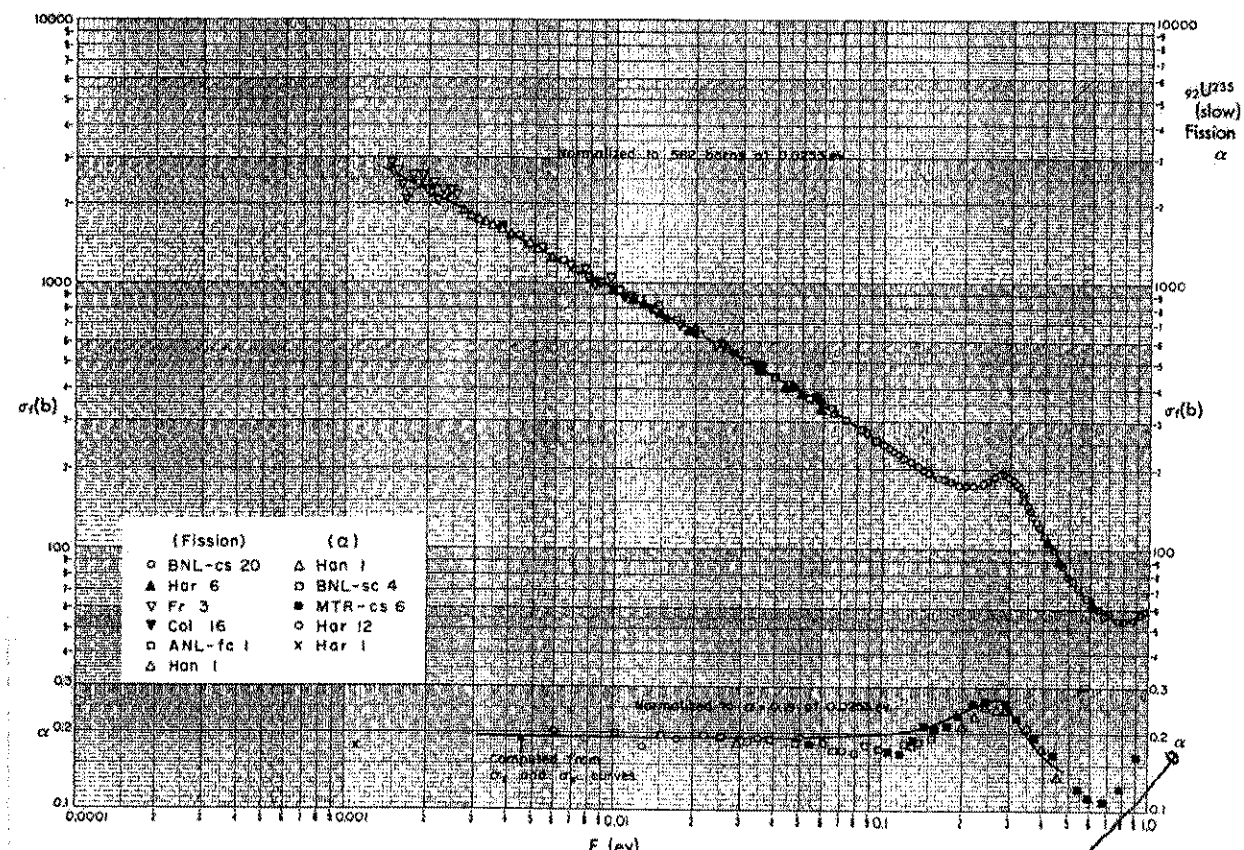
and one "barn" $= 10^{-28}\,\text{m}^2$

Thermal Neutron Fission σ for ^{235}U

10K →

barns

100 →

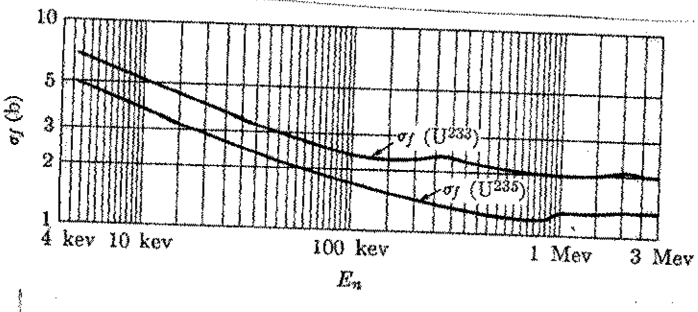


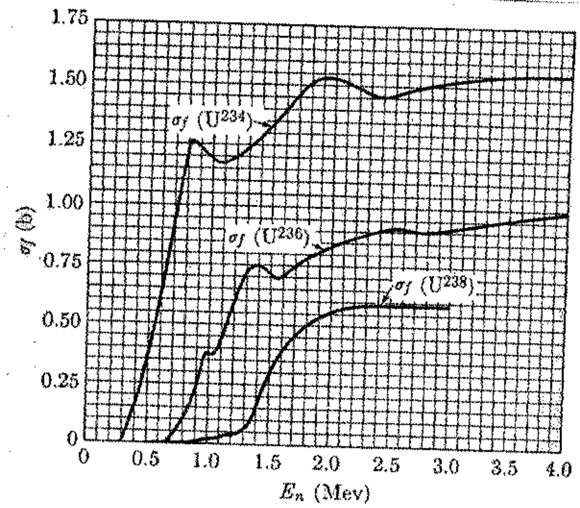
↑ 10^{-3} eV

ratio $\frac{\text{capture}}{\text{fission}}$

↑ 1eV

At higher neutron energies for different uranium isotopes





(b)

^{238}U tiny until ~1.5 MeV!

THERMAL CROSS SECTIONS OF SOME HEAVY NUCLIDES(7)*

Nuclide	Half-life	Fission cross section, σ_f, b	Total absorption cross section, σ_a, b	Activation cross section, σ_{act}, b
$_{90}Th^{227}$	18.2d	1500 ± 1000		
$_{90}Th^{229}$	7.34×10^3y	45 ± 11		
$_{91}Pa^{230}$	17.3d	1500 ± 250		
$_{91}Pa^{231}$	3.4×10^4y	$(10 \pm 5) \times 10^{-3}$		200 ± 15
$_{91}Pa^{232}$	1.3d	700 ± 100		760 ± 100
