

One ~~more~~ property of the state of an ideal gas to talk about. The entropy.

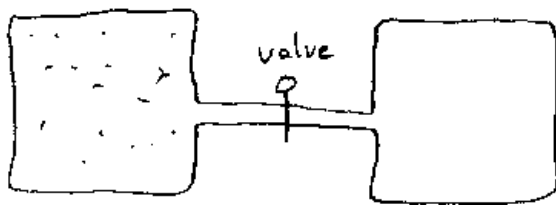
$$dQ = dW + dE_{\text{int}} \\ = PdV + C_v dT$$

FISHBANE
Chapter 20

$$\frac{dQ}{T} = \frac{Nk_B T}{V T} dV + C_v \frac{dT}{T}$$

$$\Delta S = \int_i^F \frac{dQ}{T} = Nk_B \ln(V_F/V_i) + C_v \ln(T_F/T_i)$$

Why is this useful?



Suppose I open a valve and let the gas fill the second chamber
 $\Delta E_{\text{int}} = 0, \Delta Q = 0, \Delta W = 0$

In order to get it back into the bottle, we'd have to do work on the gas. Since we got no work out of the gas, we've really lost something by opening the valve

$$\Delta S = Nk_B \ln(V_F/V_i) \quad \text{by opening the valve}$$

If we return this gas to its original state, say via an isothermal contraction, it will have its original entropy, but we had to remove heat and do work on the gas. So the entropy change of the reservoir where we took the heat is

$$Nk_B \ln(V_i/V_f) = \frac{Q}{T}$$

entropy increased.

If there is a net entropy increase in the universe to get back to our starting point, the process we were trying to reverse was "irreversible",

$$\Delta S_{universe} \geq 0$$

$$\Delta S_{universe} = 0 \text{ process reversible}$$

$$\Delta S_{universe} > 0 \text{ irreversible}$$

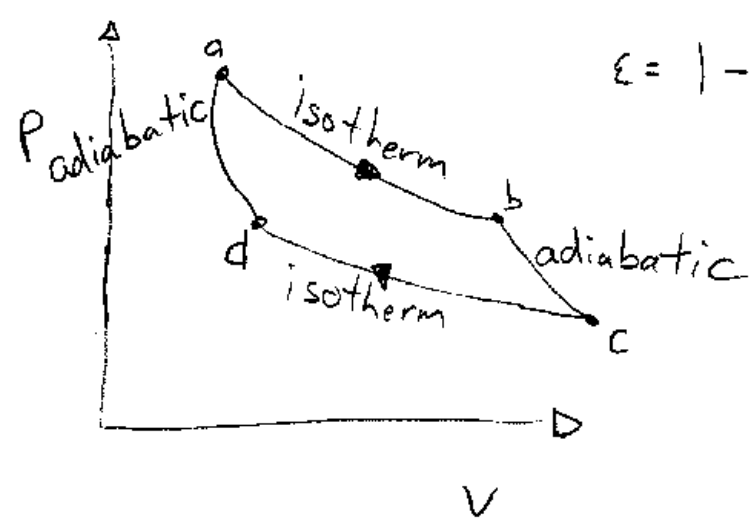
- consider our cyclic engine, if this is reversible $\Delta S = 0$
- Carnot Cycle is suppose to be most efficient between 2 temperatures

