Nucleosynthesis

How do the heavier elements come about?

So far we have been either using or creating ^1H, ^2H, ^3He, ^4He, ^8Be, ^{12}C, but our universe is composed of many heavier elements. Once ^{12}C production is able to occur, heavier elements can be created in a stepping stone sort of way:

$$^{12}C + {}^4He \rightarrow {}^{16}O \qquad Q = 7.16\,MeV$$
$$+ {}^4He \rightarrow {}^{20}Ne \qquad Q = 4.73\,MeV$$
$$+ {}^4He \rightarrow {}^{24}Mg \qquad Q = 9.32\,MeV$$

As temperatures increase, you can start fusing heavier elements

$$^{12}C + {}^{12}C \rightarrow {}^{20}Ne + {}^4He$$
$$^{16}O + {}^{16}O \rightarrow {}^{28}Si + {}^4He$$

These processes can continue to add to the stars energy production until ^{56}Fe is produced. (so should see a bunch)

Notice: 1) We have jumped over the elements ^7Li, ^9Be, ^{11}B

2) These fusions make even Z nuclei (basic building block is He)

$\Rightarrow$ doesn't explain N yet $\Leftarrow$ it's abundant

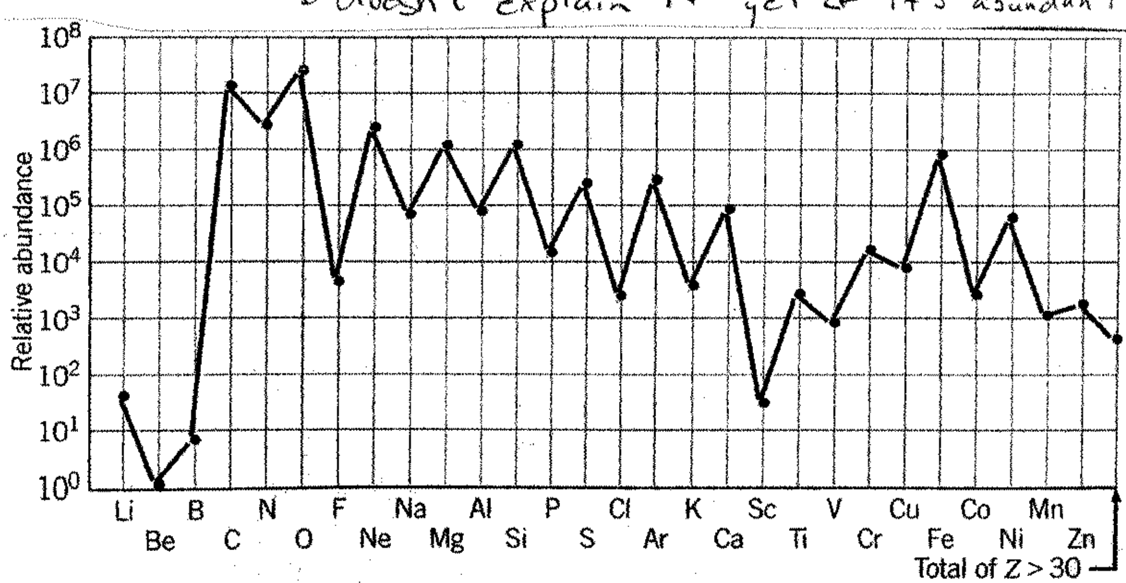

FIGURE 15.20 Relative abundances (by weight) of the elements beyond helium in the solar system

In order to make Nitrogen, a couple of reactions are suggested;

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$
$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu_e$$
$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$

{ It's tough to make Li, Be, & B out of He & H and have a star that continues to burn, takes <u>more</u> energy to form these than the other reactions }

$$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$$
$$^{17}F \rightarrow {}^{17}O + e^{+} + \nu_e$$
$$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$$

Can create other elements heavier than iron, but need neutrons.

$$^{13}C + {}^{4}He \rightarrow {}^{16}O + n$$
$$^{17}O + {}^{4}He \rightarrow {}^{20}Ne + n$$
$$^{21}Ne + {}^{4}He \rightarrow {}^{24}Mg + n$$

& we can have a stepping stone process again

$$^{56}Fe + n \Rightarrow {}^{57}Fe + n \Rightarrow {}^{58}Fe + n \Rightarrow \underbrace{{}^{59}Fe}_{t_{1/2} = 45d}$$

This is long enough to be around for more neutron shenanigans, but not too much

$$^{59}Fe + n \Rightarrow {}^{60}Fe + n \Rightarrow \underbrace{{}^{61}Fe}_{t_{1/2} = 6m}$$

get this with lots of fast neutron flux $\Rightarrow$

$$^{61}Fe \Rightarrow {}^{61}Co + e^{-} + \bar{\nu}_e$$
$$^{61}Co \Rightarrow {}^{61}Ni + e^{-} + \bar{\nu}_e$$

