Applications of the Semi-Empirical Mass Formula (SEMF)

Consider β decay as we did last time

Inside the nucleus, we are essentially changing a neutron into a proton & an electron

$$n \rightarrow p + e + \Delta\varepsilon$$

or

$$m_I c^2 \rightarrow m_f c^2 + \Delta\varepsilon$$

we can represent it this way since we created both a "new" proton and an electron

So, in order for this to occur, we need $m_I c^2 > m_f c^2$ & recall, A is constant

Now, we'll use the SEMF to try and figure out as a function of Z & A, what nucleus is stable

(for now, we'll skip the δ term)

$$m_I c^2 = (A-Z)\, m_n c^2 + Z m_H c^2 - \left(a_I A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A}\right)$$

$$= A(m_n c^2 - a_I) + a_s A^{2/3} + Z(m_H c^2 - m_n c^2) + a_c \frac{Z^2}{A^{1/3}} + a_A \frac{(A-2Z)^2}{A}$$

lets drop all terms that do not involve Z

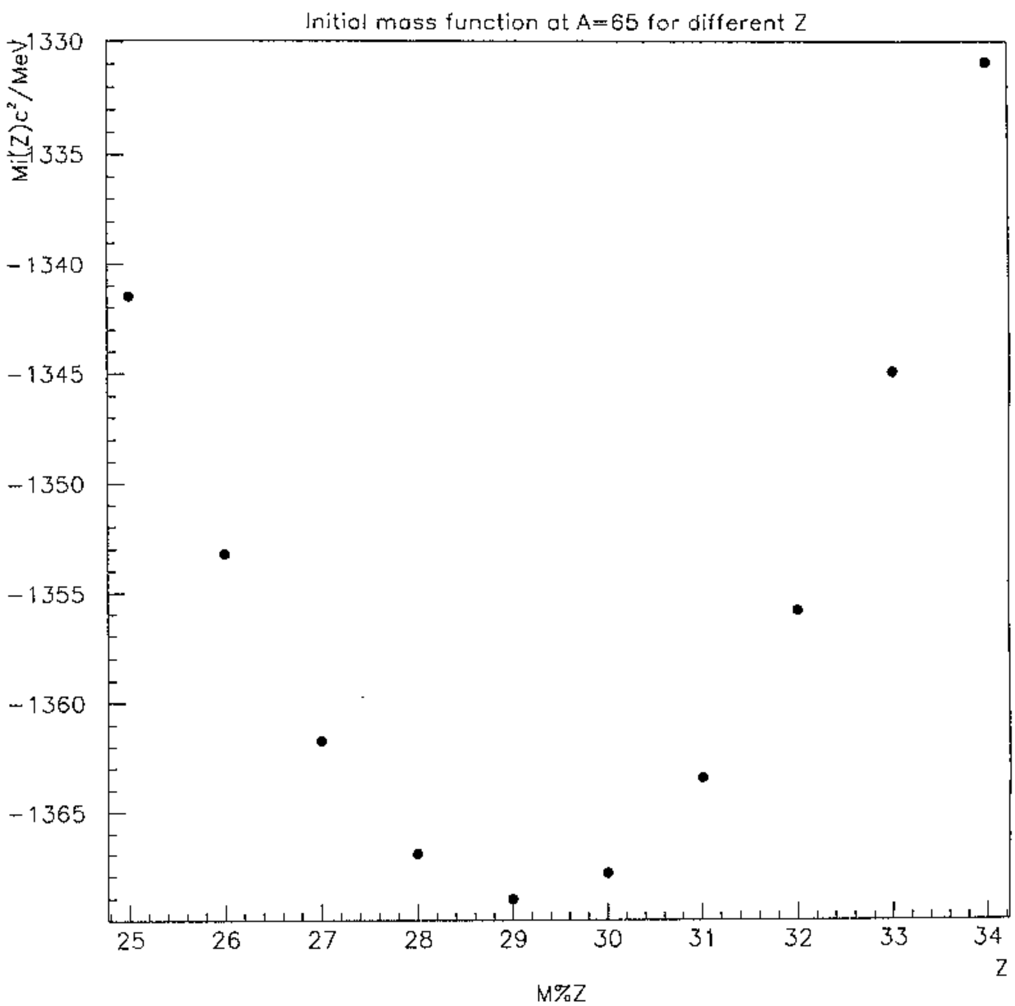
$$m_I(Z)c^2 = \underbrace{Z(m_H c^2 - m_n c^2) + a_c \frac{Z^2}{A^{1/3}} + a_A \frac{4Z^2}{A} - 4Z a_A}_{\text{equation of a parabola}}$$

