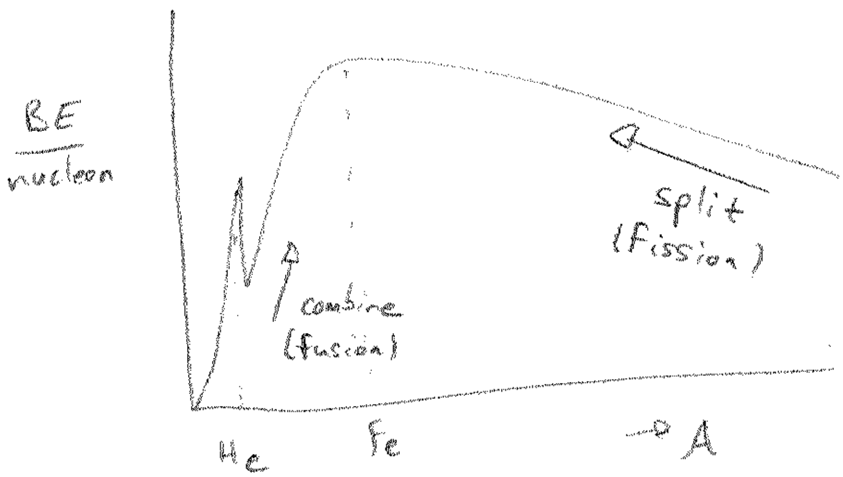
Fusion

Recall our picture of Binding Energies



Said we could trade energy for well depth

⇒ In Fission, we got about 1 MeV/nucleon used

⇒ We learned last time it can take a certain amount of energy to produce a reaction (we ignored a bit to do it, as we'll see.)

Consider now, combining nuclei to get deeper in the well. Here is one cycle that occurs in the Sun

Q(MeV)

1 $\quad {}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu_e$ $\quad$ 2 of these

and

~6 $\quad {}^2_1H + {}^1_1H \rightarrow {}^3_2He + \gamma$ $\quad$ need 2 of these

~12 $\quad {}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2\,{}^1_1H$ $\quad$ (to get one of these)

net effect

$$4\,{}^1_1H \rightarrow {}^4_2He + 2e^+ + 2\nu_e + 2\nu$$

$$Q_{tot} = 2 + 2(6) + 12 \sim 26 \text{ MeV}$$

or 6.5 MeV/nucleon much more!

Recall

Power Output of sun $\sim 4\times 10^{26}$ W

which is about 2×10^{39} MeV/s

or about 4×10^{38} protons get converted each second. (or $\sim 10^{15}$ moles/s)

$M_{sun} = 2\times 10^{33}$ g $\to$ 2×10^{33} moles

2×10^{18} s of hydrogen

or 6.3×10^{10} y

lots of time if hydrogen is the only contributor

$\Rightarrow$ So how come ${}^1_1H$ doesn't fuse of it's own accord?

$\Rightarrow$ Coulomb repulsion

Assume we need to get protons close enough to fuse (~ 1.5 fm)

$$\frac{Ke^2}{\hbar c} = \frac{1}{137}$$

$$u = \frac{Kq^2}{r} = \frac{hc}{2\pi(137)}\frac{1}{r}$$

$$\sim \frac{1.44\,\text{MeV fm}}{1.5\,\text{fm}} = 0.96\,\text{MeV}$$

$$0.48\,\text{MeV} = \frac{3}{2}\left(\frac{1}{40}\times 10^{-6}\,\text{MeV}\right)\frac{T}{300}$$

room temp ($k_B T_{room}$)

$$T = 12.8\times 10^{6}\,\text{K}!$$

(of course there's a tail & tunneling so this occurs at a lower temp)

Whats wrong with this arguement?

⇒ The "Fusion" cycle begins with a weak decay, & we argued that we need to get within 1.5 fm to get it to go, so $\sigma \sim \pi (1.5\,\text{fm})^2$

& expect

$$\frac{\#\text{ interactions}}{s} = \frac{N}{V}\sigma_I\, t \;\frac{\#\text{ incident}}{s}$$

can think of t/s as the velocity of the protons in the sun $v = c\sqrt{2(.5\,\text{MeV})/(938\,\text{MeV})}$

so, expect $\approx 10^7\,\text{m/s}$

$$\frac{\#\text{ interactions}}{s} = \frac{(2\times10^{33}\,\text{moles}\cdot 6.02\times10^{23}/\text{mole})^2}{\frac{4}{3}\pi\,(6.69\times10^8\,\text{m})^3}\,\pi(1.5\times10^{-15}\,\text{m})^2\,10^7\,\text{m/s}$$

$$= 7.25\times10^{64}/s\ !$$

so, that decay must occur only very rarely!