$_{92}U^{230}$	20.8d	25 ± 10		
$_{92}U^{231}$	4.2d	400 ± 300		
$_{92}U^{232}$	74y	80 ± 20		300 ± 200
$_{92}U^{233}$	1.62×10^5y	525 ± 4	578 ± 4	53 ± 2
$_{92}U^{234}$	2.52×10^5y	<0.65	105 ± 4	90 ± 30
$_{92}U^{235}$	7.1×10^8y	582 ± 4	683 ± 3	101 ± 5
$_{93}Np^{234}$	4.4d	900 ± 300		
$_{93}Np^{236}$	22h	2800 ± 800		
$_{93}Np^{237}$	2.2×10^6y	$(19 \pm 3) \times 10^{-3}$	170 ± 5	169 ± 6
$_{93}Np^{238}$	2.1d	1600 ± 100		
$_{93}Np^{239}$	2.3d	<1		35 ± 10
$_{94}Pu^{238}$	86.4y	16.8 ± 0.3		400 ± 10
$_{94}Pu^{239}$	2.44×10^4y	742 ± 4	1028 ± 8	286 ± 4
$_{94}Pu^{240}$	6.6×10^3y	0.030 ± 0.045	286 ± 7	250 ± 40
$_{94}Pu^{241}$	13y	1010 ± 13	1400 ± 80	400 ± 50
$_{94}Pu^{242}$	3.75×10^5y	<0.2	30 ± 2	19 ± 1
$_{95}Am^{241}$	458y	3.2 ± 0.2	630 ± 35	750 ± 80
$_{95}Am^{242}$	100y	6400 ± 500		

*For 2200 m/sec neutrons.

