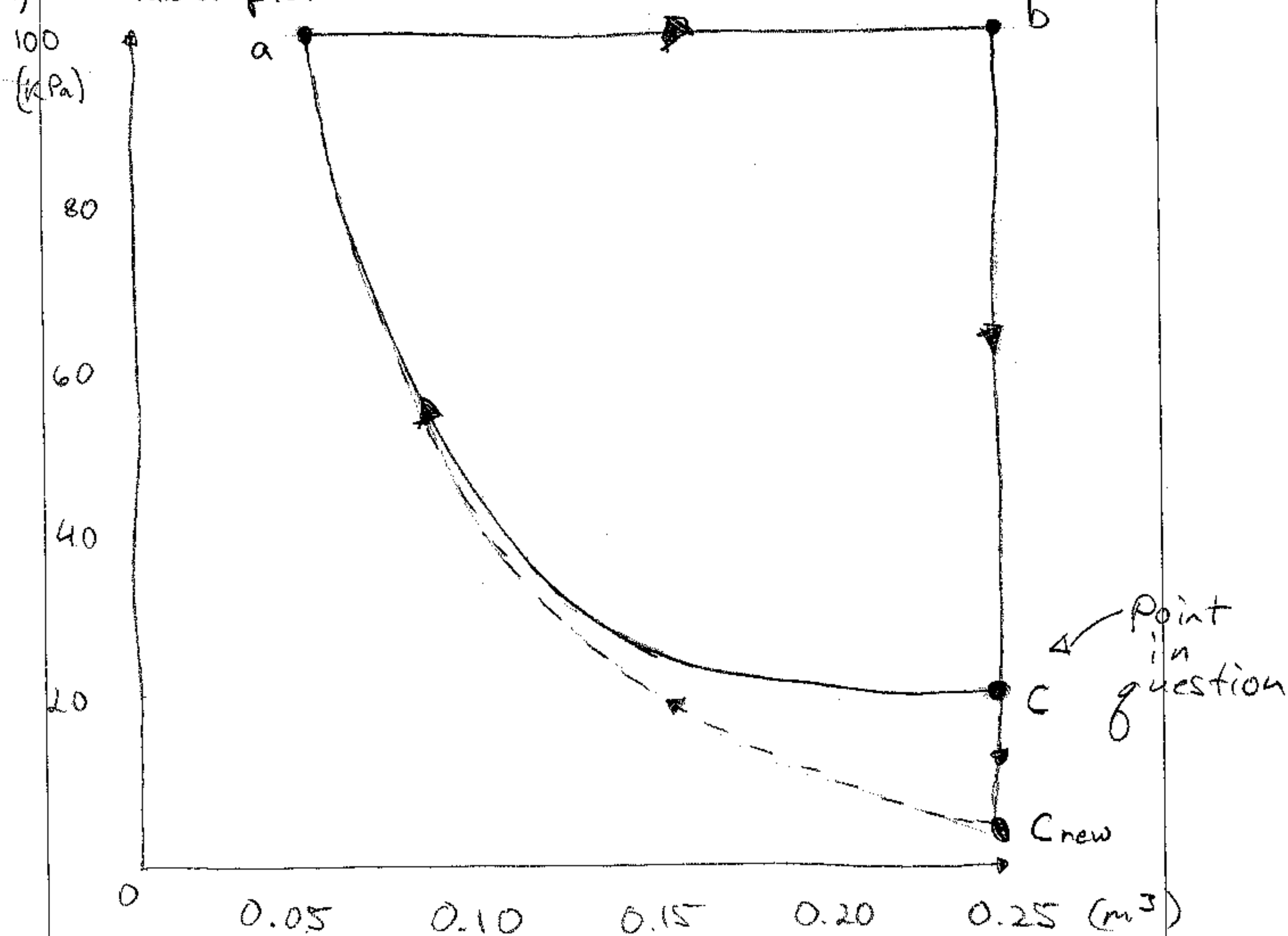
1) Draw a picture



a) check for adiabatic $\gamma = 1.667$ for ideal gas

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 100\,\text{kPa} \left(\frac{.05}{.25}\right)^{1.667}$$

$$= 6.84\,\text{kPa}\ !$$

no! not possible

b) modify point C to go to 6.84 kPa

a) i) Know total entropy in a cycle with reversible gas $\Delta S_{cycle} = 0$

$$\Delta S_{12} = \int_1^2 \frac{dQ}{T} = C_V \ln\left(\frac{T_2}{T_1}\right) + Nk_B \ln\left(\frac{V_2}{V_1}\right)$$

