Intro Thermo

- operations on ideal gasses
- operations on solids
- definitions

Intro Stat Mech

- Modern Approach to Thermo

Ideal Gas Law

Ideal monatomic gas of non-interacting (like Coulomb) particles

$$PV = nRT = N \frac{R}{N_A} T$$

- $8.31 \, J/mol\,K$
- $0.0821 \, \ell \cdot atm/mol\,K$ ← universal gas constant
- k_B (Boltzman's constant) $1.38 \times 10^{-23} \, J/K$
- n = # moles

$$R = 0.0821 \, \ell \cdot atm / mol \cdot K$$

$$\# \, moles = \frac{\# \, molecules}{N_A} = \frac{N}{N_A}$$

$$PV = \frac{N}{N_A} RT = N k_B T$$
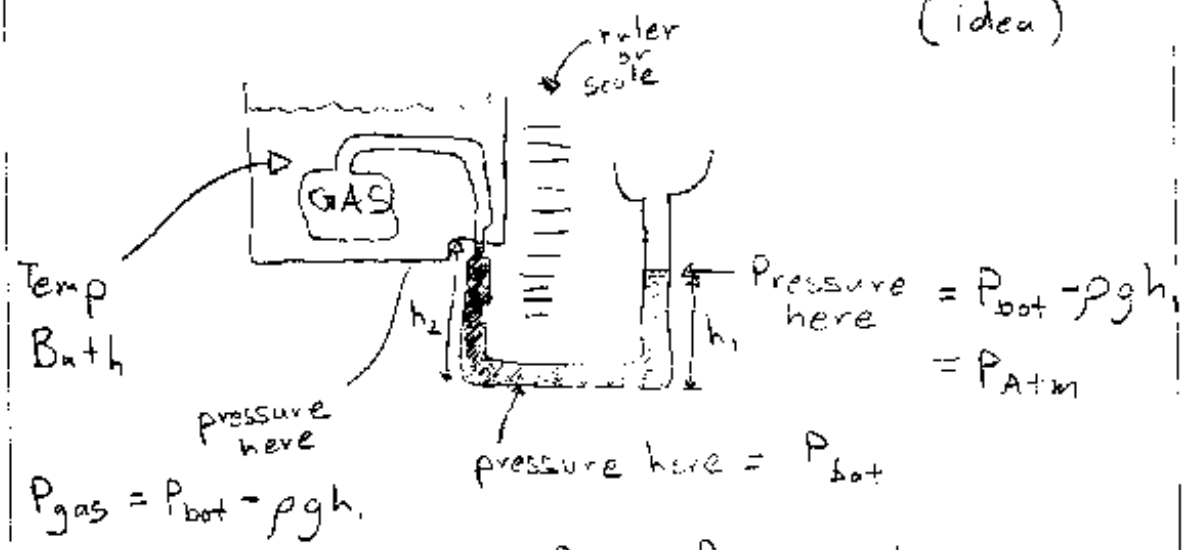
↑ Boltzman's constant

This equation is actually pretty cool. In your homework, you try to find the conversion between Kelvin & Celcius

a number of gasses will intersect at the same T on a P vs T plot

⟹ concept of an absolute 0 of Temp

Method   Constant Volume Gas Thermometer

(idea)

Temp
Bath

$P_{gas} = P_{bot} - \rho g h_2$

ruler
or
scale

pressure
here

pressure here = P_{bot}

Pressure here $= P_{bot} - \rho g h_1$
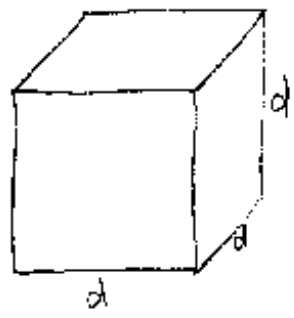$= P_{Atm}$

$P_{bot} = P_{Atm} + \rho g h_1$

$P_{gas} = P_{Atm} + \rho g h_1 - \rho g h_2$

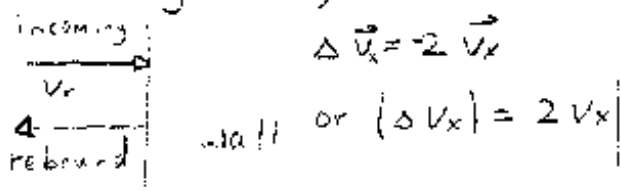try to keep h_2 constant by adding
liquid to manometer.

