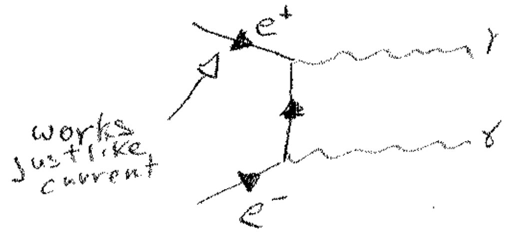
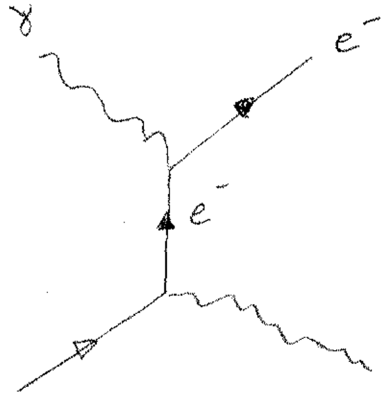
For the strong & EM interactions we are allowed to anhiliate particles

pair production



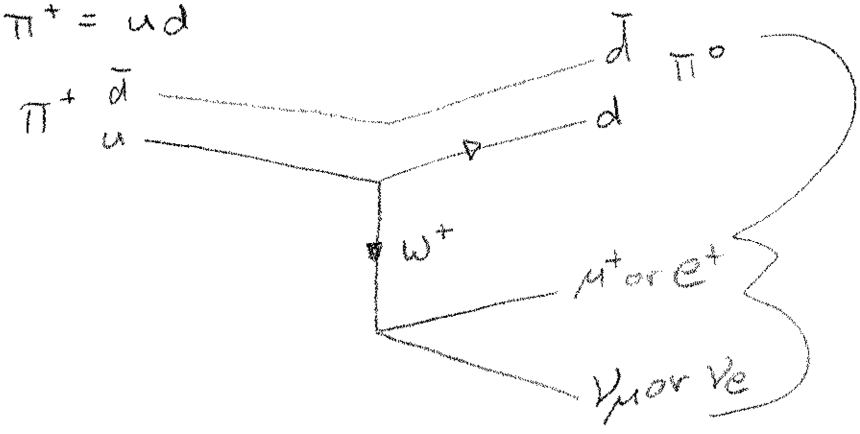
photons (or other)



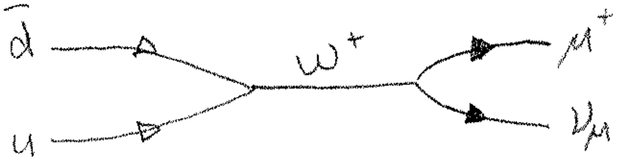
compton scattering

pion decay

$\pi^+ = u\bar{d}$



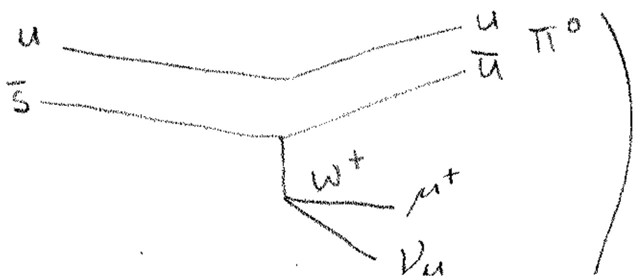
no phase space for μ, must be electron



more likely

K^+ decay

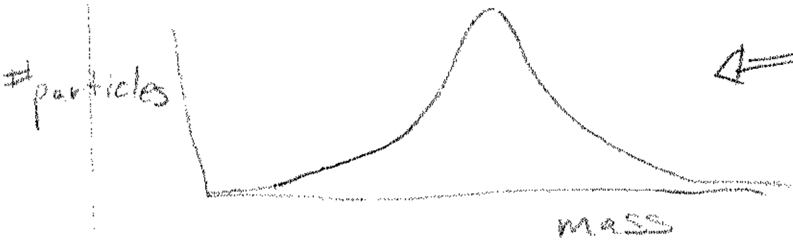
$K^+ = \bar{s}u$



perfectly ok

Strong decays

$\Rightarrow$ when you go to reconstruct the invariant mass for particles that decay via the strong force, no matter how good your apparatus is, you can still get a spread



not a gaussian shape!

Full width at $\frac{1}{2}$ max

$$P(m) = \frac{\Gamma}{2\pi} \frac{1}{(m-m_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

$\Rightarrow$ just like resonance you've seen in driven oscillators

& we identify $\Gamma = \lambda h$

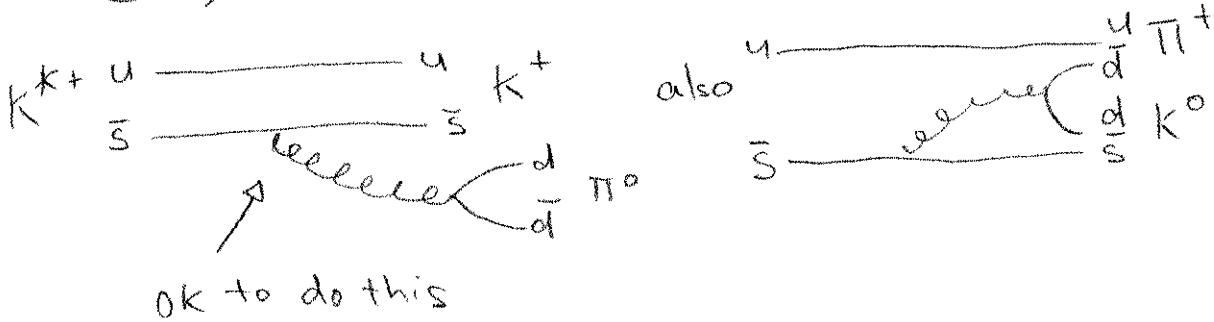
becomes finite for Δt's very small.