To have a good chain reaction that lasts awhile want:

1) good Fission σ
2) long $\frac{1}{2}$ life
   ^{233}U, ^{239}P, ^{235}U
3) since $\sigma_{TOT} = \sigma_{Fiss} + \sigma_{Act}$ want good $\sigma_{Fiss}/\sigma_{tot}$

This isn't the end of the story. In order to get these slow neutrons to get these big $\sigma_{fission}$'s we need to get them to ~room temp

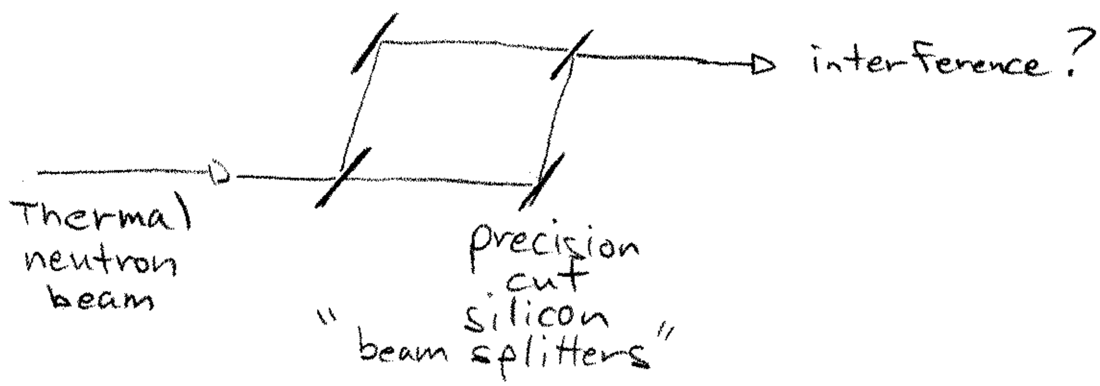
$$KE \approx \frac{3}{2} k_B T \quad \text{(really!)}$$

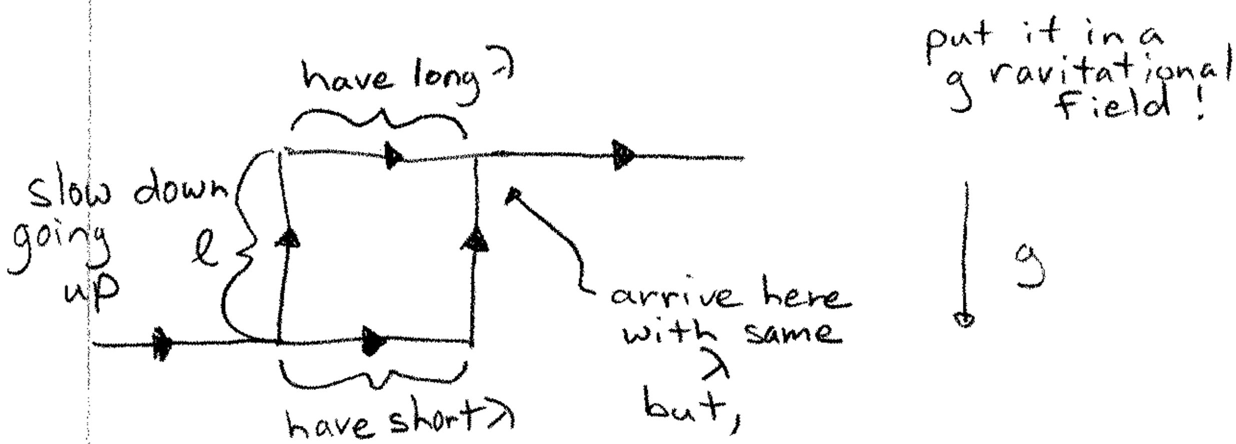
$$= \frac{3}{2}\left(8.617\times10^{-5}\,\text{eV/K}\right)(300\text{K})$$

$$= 0.039\,\text{eV} \quad \text{tiny!} \quad v \approx 2700\,\text{m/s}$$

Aside: Fun with thermal neutrons.

