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SUBJECT: Homework 1 Solution Phys 225 b

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1) a) Recall $P_0(\Delta t) = 1 - \mu \Delta t \quad P_1(\Delta t) = \mu \Delta t$

$$P_0(t+\Delta t) = P_0(t)\, P_0(\Delta t) = P_0(t)(1 - \mu \Delta t)$$

$$P_0(t+\Delta t) - P_0(t) = -\mu P_0(t) \Delta t$$

$$\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -\mu P_0(t)$$

$$\lim_{\Delta t \to 0} \Rightarrow \quad \frac{d P_0(t)}{dt} = -\mu P_0(t)$$

$$P_0(t) = e^{-\mu t}$$

b) $P_0(t)\, P_1(\Delta t) = e^{-\mu t} \mu \Delta t$

$$\int_0^\infty e^{-\mu t} \mu\, dt = \mu \left(-\frac{1}{\mu} e^{-\mu t} \Big|_0^\infty \right)$$

$$= 1$$

c) $$\langle t \rangle = \int_0^\infty t \mu e^{-\mu t} dt = \frac{1}{\mu} \int_0^\infty a e^{-a} da$$

(where $a = \mu t$, $da = \mu dt$)

$$= \frac{1}{\mu}$$

consider expression for lifetime

$$N(t) = N_0 e^{-t/\tau_0}$$

lifetime

$\frac{1}{\mu} = \tau_0$ (you can measure the lifetime by taking the average value of time in your measurement. Probably you did this in lab already!)

2)

$$R = \frac{\rho l}{A} \qquad \Delta\rho = \rho_0 \alpha_\rho \Delta T \qquad \Delta l = l_0 \alpha_e \Delta T$$

a)

$$(A + \Delta A) = (l + \Delta l)^2 \quad \text{where } A = l^2$$

$$= l^2 (1 + \alpha_e \Delta T)^2$$

$$= A(1 + 2\alpha_e \Delta T + \cancel{(\alpha_e \Delta T)^2}) \to 0$$

$$= A(1 + 2\alpha_e \Delta T)$$

$$\Delta A = A_0 (2\alpha_e) \Delta T$$

b)

$$R_0 = \frac{\rho_0 l_0}{A_0} \qquad R_{new} = R_0 \frac{(1 + \alpha_\rho \Delta T)(1 + \alpha_e \Delta T)}{(1 + 2\alpha_e \Delta T)}$$

$$= 100\Omega \frac{[1 + (3.9 \times 10^{-4}/°C)100°C][1 + \frac{2.9 \times 10^{-5}}{°C} 100°C]}{[1 + 2 \cdot \frac{2.9 \times 10^{-5}}{°C} 100°C]}$$

$$= 100\Omega \frac{(1.039)(1.0029)}{(1.0058)} = 103.60\Omega$$

$$\Delta R = 3.60\Omega$$

c) Recall,

$$(R - R_0)^2 = (R - \bar{R})^2 = \left(\frac{\partial R}{\partial \rho}\right)^2 \langle(\rho - \rho_0)^2\rangle + \left(\frac{\partial R}{\partial l}\right)^2 \langle(l - l_0)^2\rangle + \left(\frac{\partial R}{\partial A}\right)^2 \langle(A - A_0)^2\rangle$$

$$+ 2\left(\frac{\partial R}{\partial \rho}\right)\left(\frac{\partial R}{\partial l}\right)\langle(\rho - \rho_0)(l - l_0)\rangle + 2\left(\frac{\partial R}{\partial \rho}\right)\left(\frac{\partial R}{\partial A}\right)\langle(\rho - \rho_0)(A - A_0)\rangle$$

$$+ 2\left(\frac{\partial R}{\partial l}\right)\left(\frac{\partial R}{\partial A}\right)\langle(l - l_0)(A - A_0)\rangle$$

$$\frac{\partial R}{\partial \rho} = \frac{1}{\rho_0} R_0, \quad \frac{\partial R}{\partial l} = \frac{1}{l_0} R_0, \quad \frac{\partial R}{\partial A} = -\frac{1}{A_0} R_0$$

$$(\rho - \rho_0) = \Delta\rho = \rho_0 \alpha_\rho \Delta T, \quad (l - l_0) = l_0 \alpha_e \Delta T, \quad (A - A_0) = A_0 2\alpha_e \Delta T$$