1) Caveat - we do not consider odd odd or even-even nuclei (δ is always 0)

plug in values

$$\frac{M_I(Z)c^2}{MeV} = Z(-0.7825) + 0.72\left(\frac{Z^2}{A^{1/3}}\right) + 4(23.3)\frac{Z^2}{A} - 4(23.3)Z$$

try even-odd $A=65$



Stable nucleus should be $^{65}_{29}X$

check properties

$^{65}_{27}Co \quad t_{1/2} = 1.20\,s$

$^{65}_{28}Ni \quad t_{1/2} = 2.517\,h$

$^{65}_{29}Cu \quad$ stable

$^{65}_{30}Zn \quad t_{1/2} = 244.3\,d$

$^{65}_{31}Ga \quad 15.2\,m$

bit of a cheat, these really have too many protons

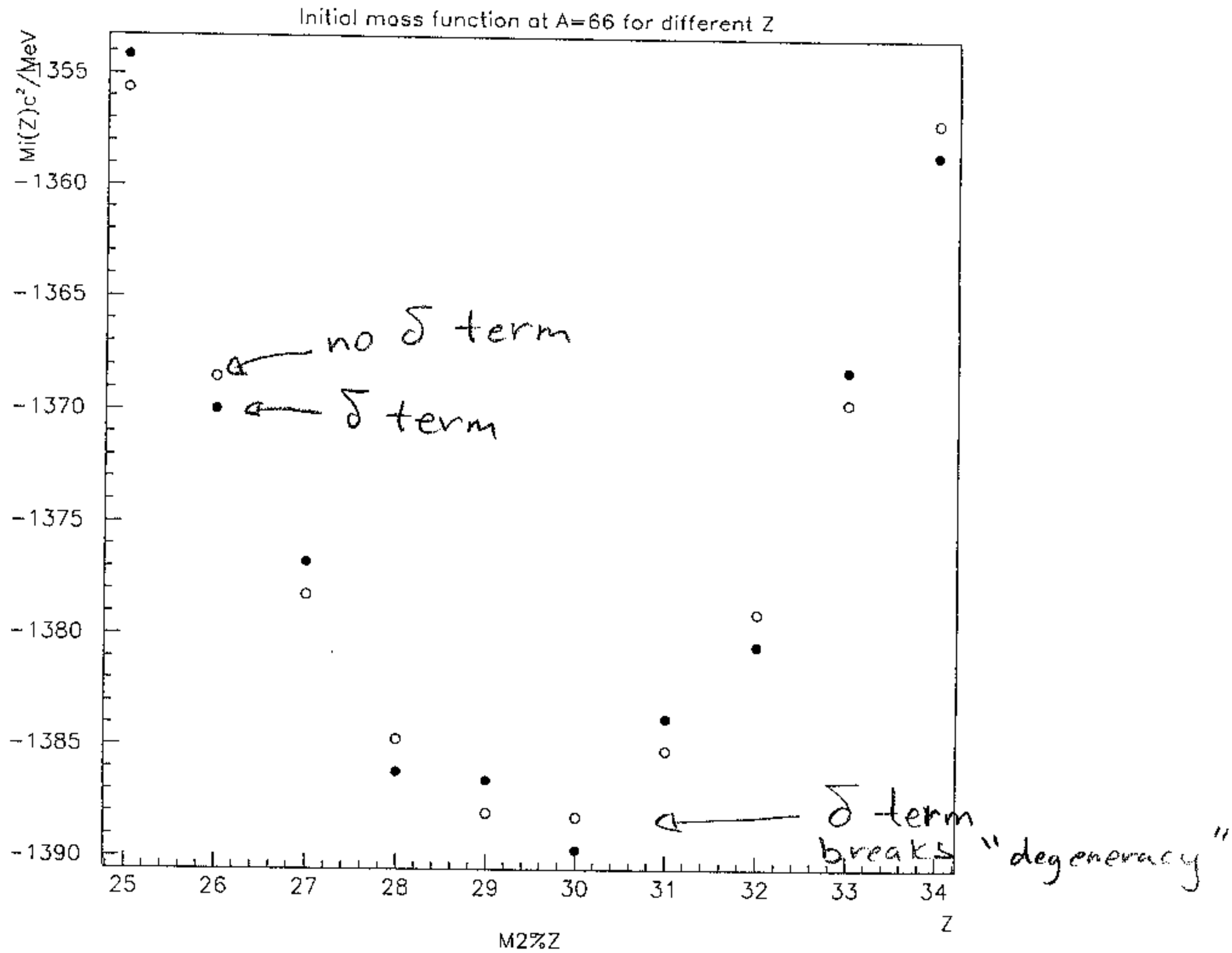
Recall for positron emission

$$m_I \Rightarrow m_F + e^+ + e^- + \Delta E/c^2$$

or $$N m_n c^2 + Z m_H c^2 - E_b \geq (N+1) m_n c^2 + (Z-1) m_H c^2 + 2 m_e c^2$$

If you look on the plot, you see that theenergy difference between $^{65}_{29}Cu$ & $^{65}_{30}Zn$ is very nearly $2 m_e c^2$. Squeezing this difference often makes a decay more difficult, and is a qualitative arguement for the difference between the lifetimes on either side of $^{65}_{29}Cu$

For nuclei with even A, we have to put in the δ term



Check $^{66}_{28}Ni$ $t_{1/2} = 54.6\,h$

$^{66}_{29}Cu$ $t_{1/2} = 5.1\,m$

$^{66}_{30}Zn$ stable

$^{66}_{31}Ga$ $9.49\,h$

When the mass difference is less than $2m_ec^2$, we can still get $p \to n$ inside a nucleus, it just occurs less frequently (sort of a tunnelling effect) than if there were $2m_ec^2$ of energy difference.