For (a-b) have $\Delta S_{ab} = C_V \ln\left(\frac{T_b}{T_a}\right) + Nk_B \ln\left(\frac{V_b}{V_a}\right)$

$$= C_p \ln T_b/T_a$$

For (b→c) have $\Delta S_{bc} = C_V \ln T_c/T_b$

(use $P_a V_a^\gamma = P_c V_c^\gamma$ or $T_a V_a^{\gamma-1} = T_c V_c^{\gamma-1}$) $\quad = C_V \ln\left(\frac{T_a}{T_b} \cdot \left(\frac{V_a}{V_c}\right)^{\gamma-1}\right)$

$$= C_V \ln\left(\frac{T_a}{T_b}\right) + C_V\left(\frac{C_p}{C_V} - 1\right) \ln\left(\frac{V_a}{V_c}\right)$$

$$= C_V \ln\left(\frac{T_a}{T_b}\right) + (C_p - C_V) \ln\left(\frac{V_a}{V_c}\right)$$

but $V_c = V_b$ and $C_p - C_V = Nk_B$

so $\Delta S_{bc} = C_V \ln\left(\frac{T_a}{T_b}\right) + Nk_B \ln\left(\frac{V_a}{V_b}\right)$

& since $\Delta S_{ab} + \Delta S_{bc} = 0$ $\Delta S_{ca} = 0$ too which we would have guessed since $dQ = 0$ in an adiabatic process

ii) The efficiency of a cycle is defined $\frac{\text{Work you get}}{\text{energy you pay for}}$

for a cycle $\Delta E_{INT} = 0 = Q_{put\ in} - |Q|_{take\ out} - W_{net}$

in the isobaric process $Q_{put\ in} = C_p \Delta T$

$W_{\substack{done \\ by\ gas}} = p\,\Delta V$

isochoric process $|Q_{take\ out}| = C_V |\Delta T|$ (no work done)

adiabatic $\Delta E_{INT} = W_{\substack{done\ by \\ gas}}$

c) hottest temp is at biggest $PV = P_B V_B$

lowest temp is at lowest $PV = P_c V_c$

$$\text{Maximum Efficiency} = 1 - \frac{T_c}{T_H} = 1 - \frac{P_c V_c}{P_B V_B} \quad \begin{matrix} \leftarrow Nk_B T_C \\ \leftarrow Nk_B T_B \end{matrix}$$

$$= 1 - \frac{(6.84\,kPa)(0.25\,m^3)}{(100\,kPa)(0.25\,m^3)}$$

$$= 0.932 \quad \text{wow!}$$

Our actual efficiency will be $\frac{W}{Q_H} = 1 - \frac{|Q_c|}{Q_H}$

Since no heat flowed during the adiabatic cycle

$$Q_H = C_p \Delta T_{ab} = \frac{5}{2}(P_b V_b - P_a V_a)$$

$$Q_c = C_v \Delta T_{bc} = \frac{3}{2}(P_c V_c - P_b V_b)$$

$$\varepsilon = 1 - \frac{3}{5} \frac{|(6.84(0.25) - 100(0.25))|}{((100)(0.25) - 100(0.05))}$$

$$= 1 - \frac{34.9\,kJ}{50.0\,kJ} = 0.30 \quad \text{yikes, better check}$$

Work should be $Q_H - |Q_c| = 15.1\,kJ$

should be $W_{ab} + W_{ca} = P_a(V_b - V_a) + \int_{V_c}^{V_a} \frac{P_c V_c^\gamma}{V^\gamma} dV$

$$= P_a(V_b - V_a) - \frac{P_c V_c^\gamma}{(\gamma - 1)}\left(\frac{1}{V_a^{\gamma-1}} - \frac{1}{V_c^{\gamma-1}}\right)$$

$$= P_a(V_b - V_a) - \frac{1}{(\gamma-1)}\left[\frac{P_c V_c^\gamma}{V_a^\gamma} V_a - P_c V_c\right]$$

$$= P_a(V_b - V_a) - \left(\frac{1}{\gamma - 1}\right)\left[P_a V_a - P_c V_c\right]$$

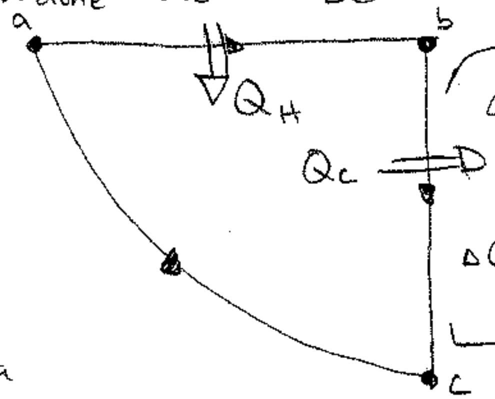
$$= P_a(V_b - V_a) - \left(\frac{C_v}{C_p - C_v}\right)\left[P_a V_a - P_c V_c\right]$$

$$= \frac{C_p(P_b V_b - P_a V_a) - C_v(P_b V_b - P_a V_a) - C_v(P_a V_a - P_c V_c)}{Nk_B}$$

$$= \frac{5}{2}(P_b V_b - P_a V_a) - \frac{3}{2}(P_b V_b - P_c V_c) \quad \text{same}$$

lets look at the whole cycle now

a - b
$$\begin{cases} \Delta E_{Int} = \frac{3}{2}\left(100\,kPa\left[0.25\,m^3 - 0.05\,m^3\right]\right) = 30\,KJ \\ W_{done} = 20\,KJ \quad \Delta Q = 50\,KJ \text{ (heat flowed into gas)} \end{cases}$$