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2) cont'd

Since all the Δ's move in the same direction, there is no randomness involved

$\langle(\rho-\rho_0)^2\rangle = \rho_0^2\alpha_\rho^2\Delta T^2 \qquad \langle(\ell-\ell_0)^2\rangle = \ell_0^2\alpha_\ell^2\Delta T^2$

$\langle(A-A_0)^2\rangle = A_0^2(2\alpha_\ell)^2\Delta T^2 \qquad \langle(\rho-\rho_0)(\ell-\ell_0)\rangle = \rho_0\alpha_\rho\Delta T\,\ell_0\alpha_\ell\Delta T$

$\langle(\rho-\rho_0)(A-A_0)\rangle = \rho_0\alpha_\rho\Delta T\,A_0(2\alpha_\ell)\Delta T$

$\langle(\ell-\ell_0)(A-A_0)\rangle = \ell_0 A_0 (2\alpha_\ell^2)\Delta T^2$

$$\Delta R^2 = R_0^2\Delta T^2\left(\alpha_\rho^2+\alpha_\ell^2+(2\alpha_\ell)^2+2\alpha_\rho\alpha_\ell-2\alpha_\rho(2\alpha_\ell)-2\alpha_\ell(2\alpha_\ell)\right)$$

$$= R_0^2\Delta T^2(\alpha_\rho^2+\alpha_\ell^2-2\alpha_\rho\alpha_\ell) = R_0^2\Delta T^2(\alpha_\rho-\alpha_\ell)^2$$

$$|\Delta R| = R_0|(\alpha_\rho-\alpha_\ell)||\Delta T| = 100\Omega(1.039-1.0029)$$
$$= 3.61\Omega$$

pretty close!

d) $\left(\delta(\Delta R)\right)^2 = \left(\left(\frac{\partial R}{\partial(\Delta T)}\right)\delta(\Delta T)\right)^2$

$$\frac{\partial R}{\partial(\Delta T)} = \alpha_\rho R_0\frac{(1+\alpha_\ell\Delta T)}{(1+2\alpha_\ell\Delta T)} + \alpha_\ell R_0\frac{(1+\alpha_\rho\Delta T)}{(1+2\alpha_\ell\Delta T)} - \frac{2\alpha_\ell R_0(1+\alpha_\rho\Delta T)(1+\alpha_\ell\Delta T)}{(1+2\alpha_\ell\Delta T)^2}$$

$$= R_0\left(\alpha_\rho\left(\frac{1.0029}{1.0058}\right)+\alpha_\ell\left(\frac{1.039}{1.0058}\right)-2\alpha_\ell\left(\frac{(1.039)(1.0029)}{(1.0058)^2}\right)\right)$$

$$= 0.036\,\Omega/°C$$

$\delta(\Delta R) = (0.036\,\Omega/°C)(0.1°C) = 0.0036\Omega$ error in ΔR

<u>note</u> can also use

$\delta(\Delta R) \approx |R_0(\alpha_\rho-\alpha_\ell)\delta(\Delta T)| = 0.00361\Omega$!

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3) Proceed as in class, form S 's

$$S = \sum_i \frac{1}{\sigma_i^2} = \frac{1}{(.05)^2}\left[\frac{1}{(1.036)^2} + \frac{1}{(.9104)^2} + \frac{1}{(.3010)^2} + \frac{1}{(0.1919)^2} + \frac{1}{(0.0953)^2}\right]$$

$$S_x = \sum_i \frac{x_i}{\sigma_i^2} = \frac{1}{(0.05)^2}\left[\frac{0.01000}{(1.036)^2} + \frac{-38.8344}{(0.9104)^2} + \frac{-189.3442}{(0.3010)^2} + \frac{-218.7916}{(0.1919)^2} + \frac{-248.5939}{(0.0953)^2}\right]$$

$$S_y = \sum_i \frac{y_i}{\sigma_i^2} = \frac{1}{(0.05)^2}\left[\frac{1}{1.036} + \frac{1}{0.9104} + \frac{1}{0.3010} + \frac{1}{0.1919} + \frac{1}{0.0953}\right]$$

