1)

a) Estimated Power Output of the sun

assume $\underbrace{\left(\text{Intensity of solar radiation}\right)}^{I_o} = \dfrac{\text{Power from cell}}{\text{area of cell}}$

(100% efficiency)

$$\text{Power of sun} = I_o \left(\text{area of sphere @ } R_{\substack{earth \\ sun}}\right)$$

$$= \frac{(3.75\,W)}{12.2\,in \times 6\,in} \cdot \left(\frac{1\,in}{0.0254\,m}\right)^2 4\pi \left(1.496 \times 10^{11}\,m\right)^2$$

$$= 2.23 \times 10^{25}\ W$$

b) maximize $\quad u(\lambda) = \dfrac{1}{\lambda^5} \dfrac{1}{e^{hc/\lambda k_B T} - 1}$ w.r.t. λ

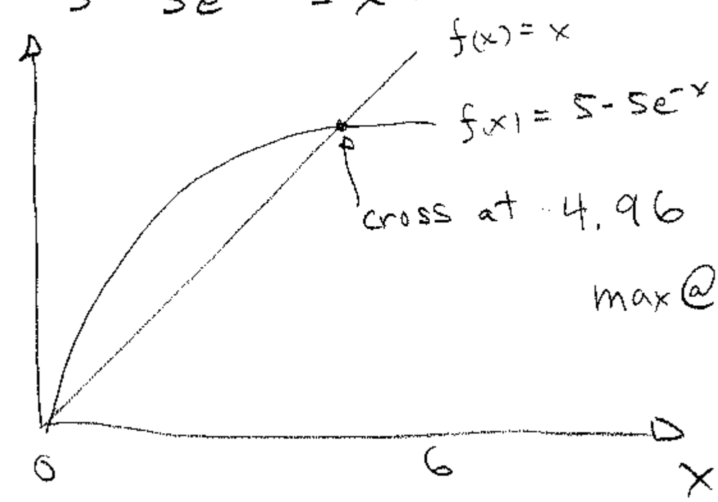
$$\frac{du(\lambda)}{d\lambda} = -\frac{5}{\lambda^6} \frac{1}{e^{hc/\lambda k_B T} - 1} + \frac{\left(\frac{hc}{\lambda^2 k_B T}\right) e^{hc/\lambda k_B T}}{\left(e^{hc/k_B T} - 1\right)^2} \frac{1}{\lambda^5}$$

@ max $\quad 0 = -\dfrac{5}{\lambda^6}\left(e^{hc/\lambda k_B T} - 1\right) + \dfrac{1}{\lambda^7}\left(\dfrac{hc}{k_B T}\right) e^{hc/\lambda k_B T}$

(T = ∞ exteremum not interesting for this)

$$= -5e^x + 5 + x e^x$$

$$5 - 5e^{-x} = x$$



max @ $4.96 = \dfrac{hc}{\lambda k_B T}$

or $\lambda_{max} T = \left(\dfrac{hc}{k_B}\right) \dfrac{1}{4.96}$

total energy density $\frac{U}{V} = \int u(\lambda)\, d\lambda$

$$\frac{U}{V} = \int_0^\infty \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1}\, d\lambda$$

let $x = \frac{hc}{\lambda k_B T}$

$$dx = -\frac{hc}{\lambda^2 k_B T} d\lambda$$

$$\frac{U}{V} = 8\pi hc \, -\int_\infty^0 \left(\frac{k_B T}{hc}\right)^3 x^3 \left(\frac{1}{e^x - 1}\right)\left(\frac{k_B T}{hc}\right) dx$$

$$= 8\pi hc \left(\frac{k_B T}{hc}\right)^4 \underbrace{\int_0^\infty x^3 \left(\frac{1}{e^x - 1}\right) dx}_{\text{look it up! } \frac{\pi^4}{15}}$$

$$= \frac{8}{15}\pi^5 \frac{k_B^4 T^4}{(hc)^3}$$

so $I_{@surface} = \left(\frac{c}{4}\right) \frac{8}{15} \pi^5 \frac{k_B^4}{(hc)^3} T^4$

$$= \left(\frac{3.0 \times 10^8 \frac{m}{s}}{4}\right) \frac{8}{15} \pi^5 \frac{\left(1.381 \times 10^{-23} \frac{J}{K}\right)^4}{\left(1240 \times 10^{-9} \times 1.602 \times 10^{-19} J\, m\right)^3} T^4$$

$$= 5.68 \times 10^{-8} \frac{W}{m^2} \frac{1}{K^4} T^4$$

@ 500nm we have

$$T = \frac{1240\, eV\, nm}{500\, nm} \frac{1}{8.617 \times 10^{-5} eV/K} \frac{1}{4.96}$$

$= 5802K$ (need this temp in light bulbs too:

So power output of sun

$$P_{sun} = \left(5.68 \times 10^{-8} \frac{W}{m^2} \frac{1}{K^4}\right)(5802\,K)^4 (4\pi)(6.96 \times 10^8 m)^2 = 3.92 \times 10^{26} W$$

c) The efficiency of this solar cell is

$$\frac{2.23\times10^{25}\,W}{3.92\times10^{26}\,W} = 0.057$$

5.7% efficient

2) Review conductivity

$$I = A\frac{N}{V}q v_D \qquad v_D = \frac{qE}{m_e}\tau$$

avg time between collisions

$$= A\frac{N}{V}q\left(\frac{qE}{m_e}\tau\right)$$

$$= \underbrace{A\frac{N}{V}\frac{q^2\tau}{m_e}}_{\text{property of the conductor, } \sigma\text{, conductivity}}\left(\frac{V}{d}\right)$$

what we're after is

$$\tau = \frac{\sigma\, m_e}{\left(\frac{N}{V}q^2\right)}$$

avg time between collisions $\approx \dfrac{\text{avg distance between collisions}}{\text{avg speed of electrons}}$

get this from fermi momentum

For gold $\sigma = \dfrac{1}{\rho} = \dfrac{1}{2.44\times10^{-8}\,\Omega m}$

resistivity

density of gold $= 19.3\times10^{3}\ Kg/m^3$

Atomic mass $= 0.19697\ Kg$

$$\frac{N}{V} = \text{Density of charge carriers} = \frac{6.02\times10^{23}}{\text{mole}}\cdot\frac{1\text{mole}}{0.19697\,Kg}\cdot\frac{19.3\times10^{3}\,Kg}{m^3}$$

$$= 5.899\times10^{28}/m^3$$

@ T = 0 K we calculated

$$\langle \varepsilon \rangle = \left\langle \frac{p^2}{2m}\right\rangle = \frac{3}{5}\frac{p_F^2}{2m} = \frac{1}{2}m\,v_{avg}^2$$

$$p_F = \frac{h}{2}\left(\frac{3}{\pi}\frac{N}{V}\right)^{\frac{1}{3}}$$

$$\langle \varepsilon \rangle = \frac{3}{5} \frac{\left(\frac{h}{2}\right)^2 \left(\frac{3}{\pi} \frac{N}{V}\right)^{\frac{2}{3}}}{2 m_e}$$

$$= \frac{3}{40} \frac{(hc)^2}{m_e c^2} \left(\frac{3}{\pi} \frac{N}{V}\right)^{\frac{2}{3}}$$

$$= \frac{3}{40} \frac{(1240 eV \times 10^{-9} m)^2}{511{,}000 eV} \left(\frac{3}{\pi} 5.899 \times 10^{28}/m^3\right)^{\frac{2}{3}}$$

$$= 3.3 eV \quad (\overset{\text{means}}{\varepsilon_f} \text{ for gold is } 5.53 eV)$$

estimate

avg distance between collisions

$$= \frac{\sigma \cdot m_e}{\left(\frac{N}{V} q^2\right)} \sqrt{\frac{2}{m_e} \langle \varepsilon \rangle} = \frac{\sigma m_e}{\left(\frac{N}{V} q^2\right)} c \sqrt{\frac{2 \langle \varepsilon \rangle}{m_e c^2}}$$

$$= \frac{\left(\frac{1}{2.44 \times 10^{-8} \frac{(J/C)}{(C/s)} m}\right) 9.1 \times 10^{-31} kg \cdot 3.0 \times 10^8 \frac{m}{s}}{(5.899 \times 10^{28}/m^3)(1.6 \times 10^{-19} C)^2} \sqrt{\frac{2(3.3 eV)}{511{,}000 eV}}$$

$$= 26.6 \times 10^{-9} m \quad (26.6 nm)$$

(I'll accept 34.3 nm too)

3) $N_1 = N_0 e^{-\Delta\varepsilon / k_B T}$

$$\text{average energy} = \frac{N_0 \varepsilon_0 + N_1 \varepsilon_1}{N_0 + N_1} = \frac{N_1 \varepsilon}{N_0 + N_1}$$

$$= \frac{(N_0 \varepsilon)\, e^{-\varepsilon / k_B T}}{N_0 + N_0 e^{-\varepsilon / k_B T}}$$

$$= \varepsilon \left(\frac{1}{e^{\varepsilon / k_B T} + 1} \right)$$

@ T very large, $e^{\varepsilon / k_B T} \Rightarrow e^0 = 1$

$\varepsilon_{avg} \Rightarrow \epsilon / 2$

@ T very small $e^{\varepsilon / k_B T} \Rightarrow e^{\infty}$

$\varepsilon_{avg} \Rightarrow 0$

$$\frac{C_v}{N} = \frac{dE}{dT} = \varepsilon \left(\frac{\varepsilon}{k_B} \right) \frac{1}{T^2} \frac{e^{\varepsilon / k_B T}}{(e^{\varepsilon / k_B T} + 1)}$$