b - c

$$\Delta E_{Int} = \frac{3}{2}\left(6.84\,kPa(0.25\,m^3) - 100\,kPa(0.25\,m^3)\right)$$
$$= -34.9\,kJ$$
$$\Delta Q = -34.9\,kJ$$
$$W = 0$$

c → a

$$\Delta E_{INt} = \frac{3}{2}\left(100\,kPa(0.05\,m^3) - 6.84\,kPa(0.25\,m^3)\right) = 4.94\,KJ$$

$$\Delta Q = 0$$
$$W = -4.94\,KJ$$

check $\Delta E_{INT} = 0$ for cycle

$$\varepsilon = \frac{20\,kJ - 4.94\,kJ}{50\,kJ} = 0.30$$

check entropies

$$\Delta S_{ab} = C_p \ln P_b V_b / P_a V_a = N k_B \frac{5}{2} \ln\left(\frac{(100\,kPa)(0.25\,m^3)}{(100\,kPa)\,0.05\,m^3)}\right)$$
$$= (N k_B)\,4.02$$

$$\Delta S_{bc} = C_v \ln\left(P_c V_c / P_b V_b\right)$$
$$= N k_B \frac{3}{2} \ln\left(\frac{(6.84\,kPa)(0.05\,m^3)}{(100\,kPa)(0.05\,m^3)}\right) = (N k_B)(-4.02)$$

$$\Delta S_{ca} = 0$$

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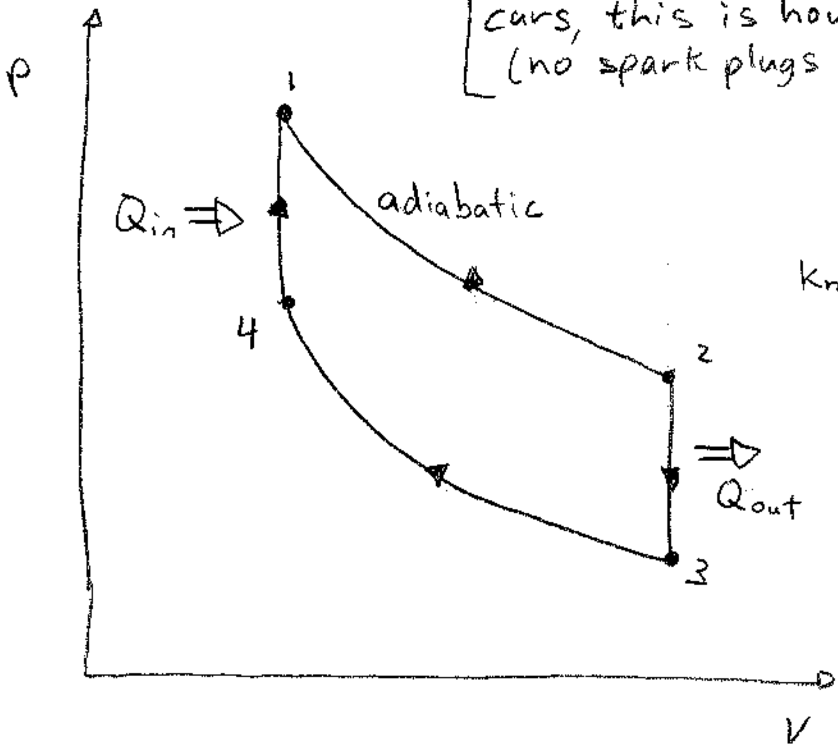
Otto Cycle

[gas can spontaneously ignite if it is compressed too much. In diesel cars, this is how the fuel burns – (no spark plugs in a diesel engine!)] ✓ higher efficiency



need to find:

efficiency in terms of temp & volume

know

$$\varepsilon = 1 - \frac{|Q_c|}{Q_H} = \frac{W}{Q_H}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_3 V_3^{\gamma} = P_4 V_4^{\gamma}$$

$$\Delta Q_{12} = 0 = \Delta Q_{34}$$

$$W_{23} = 0 = W_{41}$$

Q_H or Q we put in is only going to occur on the 4→1 transition

$$Q_H = C_v (T_1 - T_4)$$

Q_c or Q we take out only occurs on the 2→3 transition

$$|Q_c| = C_v (T_2 - T_3)$$

replace $\frac{P_x V_x}{N k_B}$

$$\varepsilon = 1 - \frac{C_v (T_2 - T_3)}{C_v (T_1 - T_4)} = 1 - \frac{(T_2 - T_3)}{(T_1 - T_4)}$$

$$= 1 - \frac{P_2 V_2 - P_3 V_3}{P_1 V_1 - P_4 V_4} = 1 - \frac{(P_2 - P_3) V_2}{(P_1 - P_4) V_1}$$

$$\boxed{P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}, \quad P_3 = P_4 \left(\frac{V_4}{V_3}\right)^{\gamma} = P_4 \left(\frac{V_1}{V_2}\right)^{\gamma}}$$

$$\varepsilon = 1 - \frac{\left(P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} - P_4 \left(\frac{V_1}{V_2}\right)^{\gamma}\right) V_2}{(P_1 - P_4) \quad V_1} = 1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$$