Homework 4

1) Activity $= \lambda N$

λ: decay constant; N: # of atoms present

Since the Ruthenium isotope is not expected to occur in pure form in a random sample, we'll assume it's all created by the Technetium

So, the number of ${}^{99}_{43}Tc$ we started with will be

$$\underbrace{\#\,{}^{99}_{43}Tc_{now}}_{N(t)} + \#\,{}^{99}_{44}Ru_{now} = N(0)$$

${}^{99}_{43}Tc$ decays like $N(t) = N(0)\,e^{-\lambda t}$

$$\text{so } \ln\left(\frac{N(t)}{N(0)}\right) = -\lambda t$$

$$t = \ln\left(\frac{N(t)}{N(0)}\right) \Big/ -\lambda$$

$$\frac{N(t)}{N(0)} = \frac{\dfrac{\#\,{}^{99}_{43}Tc_{now}}{tot}}{\dfrac{\#\,{}^{99}_{43}Tc_{now}}{tot} + \dfrac{\#\,{}^{99}_{44}Ru_{now}}{tot}} = \frac{0.012}{0.012+2.2}$$

$$\lambda = \frac{\text{Activity}}{N} \approx \frac{1200/s}{\left(6.02\times10^{23}/99g\right)\cdot 1\times10^{-6}g} = 1.973\times10^{-13}/s$$

$$t = \ln\left(\frac{0.012}{0.012+2.2}\right) \Big/ \left(-1.93\times10^{-13}/s\right) = 2.7\times10^{13}s$$

$= 8.57\times10^{5}$ years (this is a slightly used model!)

note: we can't make the other components from Tc

i) The difference in number of nucleons must be divisible by 4 for α decay

ii) can't make higher A unless there's a beam or source of big nuclei

iii) Can make the smaller A by fission I guess, but we should see the other products!

2) look at B.E. for A & Z # of nucleons

$$BE \text{ for } {}^{A-1}_{Z}X = (A-1)\,m(n)c^2 + Z\,m(H)c^2 - m({}^{A-1}_{Z}X)c^2$$

$$\text{for } {}^{A}_{Z}X = A\,m(n)c^2 + Z\,m(H)c^2 - m({}^{A}_{Z}X)c^2$$

Binding energy for one neutron and ${}^{A-1}_{Z}X$

$$BE = m(n)c^2 + m\left({}^{A-1}_{Z}X\right)c^2 - m\left({}^{A}_{Z}X\right)c^2 = S_n$$

this is the energy we need to supply to knock out that neutron

${}^{17}O$ $\quad S_n = m(n)c^2 + m\left({}^{16}_{8}O\right)c^2 - m\left({}^{17}_{8}O\right)c^2$

$= \left(1.008664924u + 15.994915u - 16.999132u\right) 931.5\,\frac{MeV}{u}$

$= 4.14\ MeV$

${}^{7}Li$ $\quad S_n = \left(1.008664924u + 6.015122u - 7.016004u\right) \frac{931.5}{u}$

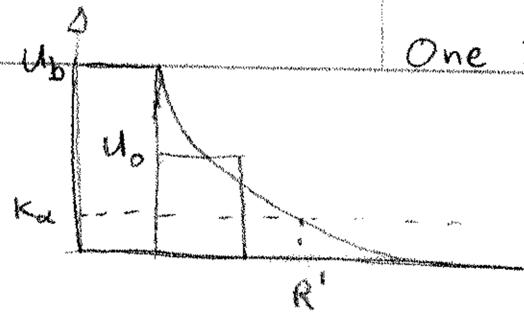
$= 7.24\ MeV$

${}^{57}Fe$ $\quad S_n = \left(1.008664924u + 55.934924u - 56.935398u\right) \frac{931.5\,MeV}{u}$

$= 7.63\ MeV$

One Interpretation

3)