We'd need to modify this by the probability that a particle consisting of 2p's will decay.

$\lambda t \Leftarrow$ time 2 p's spent when they are close enough to have a decay

Complicated problem! actually $\lambda t \sim \frac{1}{3\times10^{17}}$

still some wierdness...

So this is not an especially useful way to get terrestrial fusion. A better medium is to use ${}^2_1H$. A bit on that.

$${}^2_1H + {}^2_1H \Rightarrow {}^3_1H + {}^1_1H$$

$Q = 4\,MeV$ (1 MeV/nucleon)

takes less energy to do it

0.25 MeV (neutrons help a little)

but in the sun, these deuterons get snapped up really quicky, so that the concentration is low

(on earth $\sim 0.015\%$ D_2O in H_2O, a bit biggish to have originated from the p-p cycle)

when sun is finished with hydrogen, burns He. gets hotter $\rightarrow$ smaller get a metastable state of

$\alpha + \alpha \leftrightarrows {}^8Be$ short lived resonance

lucky, there is an increase in the cross section for $\alpha + {}^8Be \Rightarrow {}^{12}C$

sometimes decays to ground state

$\Rightarrow$ available for carbon fusion later

Other, hotter stars can have

Carbon cycle

$${}^{12}C + {}^1H \Rightarrow {}^{13}N + \gamma$$
$${}^{13}N \Rightarrow {}^{13}C + e^+ + \nu$$
$${}^{13}C + {}^1H \Rightarrow {}^{14}N + \gamma$$
$${}^{14}N + {}^1H \Rightarrow {}^{15}O + \gamma$$
$${}^{15}O \Rightarrow {}^{15}N + e^+ + \nu$$
$${}^{15}N + {}^1H \Rightarrow {}^{12}C + {}^4He$$

$4\,{}^1_1H \Rightarrow {}^4He$ again! $(+2e^+ + 2\gamma + 2\nu)$

Commercial Fusion

Recall the payoff we had for fusion last time was about 6.5 MeV/nucleon. Wouldn't it be great if we could harness this power?

Lawson's Criteria

Recall our calculation for the sun showed that there needed to be some slowing mechanism or the sun would have run away fusion.

Consider these reactions: (think of this stuff as in a gas)

Reaction	Q (MeV)	
$^1_1p + ^2_1H \Rightarrow ^3He + \gamma$	5.49	Very energetic photons [not useful for making heat in the gas]
$^2_1H + ^2_1H \Rightarrow ^4_2He + \gamma$	23.85	
$\Rightarrow ^3_2He + n$ (most likely candidates)	3.27	easier to keep charged particles trapped but, much larger Temp ⟷ Energy
$\Rightarrow ^3_1H + p$ (most likely candidates)	4.03	
$^2_1H + ^3_1H \Rightarrow ^4He + n$	17.59	
$^2_1H + ^3_2He \Rightarrow ^4He + p$	18.35	① needed for $^2_1H + ^3_2He$

② ^3_1H radioactive, tough to keep around ($t_{1/2} = 12.33y$)
3g ⇒ Activity of 3.2×10^{15} Bq
10^5 Ci: hot!

Calculations proceed as before:

$$\frac{\text{Energy released}}{\text{Volume}} = \frac{\text{Rate of reaction}}{\text{Volume}} \times \text{time over which reaction can occur}$$

t

$$\frac{\text{Rate of reaction}}{\text{Volume}} = \left(\frac{N_1}{V}\langle\sigma_{12} v_2\rangle\frac{N_2}{V}\right)\left(\text{time you can maintain the densities } \frac{N_1}{V} \text{ \& } \frac{N_2}{V}\right)$$

Energy required to get the gas hot:

$$E_{req} = \underbrace{\tfrac{3}{2}(N_1+N_2)k_BT}_{\text{nuclei}} + \underbrace{\tfrac{3}{2}(N_1+N_2)k_BT}_{\text{electrons!}}$$

$$= 3(N_1+N_2)k_BT$$

get picky, it would be $(N_{tot})(N_{tot}-1)/2$

$$\frac{\text{Rate of reaction}}{\text{Vol}} \Rightarrow \underbrace{\left(\frac{N_{tot}}{2}\right)^2}_{} \langle\sigma_{12}v_2\rangle \frac{1}{V^2}$$

