Take Home Exam III

Kinematics

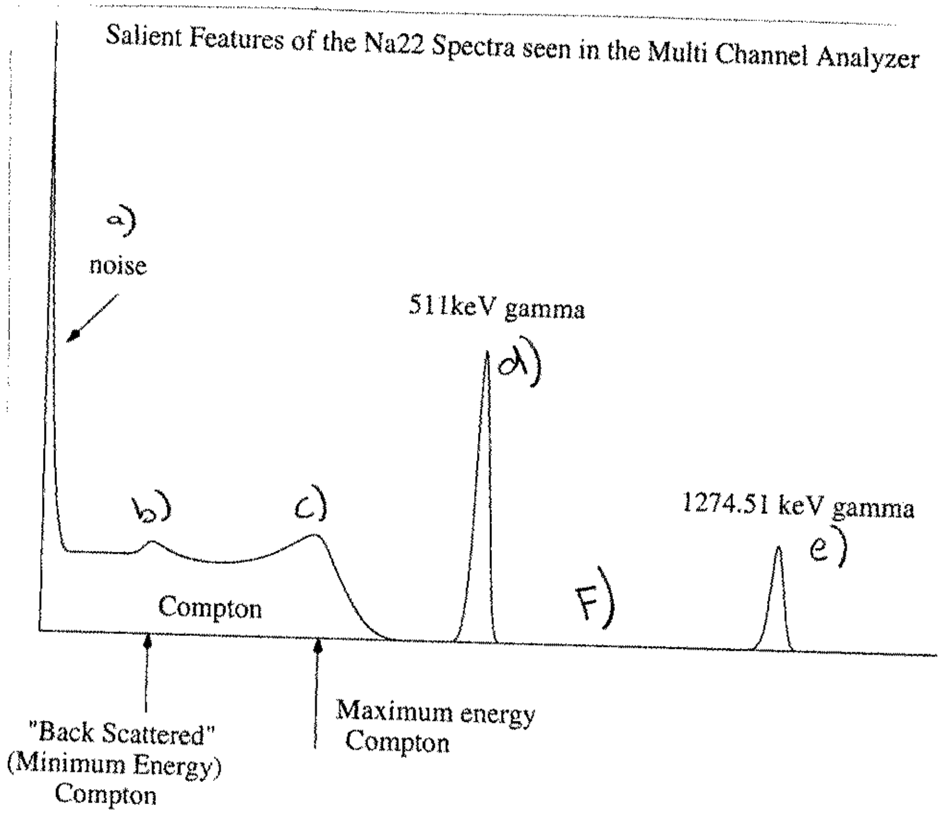
1)a) In the solid state lab, there is a decription of Compton scattering, and then later a picture of what you see in your solid state detector.
Please describe qualitatively what is happening in the picture. (Do you see any mistakes?)(10 pts)
Please derive the mathematical expression for the energy of the "horns" caused by the backscattered photon and the forward scattered electron. Be sure you described how these enhancements in the spectra can occur.(15 pts)
For the 511 KeV photon shown in the lab write up, calculate the likely positions of the "horns" below the 511KeV peak.(5pts)

1b) Please derive the expression for the energy of particle B in the decay $A \rightarrow B + C$ when the particle A is at rest. Be sure to include all the steps and show that:(20 pts)

$$E_B/c^2 = \frac{M_A^2 + M_B^2 - M_C^2}{2M_A}$$

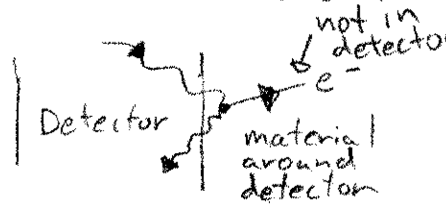
Take Home III Kinematics

1 a) Picture

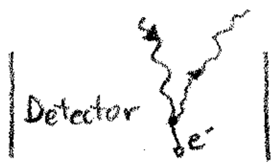


i) a) As much as we try, there is still some noise in the electronics. If it gets real bad = no experiment. If you diddled with the knobs in the experiment, you'd see it can get much worse

b) This little peak occurs when photons hit electrons outside the detector and come back in.