Plot "P" vs T
get T at known points

"Ballistic" view of an ideal gas.

consider ideal gas confined to a box

Particles of gas
constantly hitting the walls

incoming
V_r

rebound ....wall

$\Delta \vec{u}_x = -2 \vec{V_x}$

or $|\Delta V_x| = 2 V_x$

force on wall due to this
particle $= \frac{|\Delta V x|}{\Delta t}$

but Δt is tough to evaluate, so we consider
the action of many particles hitting a wall

Consider, a particle will hit this wall on average every $2d/\langle |v_x| \rangle$ seconds or so

So the average force on the wall is going to be

$$\frac{\langle \Delta p \rangle}{\langle \Delta t \rangle} = \frac{F_{avg}}{m} = \frac{|\Delta v_x|}{2d/\langle |v_x| \rangle} = \frac{2\langle v_x^2 \rangle}{2d} \quad \text{due to a single particle}$$

In our box, we don't have a favored direction, so

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$

$$\& \quad \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

$$\langle v^2 \rangle = 3\langle v_x^2 \rangle \Rightarrow F_{avg} = \frac{1}{3} \frac{m\langle v^2 \rangle}{d}$$

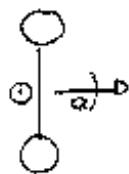$$\text{Pressure} = \frac{F}{A} \Rightarrow P_{avg} = \frac{F_{avg}}{A} = \frac{1}{3} m \langle v^2 \rangle / d^2$$

$$(P_{avg}) \, Vol = \frac{1}{3} m \langle v^2 \rangle = N k_B T$$

$$\text{Avg kinetic energy} = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} N k_B T = E_{int}$$
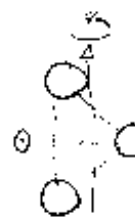
The 3 is special, it's indicative of degrees of freedom

consider has 2 rotational degrees of freedom

$$E_{int} = \frac{5}{2} N k_B T$$

 has 3 rotational degrees of freedom

$$E_{int} = \frac{6}{2} N k_B T$$

At higher temps can have vibrational degrees of freedom too ⟵ usually, more except in solids

Notice that PV has units of energy
in fact, if we push against a piston
we can do work with a gas

$$\Rightarrow \begin{matrix} \text{Work} \\ \text{done} \\ \text{by gas} \end{matrix} \Rightarrow W = \int_A^B P dV$$

So we have 2 ways to change the
energy state of a gas

heat flow $\Rightarrow$ 1) change in temperature
                  2) work

$$\Delta E_{int} = Q - W$$

heat flow   Work   (1st law of
into gas   done   thermodynamics)
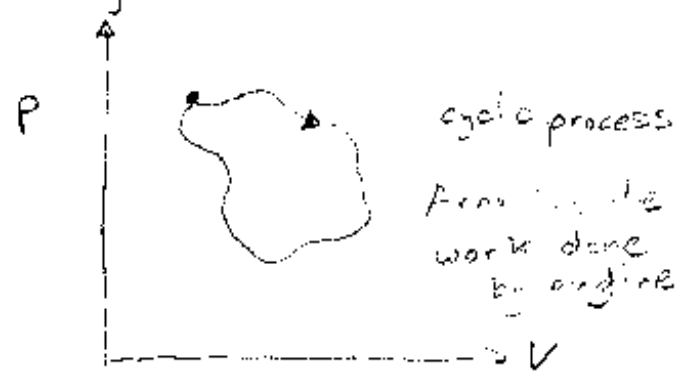   by gas

Conservation
of energy

sometimes you see it

$$\Delta E = Q + W \leftarrow \text{work done on gas}$$

Useful to discuss concept of an engine
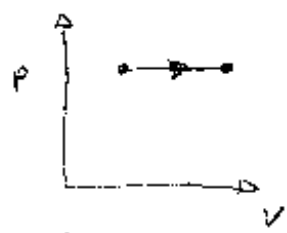
$\rightarrow$ engine $\Rightarrow$ Something that gives us work
                         for energy we put in
cyclic $\Rightarrow$ we can do the process again &
                 again