# of ways to arrange exclusive pairs } would be $\frac{N_{tot}^2}{4}$ for equal numbers of different species

so to sustain a reaction, we want

$$E_{released} > E_{req} \quad \text{or}\quad Q\left(\frac{N_{tot}^2}{2}\frac{\langle\sigma_{12}v_2\rangle}{V}\right)t > 3N_{tot}k_BT$$

$$\text{or}\quad \left(\frac{N_{tot}}{V}\right)t > \frac{6k_BT}{\langle\sigma_{12}v_2\rangle Q} \quad \left\{\frac{12k_BT}{\langle\sigma_{12}v_2\rangle Q}\right\}\ \text{diff species}$$

ex for a D-D plasma @ 100keV of energy

$\langle v\sigma\rangle = 5\times10^{-23}\,\frac{m^3}{s}$, $Q = 3.6\,MeV$

from graph

$$nt > \frac{6(0.1\,MeV)}{\left(5\times10^{-23}\,\frac{m^3}{s}\right)(3.6\,MeV)} = 3\times10^{21}\,\frac{s}{m^3}$$

ex for a ${}^2_1H + {}^3_1H$ reaction @ 60 keV (peak)

$$nt > \frac{12(0.06\,\text{MeV})}{\left(10^{-21}\frac{m^3}{s}\right)(17.59\,\text{MeV})} = 4\times10^{19}\frac{s}{m^3}$$

where $\langle v\sigma\rangle = \int P(v)\,\sigma(v)\,v\,dv$

over all speeds in the hot gas (plasma)

Tough to do!

Best I could find was

$$\frac{nt_{achieved}}{nt_{needed}} \approx 1$$

Other possibility

Smash a pellet of ${}^2_1H$ & ${}^3_1H$ (can calculate everything)

For 60 keV again (assuming this can happen)

$$t \approx \frac{r}{\langle v\rangle} = \left(\frac{r}{3.1\times10^6\,m/s}\right)$$

take a pellet that is compressed to 310 μm

$$t = \frac{3.1\times10^{-4}\,m}{3.1\times10^6\,m/s} = 100\,ps$$

Considerations
① losses in compression
② fuel cost
③ Fuel usage in t (don't use much fuel & pellets = $)
④ α capture reheats?

so, after compression, the pellet needs to have a density of

$$n = \frac{N}{V} \Rightarrow \rho = \left(\frac{2.5g}{mole}\cdot\frac{1}{6.02\times10^{23}/mole}\right)\frac{N}{V} = \frac{4\times10^{19}\,s/m^3}{1\times10^{-10}\,s}\left(\frac{2.5g}{mole}\,\frac{1}{6.02\times10^{23}/mole}\right)$$

$$= 1.66\times10^6\,g/m^3 = 1.66\times10^3\,kg/m^3$$

(liquid hydrogen ~ 38g/ℓ = 38 kg/m³ ⇒ ordinary density ~ 100 kg/m³ ²H & ³H mix)

so only × 20 compression

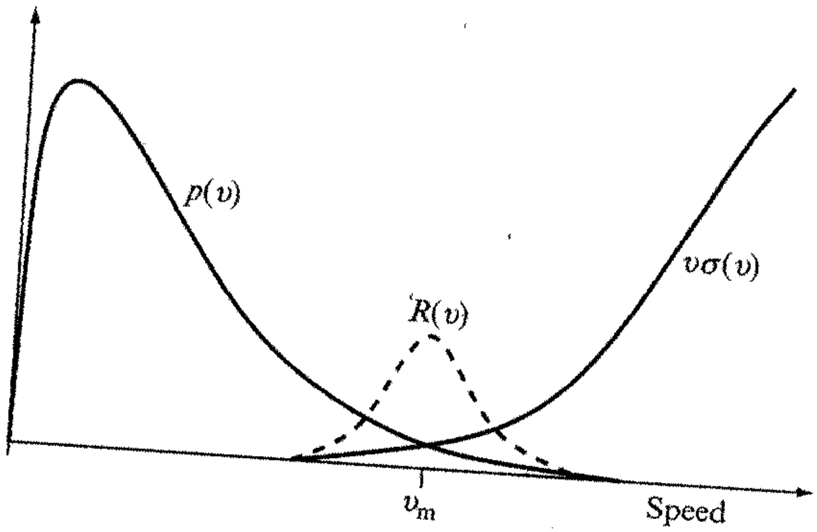


Figure 11.2 Qualitative plots showing the variation with speed of the Maxwell–Boltzma probability distribution $p(v)$ and the fusion reaction rate $v\sigma(v)$. Their product $R(v)$ (show dashed) is the integrand of Equation (11.1), which has a maximum at v_m corresponding to a effective thermal energy E_m.