c) You get this one from the forward scattered electron (the backscattered photon escapes)



d) caused by a positron emission & e^+e^- anhiliation {and all the photon being absorbed}

e) nuclear transition where the γ energy is absorbed

f) where are the compton peaks! The author forgot! (should be mking some e^+e^- pairs too...)

ii) Expression for compton scattering:

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta_{scat})$$

angle of scattered photon w.r.t. incident

$E = \frac{hc}{\lambda}$ so @ largest λ', have smallest E_γ'

$$\lambda' = \frac{hc}{m_e c^2}(1-\cos\theta_{scat}) + \lambda$$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\frac{hc}{m_e c^2}(1-\cos\theta_{scat}) + \lambda}$$

$$= \frac{hc}{\lambda} \frac{1}{\frac{hc}{\lambda}\frac{1}{m_e c^2}(1-\cos\theta_{scat}) + 1}$$

has a min when $1-\cos\theta_{scat}$ is max

or $\theta_{scat} = 180°$

$$E'_{min} = E \Big/ \left(1 + 2\frac{E}{mc^2}\right)$$

& since $E = mc^2$ in this case (e^+e^- anhil.)

$$E'_{min} = \left(m_e c^2 / 3\right) \quad \text{(lower horn)}$$

& the energy of the (KE) forward scattered electron is just

$$E_0 - E'_{min} = m_e c^2 - \frac{m_e c^2}{3} = \frac{2}{3} m_e c^2$$

(upper horn)

iii) lower horn $E_{lower} = 170.3$ KeV

$E_{upper} = 340.6$ KeV

1 b) $E_A = E_B + E_c = M_A \quad (c=1)$

$\vec{P}_A = \vec{P}_B + \vec{P}_c = 0 \qquad \vec{P}_B = -\vec{P}_c$

use relativistic invariant mass (let c=1)

$$E_A^2 - p_A^2 = M_A^2 = (E_B + E_c)^2 - (\vec{P}_B + \vec{P}_c)^2$$

$$= \overbrace{E_B^2 - p_B^2}^{M_B^2} + \overbrace{E_c^2 - p_c^2}^{M_c^2} + 2E_B E_c - 2\vec{P}_B \cdot \vec{P}_c$$

$$\left[\vec{P}_B \cdot \vec{P}_C = -P_B^2 \right]$$

$$M_A^2 = M_B^2 + M_c^2 + 2E_B E_C + 2P_B^2$$

$$\left[E_c = E_A - E_B \right]$$

$$M_A^2 = M_B^2 + M_c^2 + 2E_B(E_A - E_B) + 2P_B^2$$

$$= M_B^2 + M_c^2 + 2E_B E_A \underbrace{- 2E_B^2 + 2P_B^2}_{-2M_B^2}$$

$$M_A^2 = M_B^2 + M_c^2 + 2E_B \underset{M_A}{E_A} - 2M_B^2$$

$$M_A^2 = M_c^2 - M_B^2 + 2M_A E_B$$

$$M_A^2 + M_B^2 - M_c^2 = 2M_A E_B$$

$$E_B = \frac{M_A^2 + M_B^2 - M_c^2}{2M_A}$$

Take Home Exam III
Fusion

1)a) We have discussed several types of fusion in class. Fusion inside stars, fusion in a hot plasma and fusion due to particle collisions. One process we didn't mention is called muon catalyzed fusion. Briefly, an isotope of hydrogen has its electron replaced by a muon. This allows other isotopes of hydrogen to approach this muonic isotope much more closely.