$$\epsilon = \frac{\text{Work you got}}{\text{energy you put in}}$$

$$0 = \Delta E_{INT} = Q_{put\ in} - |Q_{absorbed}| - W_{net}$$

$$Q_{put\ in} - |Q_{absorbed}| = W_{net}$$

$$\epsilon = 1 - \frac{|Q_{absorbed}|}{Q_{put\ in}}$$

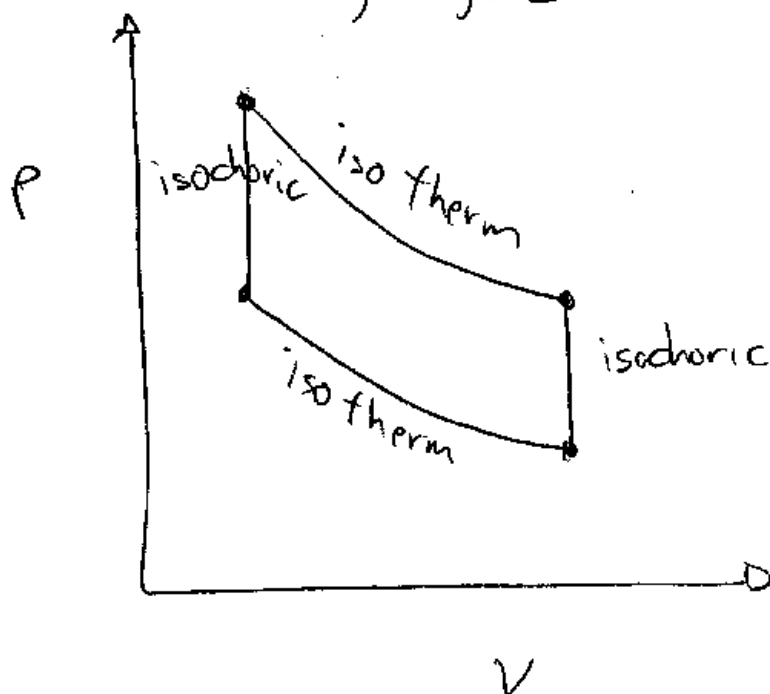


You can show that for a carnot cycle

$$\epsilon = 1 - \frac{T_c}{T_H}$$

$$\Delta S = \frac{Q_H}{T_H} - \frac{|Q_c|}{T_c} = 0 \quad \frac{Q_H}{T_H} = \frac{|Q_c|}{T_c}$$

consider this a second and compare it to our stirling engine



The heat we put in occurs over the isotherm & the isochoric

$$\text{so } \Delta S_{\text{put in}} = \frac{Q_H}{(T_H - \delta_H)}$$

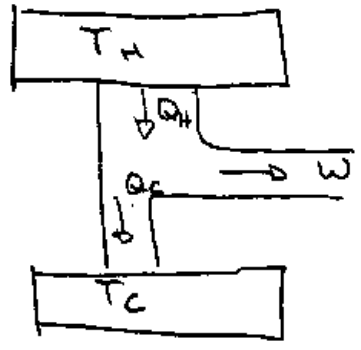
$$\text{likewise } |\Delta S_{\text{pull out}}| = \frac{|Q_C|}{(T_C + \delta_C)}$$

$$\frac{Q_H}{(T_H - \delta_H)} - \frac{|Q_C|}{T_C + \delta_C} = 0$$

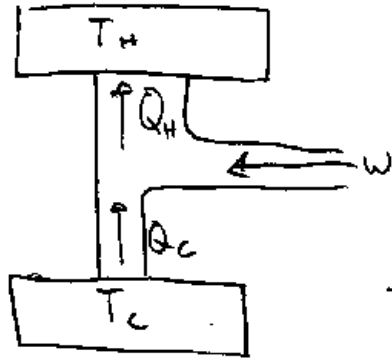
$$\text{or } \frac{|Q_C|}{Q_H} = \frac{T_C + \delta_C}{T_H + \delta_H} > \frac{T_C}{T_H}$$

so $\epsilon < \epsilon_{\text{Carnot}}$!

It is useful to have a simplified view of engines



$$\epsilon = 1 - \frac{T_C}{T_H} \quad \text{Carnot engine}$$



Carnot refrigerator ($T_C = \text{Fridge}$, $T_H = \text{room}$)
 or heat pump ($T_C = \text{Room}$, $T_H = \text{ground}$)

Talk about these using coefficient of performance (COP)

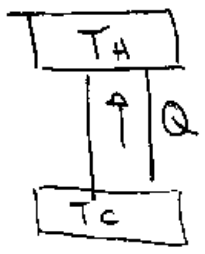
Fridge (you pay for work done)

$$\begin{aligned} \text{COP} &= \frac{\text{Heat removed}}{\text{Work done}} = \frac{|Q_C|}{Q_H - |Q_C|} \\ &= \frac{1}{\frac{T_H}{T_C} - 1} \quad \text{closer } T_H \text{ \& } T_C \text{ are, the better the COP} \end{aligned}$$

Heat pump (you pay for work done)

$$\begin{aligned} \text{COP} &= \frac{\text{Heat Added}}{\text{Work done}} = \frac{Q_H}{Q_H - |Q_C|} \\ &= \frac{1}{1 - T_C/T_H} \quad (\text{same deal}) \end{aligned}$$

can't have



$$\Delta S = -\frac{|Q|}{T_C} + \frac{|Q|}{T_H} > 0$$