Last time we calculated the radius of a white dwarf star with the mass of the sun to be 7.2×10^6 m. The white dwarf has a density of

$$(2\times10^{30}\,\mathrm{kg})\Big/\tfrac{4}{3}\pi(7.2\times10^{6}\,\mathrm{m})^3$$

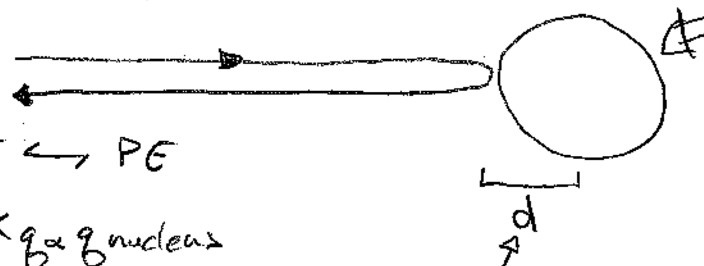
$$=1.3\times10^{9}\,\mathrm{kg/m^3}\ !$$

Around 1910, 2 graduate students conducted an experiment where they bombarded a gold foil with α particles (He nuclei)

$\#$ of nuclei $\Rightarrow$ ${}^{4}_{2}He$ $\Leftarrow$ $\#$ protons

isotope - same # protons
isotone - same # neutrons
isobar - same # neucleons
(proton or a neutron is a nucleon)

These α particles have a charge of 2 and an energy of 6 MeV. Sometimes they saw the particles bounce back! Implies a dense nucleus



gold has $Z=79$

convert $KE \leftrightarrow PE$

$$\frac{1}{2}mv^2=\frac{k q_\alpha q_{nucleus}}{d}$$

$$d=\frac{k q_\alpha q_{nucleus}}{\frac{1}{2}mv^2}$$

$$=\frac{9\times10^9\ N\frac{m^2}{C^2}\,(1.6\times10^{-19}C)^2\cdot 2\cdot 79}{6\times10^6\,V\,(1.6\times10^{-19}C)}$$

$$=3.8\times10^{-14}\,m$$

If you think of all this stuff packed together in a sphere

$$\rho_{nucleus}=197(1.67\times10^{-27}\,kg)\Big/\tfrac{4}{3}\pi(3.8\times10^{-14}\,m)^3$$
$$=1.4\times10^{15}\,kg/m^3\quad \text{wow!}$$

Means in order to conserve energy, m_H is less than its constituents. (Let's check carbon)

$$^{12}_{6}C \quad \text{"Binding Energy"} \sim 6M_H + 6M_N - 12.000u$$

$$= 6(1.007825u) + 6(1.008665u) - 12.0000u$$
$$= 9.894 \times 10^{-2} u \cdot 931.5 \frac{MeV/c^2}{u} = 92.163 \; MeV/c^2$$

or about 7.68 MeV/nucleon.

Some nuclei have a higher binding energy and some lower. We'll figure out a way to predict this energy and also when a configuration of protons and neutrons are stable. A nucleus will attempt to become more stable in several ways

Type of decay	Particle emitted	Effect on nucleus	why?
Gamma (γ)	photon	de-excitation	nucleus in excited state
Alpha Decay (α)	He nucleus	looses 2n's & 2p's	too many nucleons
Beta Decay (β^-)	electron + ?	$n \to p$	too many n's
electron capture	? + x ray	$p \to n$	too many p's
Positron emission (β^+)	positron + ?	$p \to n$	too many p's

We've already seen how a random probability can give us an exponential probability that a particle will decay, but theres another way at this problem

For a collection of unstable nuclei, (for now we'll deal with nuclei of the same type)

The rate at which decays occur is directly proportional to the number of particles that can decay

$$\frac{dN}{dt} \propto N$$

now since the number of particles decreases with time, $\frac{dN}{dt}$ is negative

$$\frac{dN}{dt} = -\lambda N$$

λ called the decay constant

λN is called the Activity (sometimes A, sometimes R)

The activity is usually listed in Curies for the sources we use in the lab, but the SI unit is called a becquerel (Bq = 1 decay /s)

$1\ Ci = 3.7 \times 10^{10}$ decays /s < what 1 gram of radium gives you

(Your lab sources are μCi)

To get the behavior of our collection of nuclei then, we integrate

$$\int \frac{dN}{N} = \int -\lambda dt$$

$$\ln(N) = -\lambda t + C$$

$$N = e^{-\lambda t + C}$$

at t=0 we've got all our nuclei

$$N(t) = N_0 e^{-\lambda t}$$

Lets look at an application

^{235}U decays with a half life of $7.04 \times 10^8 y$

^{238}U decays with a half life of $4.47 \times 10^9 y$

If both of these isotopes were created equally how old must the universe be if $\frac{\# \, ^{235}U}{\# \, ^{238}U} = 0.007$ now?

half life = Time needed for $\frac{1}{2}$ to decay

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$t_{1/2} = \left(\ln(2)/\lambda\right)$$

$$N = N_0 e^{-\ln(2) t / t_{1/2}} = N_0 e^{-\lambda t}$$

$$\frac{N_{235}}{N_{238}} = \frac{\cancel{N_0} e^{-\lambda_{235} t}}{\cancel{N_0} e^{-\lambda_{238} t}} = e^{(\lambda_{238} - \lambda_{235}) t}$$

$$t = \frac{\ln\left(\frac{N_{235}}{N_{238}}\right)}{\lambda_{238} - \lambda_{235}} = \frac{\ln(0.007)}{\frac{\ln(2)}{4.47 \times 10^9 yr} - \frac{\ln(2)}{7.04 \times 10^8 yr}}$$

$$= 5.98 \times 10^9 yr$$

Kind of interesting in it's own right. If we really think the universe is $> 12 \times 10^9$ yrs old, where did all the stuff come from?