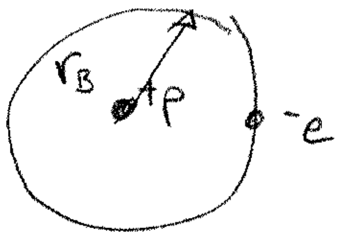
Estimate the relative separation of the hydrogen atoms in an H_2 molecule and in an H_2 molecule where one of the hydrogen atoms has acquired a muon. (You can reason this through or look at the beginning of chapter 9 in Krane for a starting place. Note: do not try to solve this exactly, reason through in an analogy to the hydrogen molecule to get an estimate.)(10 pts)

Now, calculate the temperature needed to bring 2 protons this close together. I.e. Treat your estimate above as the "turning point", and work backwards to find the temperature.(10 pts)

Ok, lets assume that this is a good temperature estimate for all the isotopes of hydrogen. Look at the graph in the lecture on fusion and pick off the highest value of $< v\sigma >$ you can find for the temperature you calculated. Now suppose there is a beaker with equal mixtures of the 2 isotopes you chose and half of the mixture is muonic. What fusion rate/volume do you expect? (I would look up the density of the liquid forms to perform the calculation)(10 pts)

Actually, a better number to get from this is the mean lifetime of a muonic atom in this soup. Relate the rate of reaction/volume to the decay constant times a density. Estimate the mean lifetime. Consider the lifetime of the muon. Estimate the number of fusion reactions a single muon can participate in before it disappears.(15 pts)

1a) Think of a hydrogen atom like a spherical ball of charge



from Gauss' Law net field out here is 0
so, expect than 2 hydrogens can get as close as $2 \times r_B$

$$r_B = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = 0.0529\,\text{nm}$$

so estimate distance between 2 H atoms is ~ 0.1 nm (according to the book, this is a bit high-ish because it is a good estimate for H_2^+! Adding another electron just brings the protons closer)

Another way is to consider $\psi(r) \propto e^{-r/a_0}$
For the electron, should be similar for the μ^-
And the reasoning from pg 272 in your book follows,

with a muon, expect $r_{B_\mu} \sim r_B \left(\frac{m_e}{m_\mu}\right) = 0.0529\,\text{nm} \left(\frac{0.511}{106}\right.$

$$= 2.55 \times 10^{-4}\,\text{nm}$$

and adding another electron should make this smaller

so

separation of protons
$\sim 4 \times 10^{-4}$ nm

b) treating this as a distance of closest approach

$$KE_1 \longrightarrow \qquad \longleftarrow KE_2$$

$$\underbrace{\qquad\qquad}_{d_{min}}$$

$$mv^2 = \frac{Kq^2}{d_{min}} \qquad \text{or} \qquad \frac{mv^2}{2} = \frac{Kq^2}{d_{min}}$$

more conservative

$$\frac{3}{2} K_B T = \frac{kq^2}{d_{min}}$$

$$K_B T \sim \frac{2}{3} \frac{Kq^2}{d_{min}}$$

$$= \frac{2}{3} \frac{\left(9 \times 10^{9} \frac{Nm^2}{C^2}\right)\left(1.6 \times 10^{-19} C\right)^2}{4 \times 10^{-13}\, m} \frac{1}{\left(1.6 \times 10^{-19} \frac{J}{eV}\right)}$$

$$\approx 2400 eV$$

$$\approx 1200 eV$$

From the lecture, looks like $\overset{t}{{}^3H}$ $\overset{d}{{}^2H}$ fusion is be[st]
you could choose have about 10^{-24} $m^3/s = \langle v\sigma \rangle$

from class notes

$$\frac{Rate}{Vol} = \frac{1}{4} \frac{N_1}{V} \frac{N_2}{V} \langle \sigma v \rangle$$

if Density of H_2 liquid $\approx \frac{0.09}{cm^3}$

T density $= \frac{3}{2} \frac{0.09g}{cm^3}$ (note, not T_2)