Γ_{apid} prociss

(or)

dominant $\Rightarrow$

$$^{59}Fe \Rightarrow {}^{59}Co + e^{-} + \bar{\nu}_e$$
$$^{59}Co + n \Rightarrow \underbrace{{}^{60}Co}_{t_{1/2} = 5y}$$
$$^{60}Co \Rightarrow {}^{60}Ni + e^{-} + \bar{\nu}_e$$

Slow process

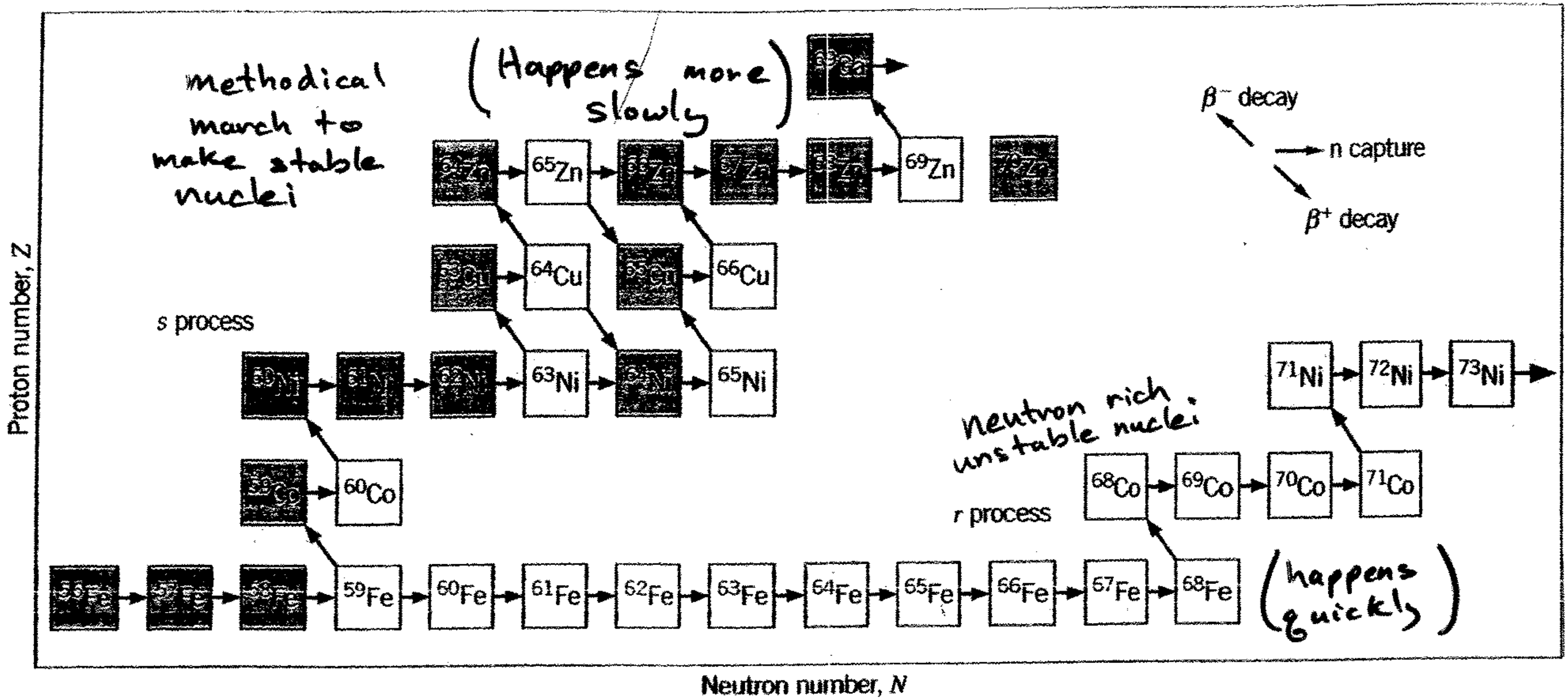

FIGURE 15.21 A section of the chart of the nuclides (Figure 12.4), showing the *s*- and *r*-process paths from ^{56}Fe. Shaded squares represent stable nuclei, and unshaded squares represent radioactive nuclei. Many *r*-process paths are possible, as the short-lived nuclei beta decay; only one such path is shown. All the nuclei in the *r*-process path are unstable and may beta decay toward the stable nuclei.

Notice in the process ^{70}Zn is not produced

$t_{1/2}$ ^{69}Zn = 56 m

& the chance for a neutron capture is 1/45d or so, so the ^{70}Zn likely comes from the "r" process.

