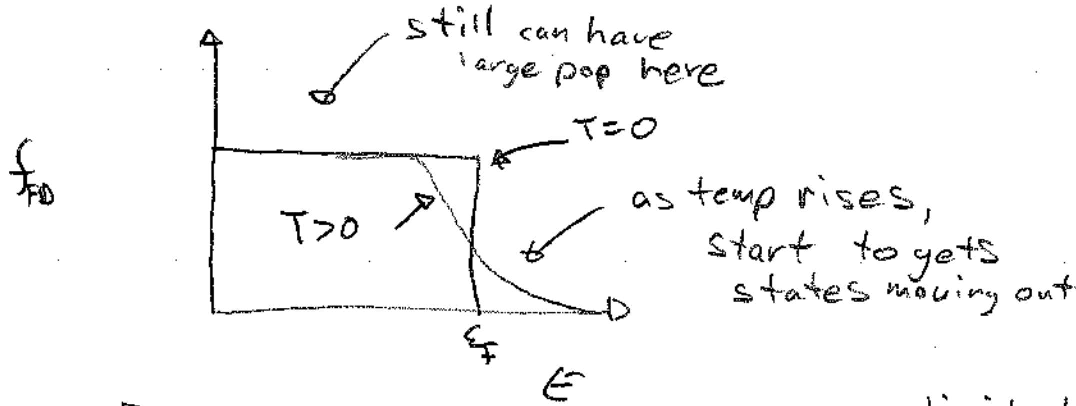
Fermi - Dirac particles

$$f_{FD}(\epsilon) = \frac{1}{e^{(\epsilon-\epsilon_F)/k_BT}+1}$$

@ low temp $f_{FD}(\epsilon) = 1 \qquad \epsilon < \epsilon_F$

$= 0 \qquad \epsilon > \epsilon_F$



ex Fermi gas at low T

$$N_{particles} = \underset{\text{spins}}{2}\ \underset{\text{positive octants}}{\frac{1}{8}}\ 4\pi \int_0^{n_F} n^2\, dn$$

limit to states at low temp

$p = \frac{h}{\lambda} \qquad \lambda_x = \frac{2L}{n_x}$

$= \left(\frac{h}{2L}\right)(n_x + n_y + n_z)$ or $\left(\frac{h}{2L} n\right)$

or $p = \frac{h}{2L} n \qquad n = \frac{2L}{h} p$

$dn = \frac{2L}{h} dp$

$\left(\frac{10^{69}\,eV}{R}\right)$

get $N_{part} = \pi \left(\frac{2L}{h}\right)^3 \int_0^{p_F} p^2\, dp$

$$p_F = \frac{h}{2}\left(\frac{3}{\pi}\frac{N}{V}\right)^{\frac{1}{3}}$$

$$\langle \epsilon_F \rangle = \left\langle \frac{p^2}{2m} \right\rangle = \frac{1}{2m} \int_0^{p_F} p^4\, dp \Big/ \int_0^{p_F} p^2\, dp$$

$$= \frac{3}{5}\frac{p_F^2}{2m}$$

<u>application</u> white dwarf star

suppose that we have a star at low temp and very high density. It is essentially a gas of electrons, protons and neutrons.

<u>consider</u> $\langle \varepsilon_f \rangle \propto \frac{1}{2m}$

The energy of the electrons will dominate the total kinetic energy since $m_e \sim \frac{M_{nucleon}}{2000}$ & the total energy of the star will be made up of the gravitational potential energy:

$$-\int_0^{M_{tot}} \frac{G M_{inside}}{r}\, dm$$ (know this will be negative in the end)

$$M_{inside} = \rho \tfrac{4}{3}\pi r^3$$

$$dm = (4\pi r^2 dr)\rho$$

$$U_{tot} = -\int_0^R G\rho^2 \tfrac{1}{3}(4\pi)^2 \frac{r^5}{r}\, dr$$

$$= -G\rho^2 \tfrac{1}{3}(4\pi)^2 \frac{R^5}{5}$$

$$= -G\left(\frac{M_{tot}}{\frac{4}{3}\pi R^3}\right)^2 \tfrac{1}{3}(4\pi)^2 \frac{R^5}{5}$$