D density $= \frac{2}{2} \frac{0.09g}{cm^3}$ (note, not D_2)

one mole of T_2 is 6 grams

one mole of D_2 is 4 grams

muonic T, D will be a little denser due to μ, but we'll ignore it for now

$$\frac{N_D}{V} = 0.09\frac{g}{cm^3} \cdot \frac{1\,mole}{2g} \cdot \frac{6.02\times10^{23}}{mole}\left(\frac{100cm}{m}\right)^3 = 2.7\times10^{28}/m^3$$

$$\frac{N_T}{V} = \frac{3}{2}\;0.09\frac{g}{cm^3} \cdot \frac{1\,mole}{3g} \cdot \frac{6.02\times10^{23}}{mole}(100cm)^3 = 2.7\times10^{28}/m^3$$

$$\frac{Rate}{Vol} = \frac{1}{4}\left(\frac{2.7\times10^{28}}{m^3}\right)^2\left(10^{-24}\frac{m^3}{s}\right)$$

$$= 1.82\times10^{32}/m^3s \quad (\text{gobble it all up!})$$

expect an activity of

$$\left(\frac{N}{V}\right)\lambda = \left(\frac{2.7\times10^{28}}{m^3}\right)\frac{1}{2.2\times10^{-6}s}$$

$$= 1.2\times10^{34}/sm^2$$

So, each muon is good for $\frac{0.18}{120} = 0.015$ interactions.

(probably more, as we've neglected tails in ψ_μ)

Another way to look at this, for one muon you expect $\frac{Rate}{Vol} = \frac{1.82\times10^{32}/m^3s}{2.7\times10^{28}} = 6741/m^3s$

or time between interactions is $1.5\times10^{-4}s$

and $\tau_\mu = 2.2\times10^{-6}s$

so, in this calculation, we could only hope to get $1/\mu$ (assuming the μ is tightly bound)

$E_{tot} \propto \frac{1}{a_0} \sim -13.6eV \cdot \left(\frac{m_\mu}{m_e}\right) > m_\mu c^2$

In reality, you can get 10 or more reactions/μ, but the calculation (which involves resonance modes in a molecule of $\mu\,{}^3H + {}^1H\,{}^2H$) is advanced (The Vesman model).

Take Home Exam III

Decays

1)a) Which of the following decays occurs and which does not. Please explain why one of them does not.(10 pts)

$\Lambda^o \rightarrow p + \pi^-$

$\Lambda^o \rightarrow \overline{p} + \pi^+$

b)Can the following decays occur? If not, why not?(4 pts each)

$\tau^+ \rightarrow \pi^+ + \pi^- + \pi^+ + \overline{\nu}_\tau$

$\overline{\Xi}^o \rightarrow \overline{\Lambda}^o + \pi^o$

$\nu_e + \overline{\nu}_e \rightarrow \nu_\mu + \overline{\nu}_\mu$

$\nu_e + \overline{\nu}_\mu \rightarrow \nu_\mu + \overline{\nu}_e$

$\Sigma^- \rightarrow K^- + K^+ + K^-$

$\overline{p} + \Sigma^- \rightarrow K^- + K^+ + K^- + \pi^+$

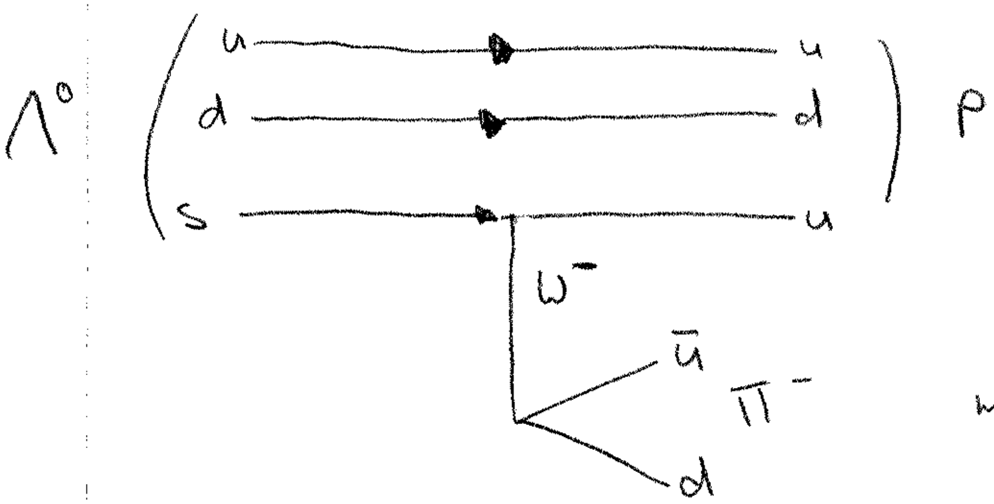
$\Xi^- \rightarrow \Xi^o + e^- + \overline{\nu}_e$