Likewise

$t_{1/2}$ ^{66}Cu = 5.10 m

^{65}Ni = 2.52 h

but

^{63}Ni = 100 y <so can get an n capture

and ^{65}Zn = 244 d this can get an n too

s - process stops @ ^{209}Bi

(know that ^{214}Bi occurs in nature though from your Solid state lab as a decay product of ^{238}U, so our planet is made up of stuff from an r process which can occur via a supernova.)

How do these states of a sun come about?

Compressed States of Matter

We have already discussed White Dwarf Stars

Recall $\langle \varepsilon_f \rangle \propto \frac{1}{mass}$

and we said that the kinetic energy in the star will be dominated by the electrons

$$E_{tot} = \langle \varepsilon_f \rangle_{tot} + \text{Grav self energy}$$

$$= N_e \frac{3}{5} \frac{1}{2m_e} \left(\frac{h}{2}\right)^2 \left(\frac{3}{\pi} \frac{N_e}{\frac{4}{3}\pi R^3}\right)^{\frac{2}{3}} - \frac{3}{5} \frac{G M_{tot}^2}{R}$$

@ equilibrium $\frac{dE_{tot}}{dR} = 0$

gave $R \sim 7000$ km for the sun

or $\langle \varepsilon_f \rangle = 122$ keV

Now, when $m_e + \langle \varepsilon_f \rangle$ gets to be about the difference in mass between the n & p or 1.3 MeV, the electrons have enough Q to make $e^- + p \to n + \nu_e$

So, eventually, the dominant KE comes from the neutrons. Since this occurs at a high energy for the electron, the electron energy $\langle \varepsilon_f \rangle$ goes more like pc

$$\langle \varepsilon_f \rangle = \langle pc \rangle = \int_0^{p_f} p^3 dp \Big/ \int_0^{p_f} p^2 dp$$

$$= \frac{3}{4} p_f c = \frac{3}{4}\left(\frac{hc}{2} \left(\frac{3}{\pi} \frac{N}{V}\right)^{1/3}\right)$$

$$E_{tot} = N_e \frac{3}{4} \frac{hc}{2} \left(\frac{3}{\pi} \frac{N_e}{\frac{4}{3}\pi R^3}\right)^{\frac{1}{3}} - \frac{3}{5} \frac{G M_{tot}^2}{R}$$

When $E_{tot} = 0$, the state is no longer bound (unstable) or when (assuming $pc \sim KE$)

$$N_e \, \frac{3}{4} \, \frac{hc}{2} \left(\frac{3}{\pi} \, \frac{N_e}{\frac{4}{3}\pi} \right)^{\frac{1}{3}} = \frac{3}{5} G \, N_e^2 m_n^2$$

$$\left(\frac{5}{3} \, \frac{1}{G m_n^2} \right) \left(\frac{3}{4} \, \frac{hc}{2} \right) \left(\frac{3}{\pi} \, \frac{1}{\frac{4}{3}\pi} \right)^{\frac{1}{3}} = N_e^{\frac{2}{3}}$$

or $$N_e = \left[\frac{5}{3} \, \frac{1}{\left(6.67\times10^{-11}\, N \frac{m^2}{kg^2}\right)\left(1.67\times10^{-27} kg\right)^2} \left(\frac{3}{4} \, \frac{(1240)(1.6\times10^{-19} J)(\times 10^{-9} m)}{2} \right) \left(\frac{9}{4\pi^2} \right)^{1/3} \right]^3$$

$= 8.2 \times 10^{57}$

This is actually about a factor of 4 higher than the correct limit computed by Chandrasekhar. Mostly I fudged. $KE = \sqrt{p^2c^2 + m^2c^4} - mc^2$ and you get some pretty nasty integrals.

The correct limit is $N_e = 1.71 \times 10^{57}$

or 1.44 m_{sun} in mass

(A crude correction @ $KE = 0.789$ MeV gives still ×2 difference, refs. I found on the subject approach this from the viewpoint of pressure) eq. $PV = N k_B T = \frac{2}{3} E$

$$P = \frac{2}{3} \frac{\langle E_f \rangle}{V} \text{ gas} \quad \& \quad \frac{G M m_n}{R^2} \, \frac{1}{4\pi R^2} \text{ on nucleon}$$

equilibrium is maintained as long as the pressures cancel or

$$\cancel{\frac{1}{V}} \, \frac{2}{4} \, \frac{hc}{2} \left(\frac{3}{\pi} \frac{N_e}{V} \right)^{1/3} = \frac{G N_e m_n^2}{3R} \, \cancel{\frac{1}{V}}$$

& in the above we'd replace $\frac{5}{3}$ with 2

$\Rightarrow$ worse!