$$S_{xx} = \sum_i \frac{x_i^2}{\sigma_i^2} = \frac{1}{(0.05)^2}\left[\frac{(0.01000)^2}{1.036^2} + \frac{(38.8344)^2}{(0.9104)^2} + \frac{(189.3442)^2}{(0.3010)^2} + \frac{(218.7916)^2}{(0.1919)^2} + \frac{(248.5939)^2}{(0.0953)^2}\right]$$

$$S_{xy} = \sum_i \frac{x_i y_i}{\sigma_i^2} = \frac{1}{(0.05)^2}\left[\frac{0.01000}{1.036} + \frac{-38.8344}{0.9104} + \frac{-189.3442}{0.3010} + \frac{-218.7916}{0.1919} + \frac{-248.5939}{0.0953}\right]$$

$S = 60175$, $S_x = -1.418 \times 10^7$, $S_y = 8436.06$

$S_{xx} = 3.40076 \times 10^9$, $S_{xy} = -1.76815 \times 10^6$

$DET = S S_{xx} - S_x^2 = 3.56986 \times 10^{12}$

$a = (S_{xx} S_y - S_x S_{xy})/DET = 1.01315$

$b = (-S_x S_y + S S_{xy})/DET = 0.00370454$

$\sigma_a^2 = S_{xx}/DET = 9.52634 \times 10^{-4}$

$\sigma_b^2 = S/DET = 1.68564 \times 10^{-8}$

$\sigma_{ab}^2 = -S_x/DET = 3.97214 \times 10^{-6}$

$P = a + bT$

$y = a + bx$, $x = \frac{y-a}{b}$

$$\sigma_x^2 = \left(\frac{\partial x}{\partial a}\sigma_a\right)^2 + \left(\frac{\partial x}{\partial b}\sigma_b\right)^2 + 2\frac{\partial x}{\partial a}\frac{\partial x}{\partial b}\sigma_{ab}^2$$

$$= \left(\frac{-x_0 \sigma_a}{a}\right)^2 + \left(\frac{x_0}{b}\sigma_b\right)^2 + 2\left(-\frac{x_0}{a}\frac{x_0}{b}\right)\sigma_{ab}^2$$

to find T_{abs}, set $P=0$ $\quad T_{abs} = -a/b = \frac{1.0135}{0.00770454} = -273.489$

$$\delta T_{abs} = \sqrt{(273.489)^2 \frac{9.52634 \times 10^{-4}}{(1.01315)^2} + \left(\frac{273.489}{0.00370454}\right)^2 1.68564 \times 10^{-8} - 2\left(\frac{273.489 \cdot 273.489}{(1.01315)(0.00370454)}\right) 3.97214 \times 10^{-6}}$$

so $T_{abs} = -273.489 \pm 1.7233 \Rightarrow$ agrees well $(-273.15^\circ C)$

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checks: 1) Chi-Squared

$$\chi^2 = \sum_i \frac{(y_i - (a + b x_i))^2}{\sigma_i^2}$$

$$S_{yy} = 5\left(\frac{1}{0.05}\right)^2$$

$$\chi^2 = \sum_i \frac{y_i^2}{\sigma_i^2} - 2a\sum \frac{y_i}{\sigma_i^2} - 2b\sum\frac{x_i y_i}{\sigma_i^2} + a^2\sum_i \frac{1}{\sigma_i^2}$$

$$+ 2ab\sum_i \frac{x_i}{\sigma_i^2} + b^2\sum \frac{x_i^2}{\sigma_i^2}$$

$$\chi^2 = S_{yy} - 2aS_y - 2bS_{xy} + a^2 S + 2abS_x + b^2 S_{xx}$$

$$= 2000 - 2(1.01315)(8436.06) - 2(0.00370454)(-1.76815\times10^6)$$

$$+ (1.01315)^2\, 60175 + 2(1.01315)(0.00370454)(-1.418\times10^7)$$

$$+ (0.00370454)^2(3.40076\times10^9)$$

$$= 3.19141$$

$$\chi^2/\text{degree of Freedom} = \frac{3.1914}{5-2} = 1.0638$$ pretty close to 1, not too bad