$U_o = \frac{1}{2}(U_b + K_\alpha)$, $U_o - E = \frac{1}{2}(U_b - K_\alpha)$

$L = \frac{1}{2}(R' - R)$

want $t_{1/2}$ for ^{232}Th & ^{218}Th

$\{U_b = 30\,\text{MeV}\}$ $\{K_\alpha = 4.01\,\text{MeV},\ 9.85\,\text{MeV}$

measured

$^{232}t_{1/2} = 1.4\times10^{10}\,y$

$^{218}t_{1/2} = 1.1\times10^{-7}\,s$

$$R = (1.2\times10^{-15}\,m)\,A^{1/3} = 7.37\,fm,\ 7.22\,fm$$

$$R' = \left(\frac{K q^2 (2)(Z-2)}{K_\alpha}\right) = \frac{\left(9\times10^9 \frac{Nm^2}{C^2}\right)(1.6\times10^{-19}C)^{\cancel{2}}(2)(90-2)}{4.01\times10^6 \cancel{(1.6\times10^{-19}C)}\,V}$$

$$= 6.32\times10^{-14}\,m \quad {}^{232}Th$$

$$= 6.32\times10^{-14}\,m \cdot \frac{4.01}{9.85} = 2.57\times10^{-14}\,m \quad \text{for } {}^{218}Th$$

$$\lambda = \frac{v}{2R} e^{-2KL}$$

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{\ln(2)\,2R}{v} e^{2KL}$$

^{232} $L = \frac{1}{2}(63.2 - 7.37)\,fm = \frac{55.83\,fm}{2}$

^{218} $L = \frac{1}{2}(25.7 - 7.22)\,fm = \frac{18.48\,fm}{2}$

$$v = \left(\sqrt{2K_\alpha/mc^2}\right)c$$

^{232}Th $= \left(\sqrt{\frac{2(4.01)\,\text{MeV}}{3727.3803\,\text{MeV}}}\right)c = 1.39\times10^7\,m/s$

^{218}Th $= 2.18\times10^7\,m/s$

$$K = \sqrt{\left(\frac{2m}{\hbar^2}\right)(U_o - E)} = \sqrt{\frac{8\pi^2 mc^2}{(hc)^2}\left(\frac{1}{2}(U_b - K_\alpha)\right)}$$

$$^{232}K = \sqrt{\frac{8\pi^2(3727.3803\,\text{MeV})}{(1240\times10^{-6}\,\text{MeV nm})^2}\frac{1}{2}(30\,\text{MeV} - 4.01\,\text{MeV})}$$

$$= 1.58\times10^6/nm = 1.58/fm$$

$$^{218}K = 1.39\times10^6/nm = 1.39/fm$$

$$^{232}t_{1/2} = \frac{\ln(2)(2)(7.37\times10^{-15}m)}{1.39\times10^7\,m/s} \frac{e^{2(1.58/fm)(27.92\,fm)}}{(365\,d/y)(24\,h/d)(3600\,s/h)} = 4.83\times10^9\,y$$ not too bad

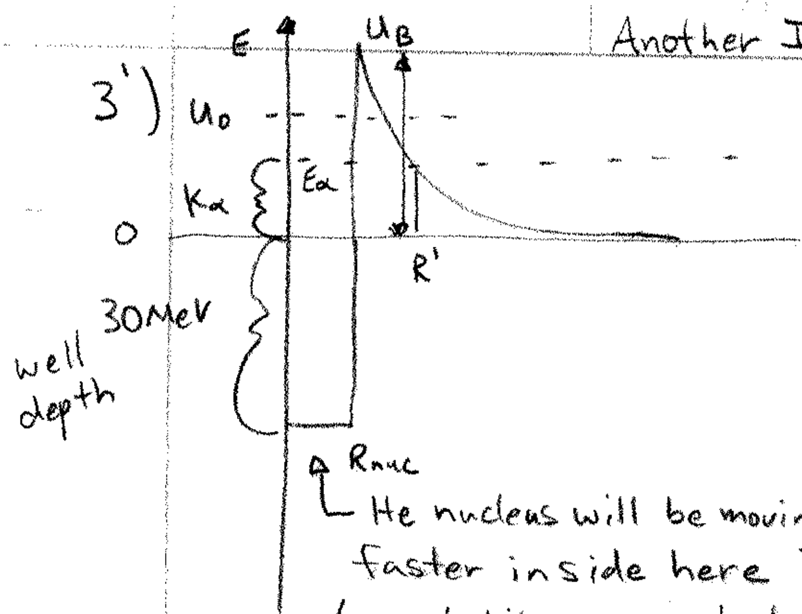
$$^{218}t_{1/2} = \frac{\ln(2)(2)(7.22\times10^{-15}m)}{2.18\times10^7\,m/s} e^{2(1.39)(9.24)} = 6.57\times10^{-11}\,s$$ bit off!

see class notes for "better" job

but $R/R' \sim 1$ for ^{218}Th!

