Refresher, Relativistic mechanics

$$E^2 = p^2c^2 + m^2c^4 \qquad E_{final} = E_{initial}$$

$$\vec{P}_{final} = \vec{P}_{initial}$$

$$E = m_oc^2 + E_{kinetic}$$

$$\cancel{(m_oc^2)^2} + 2E_{kinetic}(m_oc^2) + E_{kinetic}^2 = p^2c^2 + \cancel{m^2c^4}$$

$$E_{kinetic} + \frac{E_{kinetic}^2}{2m_oc^2} = \frac{p^2\cancel{c^2}}{2m_o\cancel{c^2}}$$

for $E_{kinetic} << m_oc^2$

$$E_{kinetic} \sim \left(\frac{p^2}{2m_o}\right)$$

<u>ex</u> 1 MeV proton

$$\left(\left(938.3\frac{MeV}{c^2}\right)c^2 + 1\,MeV\right)^2 = p^2c^2 + \left(938.3\frac{MeV}{c^2}c^2\right)^2$$

$$\left(939.3\frac{MeV}{c^2}c^2\right)^2 - \left(938.3\frac{MeV}{c^2}c^2\right)^2 = p^2c^2 = 1877.6\,MeV^2$$

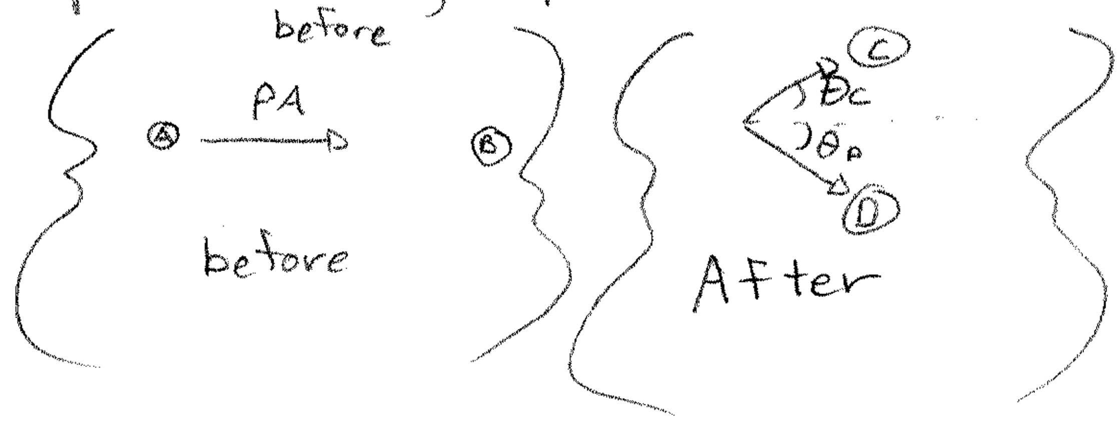
$$\frac{p^2c^2}{2m_oc^2} = \frac{1877.6\,MeV^2}{2(938.3)MeV} = 1.00053\,MeV$$

not too bad

and the fraction difference between actual and estimate

$$1 - \frac{(p^2/2m)}{E_{kin}} = \frac{E_{kin}}{2mc^2} \sim \frac{p^2/2m}{2mc^2}$$

<u>Ex</u> 2 particles making 2 particles



$E_{before} = E_{After}$

$m_A c^2 + m_B c^2 + KE_A = m_C c^2 + m_D c^2 + KE_C + KE_D$

if these are nuclei, we can use their masses as atoms by adding the appropriate number of electrons to each side.

note: if $m_A c^2 + m_B c^2 > m_C c^2 + m_D c^2$

$KE_C + KE_D > KE_A$ - get more kinetic energy out
- can transmutate species

if $m_A c^2 + m_B c^2 < m_C c^2 + m_D c^2$

$KE_A > KE_C + KE_D$

KE_A some minimum energy needed to make this reaction occur

if we convert our problem to the center of mass (particles moving slow)