$$= -\frac{3}{5}\frac{G M_{tot}^2}{R}$$

$$E = N_e \frac{3}{5}\frac{1}{2m_e}\left(\frac{h}{2}\right)^2\left(\frac{3}{\pi}\frac{N}{\frac{4}{3}\pi R^3}\right)^{\frac{2}{3}} - \frac{3}{5}\frac{G M_{tot}^2}{R}$$

consider $M_{sun} = 2\times 10^{30}\,kg$

"Atom" 1e:1p:1n $\quad N_e = \frac{1}{2}\frac{M_{sun}}{M_{neutron}} = \frac{1}{2}\frac{2\times 10^{30}\,kg}{1.67\times 10^{-27}\,kg}$

$$= 6\times 10^{56}$$

$m_e c^2 = 511{,}000\,eV$

$hc = 1239 \times 10^{-15}\,eV\,m$

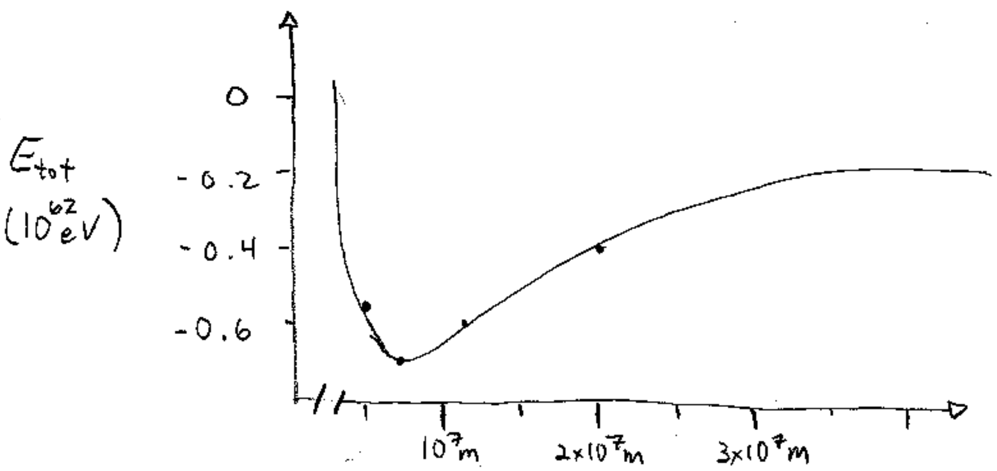
$G = 6.67 \times 10^{-11} \left(\frac{m^3}{kg\,s^2}\right)$

$$E = \frac{3}{5}\,\frac{6\times10^{56}}{2(511{,}000\,eV)}\left(\frac{1239\times10^{-9}\,eVm}{2}\right)^2\left(\frac{9}{4\pi}\,6\times10^{56}\right)^{\frac{2}{3}}\frac{1}{R^2}$$

$$-\ \frac{3}{5}\left(6.67\times10^{-11}\frac{m^3}{kg\,s^2}\right)\left(2\times10^{30}kg\right)^2\frac{1}{R}\times\left(\frac{1\,eV}{1.6\times10^{-19}J}\right)$$

$$= 3.589\times10^{75}\,eV\,m^2\,\frac{1}{R^2} - \frac{10^{69}\,eVm}{R}$$

Plot this



Find minima = equilibrium

$$\frac{dE}{dR} = 0 = -2\left(3.6\times10^{75}\,eVm^2\right)\frac{1}{R^3} + \frac{(10^{69}\,eVm)}{R^2}$$

$$R = \frac{2\left(3.6\times10^{75}\,eVm^2\right)}{10^{69}\,eVm} = 7.2\times10^{6}\,m$$

or about 7000 Km

(sun is about 100 times this size)

This isn't the end of the story...

when ε_F for the electrons is about

$$(m_n c^2 - m_p c^2) = 1.3\ \text{MeV}$$

It is actually more favorable for the electron to combine with a proton and make a neutron.

In our case

$$N_e \varepsilon_F = \left(\frac{3.6 \times 10^{75}\ \text{eV}\,\text{m}^2}{(7.2 \times 10^{6}\ \text{m})^2} \right)$$

$$\varepsilon_F = 0.115\ \text{meV}$$

Notice that this means the electrons are a bit relativistic for this to occur, and we'll need to modify our approach.