Suppose that you have a decay chain.

Particle$_1$ → Particle$_2$

Particle$_2$ → X
└ detect this

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

rate at which N_2 appears

rate at which N_2 disappears

guess a solution $N_2(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t}$

@ $t=0$ $N_2(0) = 0$ $A = -B$

$$N_2(t) = A\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$\frac{dN_2}{dt} = A\left(-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}\right)$$

$$= \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 (A)\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

group like terms

$$A(-\lambda_1) e^{-\lambda_1 t} = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 A e^{-\lambda_1 t}$$

simplify

$$A\lambda_2 e^{-\lambda_2 t} = A\lambda_2 e^{-\lambda_2 t}$$

$$-A\lambda_1 = \lambda_1 N_{10} - \lambda_2 A$$

$$A(\lambda_2 - \lambda_1) = \lambda_1 N_{10}$$

$$A = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}$$

$$N_2(t) = \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) N_{10}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

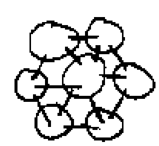
if you have some N_2 to start with

$$N_2(t) = \left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right) N_{10}\left(e^{-\lambda_1} - e^{-\lambda_2 t}\right) + N_{02} e^{-\lambda_2 t}$$

Last time we talked about binding energy in a nucleus and that it has some general properties due to the nature of the strong force, coulomb repulsion of the protons, and the fact that the neutrons and protons are fermions.

1st, lets consider larger nuclei where things are likely to work better on average. Big is about $A = 20$ for our purpose.

Next, consider what it must look like in a big nucleus



The nucleons on the outside have less bonds

The nucleons on the inside has lots of "bonds"

assume # bonds $\propto$ binding energy

Binding energy $\propto$ # nucleons on the inside × # inside bonds + # nucleons on the outside × # outside bonds

or # nucleons (# bonds on the inside) − correction for nucleons on the surface

$$\text{Binding Energy} \propto a_I A - a_S A^{2/3}$$

a_I: interior binding coefficient

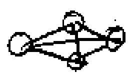
a_S: surface correction term

recall that nucleons are closely packed & the formula $r = r_0 A^{1/3}$ works pretty well for $r_0 = 1.2\,\text{fm}$

Now we need a way to deal with the coulomb repulsion. This we can add in by considering the potential energy of a collection of charged particles in a pairwise fashion (neglecting different r's, well combine everything in $\frac{1}{r_{avg}} \propto \frac{1}{A^{1/3}}$)

how many ways can you choose 2 distinct pairs of items from a collection of Z distinct objects? (need to be distinct) ordered pair $\rightarrow$ $\frac{Z!}{2!\,(Z-2)!}$ ← $N!$ ways to choose N items; ← only interested in a pair

Or you can just work it out

	# Pairwise	# protons	
	1	2	
	3	3	get 2 more
	6	4	get 3 more
	10	5	get 4 more
	⋮	⋮	

or $1 + 3 + 6 + 10 + \cdots$

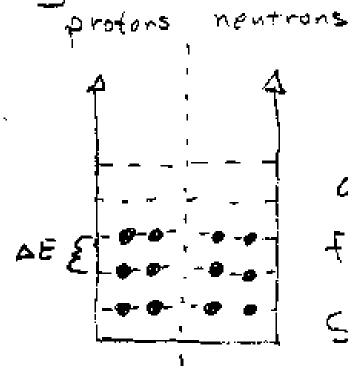
$\frac{1}{2}(Z)(Z-1)$ Z

$$\sim \frac{Z^2}{2}$$

so our binding energy becomes

$$E_{bind} \propto a_I A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}}$$

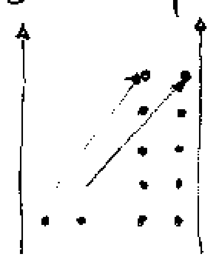
The next term we have to worry about can be thought of 2 ways. One is a simple energy arguement, one is using our fermi gas model. Consider a very simple model of energy levels in a nucleus



recall that we have a mechanism to change from protons ⟷ neutrons So, if we pile on lots of neutrons, for instance, it is likely some of them will convert into protons to get a higher binding energy. (deeper in the well)

Consider the energy cost to promote 2 protons to become neutrons.

$2\Delta E$ { to promote 2more



$2(3\Delta E)$

{ to promote 2 more $2(5\Delta E)$

(# proton pairs)	(# protons) promoted	Energy cost
1	2	$2\Delta E$
2	4	$2\Delta E + 6\Delta E = 8\Delta E$
3	6	$18\Delta E$
4	8	$32\Delta E$
$\frac{X}{2}$	X	$2(\#\text{ proton pairs})^2$

$\#\text{ proton pairs} = (\#\text{ neutrons} - \#\text{ protons})\frac{1}{4}$