$m_A v_A = (m_A + m_B) V_{cm}$

so, if we were travelling with V_{cm}, we'd see

before $A \xrightarrow{(V_A - V_{cm})} \qquad \xleftarrow{V_{cm}} B$ $\left(P_{tot} = \vec{P}_A + \vec{P}_B = 0 \text{ in this frame} \right)$

$$m_A c^2 + m_B c^2 + \frac{1}{2} m_A (V_A - V_{cm})^2 + \frac{1}{2} m_B V_{cm}^2 = m_C c^2 + m_D c^2$$

at the point where C & D are being produced

After C D $P = 0$ still

so, in order to produce C & D in the

Laboratory frame:

$$\text{let} \quad Q = m_A c^2 + m_B c^2 - m_C c^2 - m_D c^2$$

$$\frac{1}{2} m_A \left(V_A^2 - 2 V_A V_{cm} + V_{cm}^2 \right) + \frac{1}{2} m_B V_{cm}^2 = -Q$$

$$\frac{1}{2} m_A V_A^2 - m_A V_A \frac{m_A V_A}{(m_A + m_B)} + \frac{1}{2} m_A \left(\frac{m_A V_A}{m_A + m_B} \right)^2 + \frac{1}{2} m_B \left(\frac{m_A V_A}{m_A + m_B} \right)^2 = -Q$$

$$\frac{1}{2} m_A V_A^2 \left(1 - 2 \frac{m_A}{m_A + m_B} + \frac{m_A^2}{(m_A + m_B)^2} + \frac{m_B m_A}{(m_A + m_B)^2} \right) = -Q$$

$$\frac{1}{2} m_A V_A^2 \left(1 - 2 \frac{m_A}{m_A + m_B} + \frac{m_A \cancel{(m_A + m_B)}}{(m_A + m_B)^{\cancel{2}}} \right) = -Q$$

$$\frac{1}{2} m_A V_A^2 \left(1 - \frac{m_A}{m_A + m_B} \right) = -Q$$

$$\frac{1}{2} m_A V_A^2 \left(\frac{m_A + m_B}{m_A + m_B} - \frac{m_A}{m_A + m_B} \right) = -Q$$

$$\frac{1}{2} m_A V_A^2 \left(\frac{m_B}{m_A + m_B} \right) = -Q$$

beam

$$KE_{thresh} = -Q \left(1 + \frac{m_A}{m_B} \right)$$

target

takes more KE if target particle is lighter than beam particle.

ex 13.5 $\quad {}^1_1H + {}^3_1H \rightarrow {}^2_1H + {}^2_1H$

^3H target
$$Q = (1.007825u + 3.016049u - 2 \times 2.014102u)\, 931.5\,\text{MeV}/u$$
$$= -4.033\,\text{MeV} \qquad KE_{thresh} = 4.033\,\text{MeV}\left(1 + \tfrac{1}{3}\right) = 5.38\,\text{MeV}$$

^1H target
$$KE_{thresh} = 4.033\,\text{MeV}\left(1 + \frac{3}{1}\right) = 16.1\,\text{MeV}$$

Issue: Bang for your buck:

It is better to collide in the center of mass

suppose you wanted to create one great big huge particle X & $m_A = m_B$

$$m_A c^2 + m_A c^2 + 2KE_A = M_{big} c^2$$

$$KE_{tot} = 2KE_A = (M_{big} - 2m_A)c^2$$

$$KE_A = \frac{(M_{big} - 2m_A)c^2}{2}$$

if we use one particle as a target

$$KE_A = (M_{big} - 2m_A)c^2 \left(1 + \frac{m_A}{m_A}\right)$$

$$KE_{tot} = KE_A = 2(M_{big} - 2m_A)c^2$$

(twice as much)

If you get to relativistic energies, we can use a relativitic invariant to translate from one coordinate system to another ($KE_{cm} \gg m_A c^2$)

$$\left(\sum E_i\right)^2 - \left(\sum_i \vec{p}_i\right)^2 = \text{constant}$$

$$\underset{\text{initial}}{\sum_i E_i} = \underset{\text{final}}{\sum_j E_j} \qquad \underset{\text{initial}}{\sum_i \vec{p}_i} = \underset{\text{final}}{\sum_j \vec{p}_j} \qquad \Rightarrow \text{constant} = M_{big}^2 c^4 \text{ in cm frame}$$

in cm frame $\left(\overbrace{2KE_{cm}}^{KE_{tot}} + 2m_A c^2\right) = M_{Big} c^2$

& $(E_A + m_A c^2)^2 - p_A c^2 = M_{Big}^2 c^4$

$$\underbrace{E_A^2 - p_A^2 c^2}_{m_A^2 c^4} + 2E_A m_A c^2 + m_A^2 c^4 = M_{big}^2 c^4$$

$$(KE_{tot} + 2m_A c^2)^2 = 2KE_A m_A c^2 + 4m_A^2 c^4$$

$$KE_{tot}^2 + 4KE_{tot} m_A c^2 + 4m_A^2 c^4 = 2KE_A m_A c^2 + 4m_A c^2$$

so, in the lab frame, you need $\left(\frac{KE_{tot}^2 + 4KE_{tot} m_A c^2}{2m_A c^2}\right)$ lots more!