Another Interpretation

3')



He nucleus will be moving faster inside here (just like a rocket having v_{esc} at earths surface has v = 0 just outside the earths potential well

very deceptive part of problem!

$$U_b = \frac{1}{4\pi\varepsilon_0}\frac{q^2(2)(Z-2)}{R_{nuc}}$$

$$v = \sqrt{(30\text{MeV} + K_\alpha)2/m}$$

232 $U_b = K_\alpha \frac{R'}{R_{nuc}} = 4.01\text{MeV}\left(\frac{63.2}{7.37}\right)$
$= 34.39\text{MeV}$

218 $U_b = 9.85\text{MeV}\left(\frac{25.7}{7.22}\right) = 35.06$ MeV

if U_0 is midway between U_b & E_α

$$U_0 = \frac{(30\text{MeV} + U_b) + (30\text{MeV} + K_\alpha)}{2}$$

$$= 30\text{MeV} + \frac{1}{2}(U_b + K_\alpha)$$

$$E = 30\text{MeV} + K_\alpha$$

$$U_0 - E = \frac{1}{2}(U_b - K_\alpha)$$

232 $v = c\sqrt{(34\text{MeV})2/3727.3803\text{MeV}}$
$= 4.05\times10^7 \text{m/s}$

218 $v = c\sqrt{39.85\text{MeV}(2)/3727.38\text{MeV}}$
$= 4.39\times10^7 \text{m/s}$

232 $K = \frac{1.58}{\text{fm}}\cdot\sqrt{\frac{(34.39-4.01)}{30-4.01}}$
$= \frac{1.70}{\text{fm}}$

218 $K = \frac{1.39}{\text{Fm}}\sqrt{\frac{(35.06-9.85)}{(30-9.85)}}$
$= \frac{1.55}{\text{Fm}}$

232 $t_{1/2} = \frac{\ln(2)(2)(7.37\times10^{-15}\text{m})}{4.05\times10^7 \text{m/s}}\,\frac{e^{2(1.7/\text{fm})(27.92\text{Fm})}}{3.15\times10^7 \text{ s/y}} = 1.35\times10^{12}$ y

218 $t_{1/2} = \frac{\ln(2)(2)(7.22\times10^{-15}\text{m})}{4.39\times10^7 \text{m/s}}\,e^{2(1.55)(9.24)} = 6.27\times10^{-10}$ s

to do a better job, we really need to know the shape of the potential. (then we could do tricks like in class) 232 $t_{1/2} = 1.2\times10^{14}$ y
class 218 $t_{1/2} = 2.2\times10^{-6}$ s

4) Rate of decay goes like # of Atoms that can decay

$$R(t) = R_0 e^{-\lambda t} = R_0 e^{-\ln(2)\, t/t_{1/2}}$$

$$R_0 = \lambda N_0$$

$$\frac{R(0)}{R(t)} = \frac{R_0}{R(t)} = e^{\ln(2)\, t/t_{1/2}}$$

$$\ln\left(\frac{R_0}{R}\right) = \ln(2)\, t/t_{1/2}$$

$$t = t_{1/2} \frac{\ln\left(\frac{R_0}{R}\right)}{\ln(2)} = 5730\,y \frac{\ln\left(\frac{12.4}{3.5}\right)}{\ln(2)}$$

$$= 1.05 \times 10^4\,y \quad (3.3 \times 10^{11}\,s)$$

(can also ask how many half lives is this $\left(\frac{1}{2}\right)^N = \frac{3.5}{12.4}$ $N = 1.825$

$t = N(t_{1/2}) =$ same