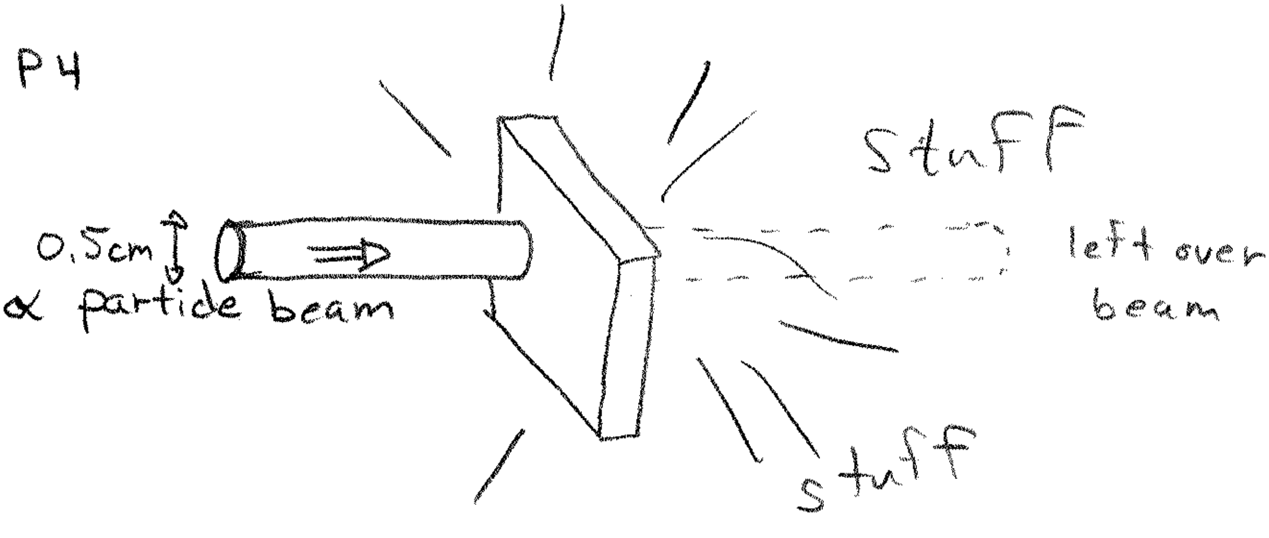
Homework 5

Krane

C. 13 P 4, 8, 15, 21, 31

P 4



recall our scattering expression from class

$$\frac{\#\text{ scattered particles}}{s} = \begin{pmatrix}\text{Probability}\\ \text{of}\\ \text{scattering}\end{pmatrix} \frac{\#\text{ incident particles}}{s}$$

& the Probability $= \dfrac{\text{effective area}}{\text{beam area}}$

$$\text{effective area} = \underbrace{\sigma}_{\text{cross section}} \underbrace{\frac{N}{V}}_{\#\text{ of nuclei/Volume}} \underbrace{t}_{\text{target thickness}} \underbrace{A}_{\text{beam area}}$$

For ^{63}Cu density is $8.92\times10^{3}\,\frac{kg}{m^3}\cdot\underbrace{\left(\frac{63}{(63\cdot.692+65\cdot.308)}\right)}_{\text{my estimate based on abundance}}$

$= 8.83\times10^{3}\,\frac{kg}{m^3}$

$$\frac{N}{V} = \left(\frac{6.02\times10^{23}}{mole}\right)\cdot\frac{1\,mole}{0.063\,kg}\cdot 8.83\times10^{3}\,\frac{kg}{m^3} = 8.44\times10^{28}/m^3$$

$$\frac{\#\text{ incident particles}}{s} = I_\alpha/\text{charge of }\alpha = \left(7.5\times10^{-6}\,\frac{C}{s}\right)/(2\cdot1.6\times10^{-19}\,C)$$

$$= 2.34\times10^{13}/s$$

so, $\dfrac{\#\text{ neutrons emitted}}{s} = \underbrace{(1.25\times10^{-28}\,m^2)}_{\sigma}\underbrace{(8.44\times10^{28}/m^3)}_{N/V}\underbrace{(2.5\times10^{-6}\,m)}_{t}(2.34\times10^{13}/s)$

$= 6.17\times10^{8}/s$

assumes no absorption in target & no attenuation of the beam (good assumption, not much stuff comes out relatively)