Consider the following exercise





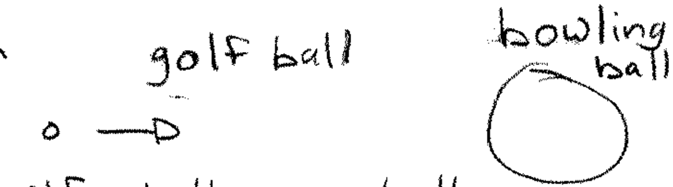
took them longer to take the high road!

phase difference is same for going up but

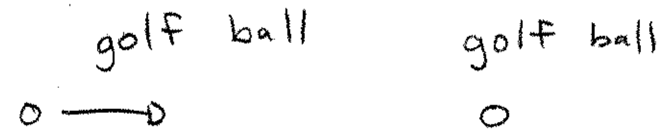
$\phi_{\text{high road}} = l\left(\frac{2\pi}{\lambda_{up}}\right) \qquad \phi_{\text{low road}} = l\left(\frac{2\pi}{\lambda_{down}}\right)$

& you can find $\Delta\phi$ with ΔKE, biggish for $l = 2\text{cm}$!

In order to slow them down, we want to collide the neutrons ejected during fission. This collision process should proceed so as to have the neutrons slow down quickly. Happens if the moderator nuclei are about the same mass as the neutron



golf ball essentially just has $|\Delta p| = 2p$



loses almost all its momentum

Some moderators

	σ_{abs} (b)	⟨# collision⟩ to get to room temp	σ_{scatt}
^1H	1/3	18	49.2
^2H	~1/2000	25	10.6
^4He	0	43	—
^{12}C	~1/300	110	4.7
^{238}U	2.75	2200	8.3
O	0	~150	—

so H_2O isn't a total disaster, just need more ^{235}U in the mix

A <u>very</u> simplified model of fission for a reactor of infinite extent.

Pure ^{235}U

probability to get a fission from a thermal neutron $= \dfrac{\sigma_f}{\sigma_f + \sigma_a}$

on average we get 2.4 neutrons / fission of ^{235}U so for each fission, we get

reproduction factor $K = 2.4 \dfrac{\sigma_f}{\sigma_f + \sigma_a}$ more fusions & a runaway reaction

Naturally occuring ^{238}U & ^{235}U

$$\sigma_f' = 0.0072\,\sigma_f^{235} + 0.9928\,\sigma_f^{238}$$
$$= 0.0072(582b) + 0.9928(0) = 4.19\,b$$
$$\sigma_a' = 0.0072\,\sigma_a^{235} + 0.9928\,\sigma_a^{238}$$
$$= 0.0072(101b) + 0.9928(2.75b) = 3.45\,b$$

$$K' = 2.4 \frac{4.19b}{4.19b + 3.45b} = 1.32$$

much less, want $K=1$ to be critical
$K>1$ super critical yikes
$K<1$ sub critical

loss mechanisms

1) leakage
2) absorbtion by water
3) absorbtion by ^{238}U @ higher n energy (resonance absorbtion)
4) control rods (Reaction so sensitive <1% emitted after 10s can control the reactor)

Example $k = 1.001$

emitted neutrons take some time to slow down & get recaptured $O(1ms)$

So, in each ~1ms the rate of neutrons released increases by a factor of 1.001 (release 1.001 times more n's every 1ms) so in N generations the rate doubles

$$(1.001)^N = 2 \qquad N = \frac{\ln 2}{\ln(1.001)} = 693.4$$

this is 0.7s or so! that reactor is darn sensitive.

Luckily, some of the reactants are delayed in time, so if $k = 1.001$

$$t_{avg} \sim 0.99(1ms) + .01(10s)$$
$$= 0.101s \text{ for each new generation}$$

and you've got about a minute before the rate of neutron release doubles

(Rate of release $\propto$ Power)