Ideal gas

cyclic process

Area inside
work done
by engine

To get a handle on what's happening, we
try to restrict ourselves to processes that
are easier to calculate

  isobaric - Process at
            constant Pressure

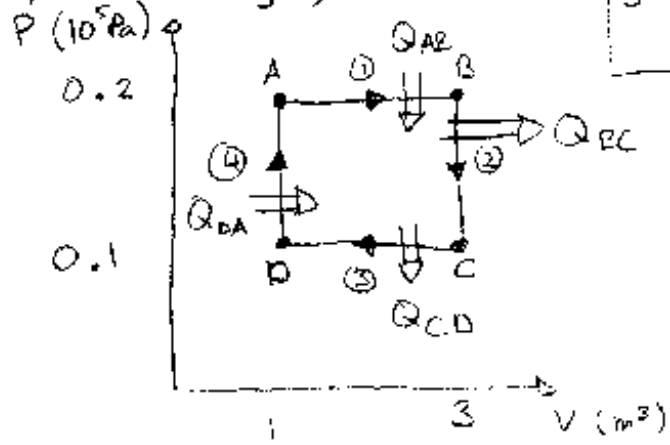 isochoric - Process at
             constant volume

adiabatic - $\Delta Q = 0$
            (insulated)

iso thermal $\Delta T = 0$

For the most part,
we will consider
cases where things
tend to take place
so as to effect the
whole gas. Or at
each step, the
gas is in equilibrium

Consider this engine
(ideal, monatomic gas)

$P (10^5 Pa)$

0.2

0.1

Q_{AB}
Q_{BC}
Q_{DA}
Q_{CD}

A ① B
④   ②
D ③ C

3    $V (m^3)$

① Gas does work at constant Pressure, need to add
   energy (heat) to keep pressure constant

$$\Delta E_{int} = \frac{3}{2} N k_B \Delta T_{AB} = \Delta Q_{AB} - P_A \Delta V_{AB}$$

rewrite as
a Temp. change $N k_B \Delta T_{AB}$

So $\Delta Q_{AB} = Q_{AB} = \frac{5}{2} N k_B \Delta T_{AB}$

② Gas changes pressure, volume is constant $W = 0$

$$\Delta E_{INT} = -|Q_{BC}| \quad (\text{take out energy to get volume smaller})$$

$$= \frac{3}{2} N k_B \Delta T_{BC} \qquad V \Delta P = N k_B \Delta T_{BC}$$

③ Do work on gas to shrink it, need to remove some energy to keep pressure constant

$$\Delta Q_{CD} = -\frac{5}{2} N k_B \Delta T_{CD}$$

$$W_{CD} = -N k_B \Delta T_{CD} = P_C \Delta V_{CD}$$

$$\Delta E_{int} = -\frac{3}{2} N k_B \Delta T_{CD}$$

④ gas changes pressure at constant volume

$$\Delta E_{int} = \frac{3}{2} N k_B \Delta T_{DA}$$

For the whole cycle, the work we get out

$$W_{tot} = P_A(V_B - V_A) + P_C(V_A - V_B)$$

$$= (P_A - P_C)(V_B - V_A)$$

$$= (0.2 - 0.1)(10^5 \tfrac{N}{m^2})(3m^3 - 1m^3) = 2 \times 10^4 \text{ J}$$

And the net heat for the cycle was,

$$Q_{tot} = Q_{AB} - |Q_{BC}| - |Q_{CD}| + Q_{DA}$$

$$= \frac{5}{2} N k_B (T_B - T_A) + \frac{3}{2} N k_B (T_C - T_B) + \frac{5}{2} N k_B (T_D - T_C)$$

$$+ \frac{3}{2} N k_B (T_A - T_D)$$

$$= N k_B [(T_B - T_A) + (T_D - T_C)]$$

$$= P_A(V_B - V_A) + P_C(V_A - V_B)$$

But the heat we added was $Q_{AB} + Q_{DA}$

$$Q_{added} = \frac{5}{2} P_A(V_B - V_A) + \frac{3}{2}(P_A - P_D)V_A$$

$$= \frac{5}{2}(0.2)(10^5 \tfrac{N}{m^2})(3m^3 - 1m^3) + \frac{3}{2}(0.2 - 0.1)(10^5 \tfrac{N}{m^2})(1m^2)$$

$$= 10^5 \text{ J} + 0.3 \times 10^5 \text{ J} = 1.3 \times 10^5 \text{ J}$$

Efficiency $= \dfrac{\text{what we got}}{\text{what we pay for}} = \dfrac{2 \times 10^4 \text{ J}}{1.3 \times 10^5 \text{ J}} = 15.4\%$