$\sim 10^{-23}$s is strong force time

$$\Delta E = \frac{h}{2\pi} \frac{1}{\Delta t} = \frac{1240\,\text{MeV fm}}{2\pi\,(3.0\times10^{23}\,\frac{\text{fm}}{\text{s}})(10^{-23}\,\text{s})}$$

$$= 66\,\text{MeV}$$

ex $K^{*+}(890)$ has $\Gamma = 50$ MeV decays to $K^+\pi^0$



More on Resonance

Why do we see a spread in mass when a particle decays via the strong force?

If you think of the particle as a wave, then a finite lifetime cuts off the extent of this wave.

if we start with

$$\psi(t) = A_0 \sin\left(\frac{E_0 t}{\hbar}\right) \qquad \omega_0 = \frac{E_0}{\hbar}$$

we have to modify this

$$\psi(t) = A_0 \sin\left(\frac{E_0 t}{\hbar}\right) e^{-\lambda t/2}$$

because we want $|\psi(t)|^2 \propto \underbrace{\text{number}}_{|\psi(0)|^2}\left(e^{-\lambda t}\right)$

now, we ask the question, what does this ψ look like as a function of energy?

$$\psi(E) = ?$$

Well, lets start by asking, what happens if you cut off a travelling wave?

in terms of frequency

$$f(\omega) \propto \int_{-\left(\frac{A\pi}{\omega_0}\right)}^{A\left(\frac{\pi}{\omega_0}\right)} \sin(\omega_0 t) \sin(\omega t)\, dt$$

tells us how big the window is

Filters out ω dependence

in general, this could be a complicated function of many sines, cosines etc.

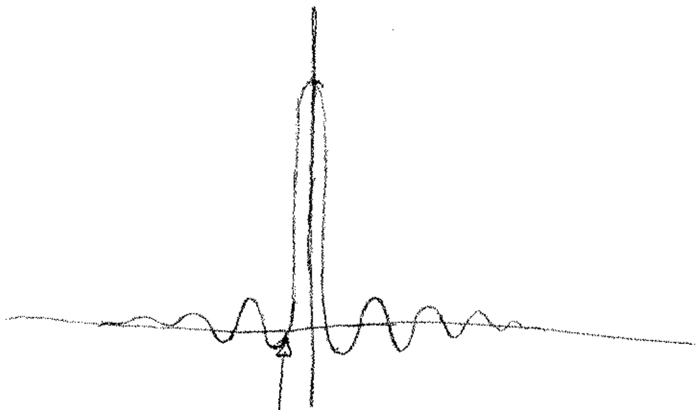
here's why $\sin(a)\sin(b) = \cos(a-b) - \cos(a+b)$

$$f(\omega) \propto \frac{\sin\left[(\omega_0 - \omega) A \frac{\pi}{\omega_0}\right]}{(\omega_0 - \omega)} - \frac{\sin\left[(\omega_0 + \omega) \frac{A\pi}{\omega_0}\right]}{\omega_0 + \omega}$$

for a particle with very high frequency or with a window small compared to the frequency

$f(\omega)$ looks like $\quad \dfrac{\sin(\text{big} - t)}{(\text{big} - t)}$

which looks like



goes to zero @ $\quad \dfrac{\omega_0 - \omega}{\omega_0} = \pm \dfrac{1}{A}$

If you consider the central peak as an error $\Rightarrow \Delta\omega = \left(\dfrac{\omega_0}{A}\right) \Rightarrow \Delta E = \hbar\Delta\omega$

& if you think of the window as Δt

$$\Delta E\,\Delta t = \hbar\left(\frac{\omega_0}{A}\right)\left(\frac{2A\pi}{\omega_0}\right) = h$$

why we say everything is a wave, you get the uncertainty principle back. And this principle of "analyzing" a periodic signal with another periodic signal is a very powerful one.
$\Rightarrow$ used in lock-in amplifiers for instance.

For our particle: $\quad \psi(E) \propto \displaystyle\int_0^\infty \sin\left(\frac{E_0}{\hbar}t\right)\sin(\omega t)\, e^{-\lambda t/2}\, dt$

or, as in your book $\quad \psi(E) \propto \displaystyle\int_0^\infty e^{-i\omega_0 t} e^{i\omega t} e^{-\lambda t/2}\, dt$

which has solutions

$$\int e^{\left(i(\omega-\omega_0)-\frac{\lambda}{2}\right)t}\,dt$$

that look like

$$\psi(E) \sim \frac{1}{(\omega-\omega_0)+\frac{i\Gamma}{2}}$$

so that

$$|\psi(E)|^2 = \frac{B}{(\omega-\omega_0)^2+\left(\frac{\lambda}{2}\right)^2}$$

so the Γ we associated with full width at half max, is the lifetime of the particle.