P8

The rate at which an isotope is produced

$$R = \begin{pmatrix}\text{Probability of}\\ \text{interaction}\end{pmatrix} \text{Rate of incident beam}$$

assuming the beam covers the whole sample $\Longrightarrow$

$$= \underbrace{\begin{pmatrix}\#\text{ of nuclei present}\\ \text{in beam}\end{pmatrix}}_{\dfrac{\text{mass of sample}}{\text{molar mass of sample}}} \underbrace{\sigma_I}_{\substack{\text{interaction}\\ \text{cross section}}} \underbrace{\frac{\#\text{ incident beam particles}}{\text{s (Area of beam)}}}_{\text{beam flux, } \phi}$$

The number of isotopes present after an irradiation time t_0 is

$$N(t_0) = \frac{R}{\lambda}\left(1 - e^{-\lambda t_0}\right) \quad \text{like a charging up capacitor}$$

& the activity is $\lambda N(t_0) = R\left(1 - e^{-\lambda t_0}\right)$

$$\lambda N(t_0) = \left(N_A \frac{m}{M}\right) \sigma_I\, \phi \left(1 - e^{-\lambda t_0}\right)$$

$$m = \frac{M \lambda N(t_0)}{N_A \sigma_I\, \phi} \frac{1}{\left(1 - e^{-\lambda t_0}\right)}$$

^{60}Co $\lambda = \dfrac{\ln(2)}{(5.27\,y)\left(31.5\,\frac{Ms}{y}\right)} = 4.175\times10^{-9}/s$

^{51}Ti $\lambda = \dfrac{\ln(2)}{5.8\,m \cdot \frac{60\,s}{m}} = 1.99\times10^{-3}/s$

For ^{60}Co λt_0 is tiny, so we can taylor expand

$$m = \frac{M(\lambda N(t_0))}{N_A \sigma_I\, \phi} \frac{1}{\lambda t_0} \qquad \left\{\lambda t_0 = (4.175\times10^{-9}/s)(150\,s)\right\} = 6.26\times10^{-7}$$

$$= \left(\frac{59\,g}{mole}\right)\left(12/s\right) \Big/ \frac{6.02\times10^{23}}{mole} \cdot 19\times10^{-28}\,m^2 \cdot \frac{3.0\times10^{16}}{s\cdot m^2} \cdot 6.26\times10^{-7}$$

$$= 3.3\,\mu g$$

For ^{50}Ti (5.25% of Ti present)

$$m = \frac{M(\lambda N(t_0))}{N_A \sigma_I\, \phi} \frac{1}{(1 - e^{-\lambda t_0})} = \frac{\left(\frac{50\,g}{mole} \cdot \frac{105}{s}\right)}{\frac{6.02\times10^{23}}{mole} \cdot 1.4\times10^{-29}\,m^2 \cdot \frac{3.0\times10^{16}}{s \cdot m^2}\left(1 - e^{-0.2985}\right)}$$

$$= 8.05\times10^{-8}\,g$$

$$\text{mass of Ti present} = \frac{8.05\times10^{-8}\,g}{.0525} = 1.53\,\mu g$$

P 15 Energy is Conserved
momentum is conserved

^2H is moving non-relativistically

before

$^2H \xrightarrow[5.000\,MeV]{} \quad {}^3He$

After

$? \longrightarrow {}^1_1p$ (treat as ^1_1H)
$\longrightarrow {}^4He$

Energy $\quad m(^2H)c^2 + m(^3He)c^2 + \underbrace{5.000\,MeV}_{KE_0} = m(^1_1p)c^2 + KE_p + m(^4He)c^2 + KE_{He}$

momentum $\quad m(^2H)\,v_0 = p_0 = \overbrace{m(^1_1p)v_p}^{p_p} + \overbrace{m(^4He)v_{He}}^{p_{He}}$

Think about this a second. If this collision takes place in the center of mass, we can have one of 2 situations

$^2H \longrightarrow \longleftarrow {}^3He \Longrightarrow {}^1_1p \longleftarrow \longrightarrow {}^4He$
or
$^4He \longleftarrow \longrightarrow {}^1_1p$

lets get some stuff out of the way:

$Q = (2.014102u + 3.016029u - 1.007825u - 4.002603u)\dfrac{931.5\,MeV}{u}$
$= 18.35\,MeV$ (lots of extra energy)

so $\quad 23.35\,MeV = KE_p + KE_{He} = \dfrac{p_p^2}{2m_p} + \dfrac{p_{He}^2}{2m_{He}}$

$\sqrt{2m_{^2H}KE_0} = p_0 = 136.97\,\dfrac{MeV}{c} = p_p + p_{He}$

$p_{He}c = 136.97\,MeV - p_pc$

$23.35 = \dfrac{p_p^2c^2}{2m_pc^2} + \dfrac{(136.97 - p_pc)^2}{2m_{He}c^2}$ all in MeV

mult both sides by $2m_pc^2$ $\quad 43{,}841 = p_p^2c^2 + 0.25179(18760.7 - 273.94p_pc + p_p^2c^2)$

or $\quad 0 = 1.25179p_p^2c^2 - 68.975p_pc + 39{,}150.8$

$p_pc = \dfrac{68.975 \pm \sqrt{68.975^2 + 4(1.25179)(39{,}150.8)}}{2(1.25179)}$

$= 206.53\,MeV, -151.43\,MeV \Rightarrow KE_p = 22.72\,MeV, 12.2\,MeV$

so $KE_\alpha = 23.35\,MeV - KE_p = 0.63\,MeV, 11.1\,MeV$

P21

Calculate neutron separation energies → excitation energies

^{236}U $\quad S_n = [1.008665u + 235.043922u - 236.045561u]\frac{931.5 MeV}{u}$

$= 6.54 MeV$ = energy available to excite ^{236}U

^{238}U $\quad S_n = [1.008665u + 238.050784u - 239.054289u]\frac{931.5 MeV}{u}$

$= 4.81 MeV$ = energy available to excite ^{239}U

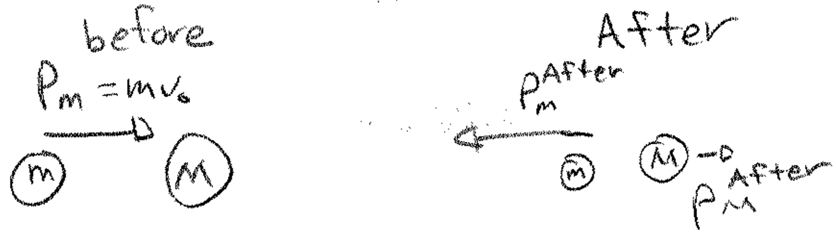
If we need about 6.54 MeV to initiate fission in a large nucleus, ^{238}U will need an additional ~1.7 MeV of energy to get there

^{239}Pu $\quad S_n = [1.008665u + 239.052156u - 240.053807u]\frac{931.5 MeV}{u}$

$= 6.53 MeV$

has a high excitation energy too, probably has fission at low neutron energies.

P3 1



in center of momentum frame, particles

$|{}^{cm}\vec{P}_m| = |{}^{cm}\overleftarrow{P}_M|$ before

$|{}^{cm}\overleftarrow{P}_m| = |{}^{cm}\vec{P}_M|$ after

just reverse direction
but keep same $|P|$

$$m v_0 = (m + M) V_{cm}$$

before $\quad {}^{cm}P_m = m(\overbrace{V_0 - V_{cm}}^{V'_{before}})$, $\quad {}^{cm}P_M = M(-V_{cm})$

after $\quad {}^{cm}P_m = m(\overbrace{V_{cm} - V_0}^{V'_{after}})$, $\quad {}^{cm}P_M = M(V_{cm})$

translate to lab frame for

$$V^{After} = V_{lab} = V'_{after} + V_{cm} = 2V_{cm} - V_0$$

$$= \frac{2m(V_0)}{m+M} - V_0 = \frac{2m(V_0) - (m+M)V_0}{m+M}$$

$$= \frac{m-M}{m+M} V_0$$

$$\Delta KE = \frac{1}{2} m V_0^2 - \frac{1}{2} m \left(\frac{(m-M)}{m+M}\right)^2 V_0^2$$

$$= \frac{1}{2} m V_0^2 \left(\frac{(m+M)^2 - (m-M)^2}{(m+M)^2}\right)$$

$$= KE_0 \frac{4mM}{(m+M)^2} = KE_0 \frac{4m/M}{(1 + m/M)^2}$$