c) Please draw the Feynman diagram for the following 2 interactions: (you may find your notes or the homework solutions helpful)(6 pts each)

$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

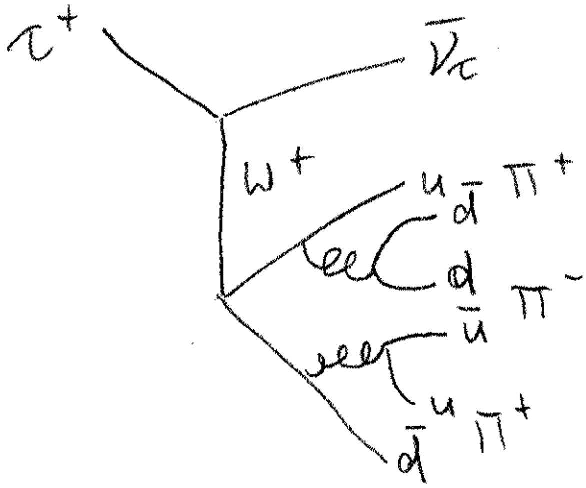
1a) $\Lambda^0 = \begin{smallmatrix} u \\ d \\ s \end{smallmatrix}$ for a weak decay (needed because we change strangness) we have a reaction that looks like



no way to make $\bar{u}\bar{d}\bar{u}$

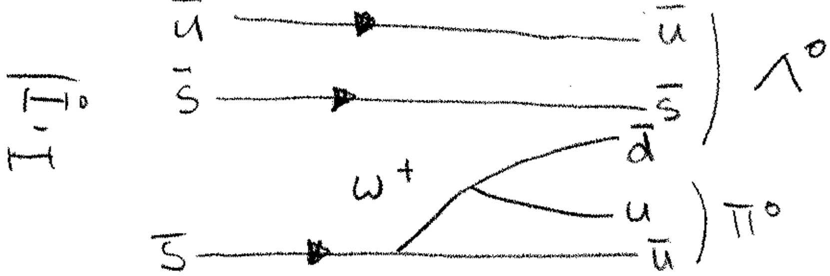
$\Lambda^0 \to \bar{p} + \pi^+$ violates baryon number conservation, even though charge conservation works

b) $\tau^+ \to \pi^+ \pi^- \pi^+ \bar{\nu}_\tau$



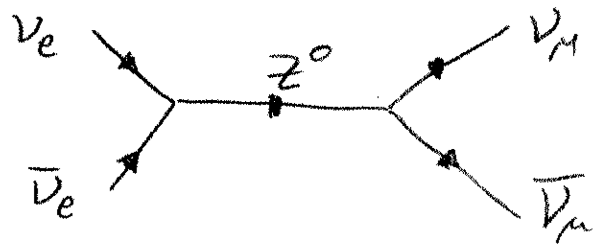
this is Ok!

$\Rightarrow \overline{\Xi}^0 \Rightarrow \overline{\Lambda}^0 \, \pi^0$ need to change one $\bar{s}$



looks OK

$\Rightarrow \nu_e + \bar{\nu}_e \to \nu_\mu + \bar{\nu}_\mu$

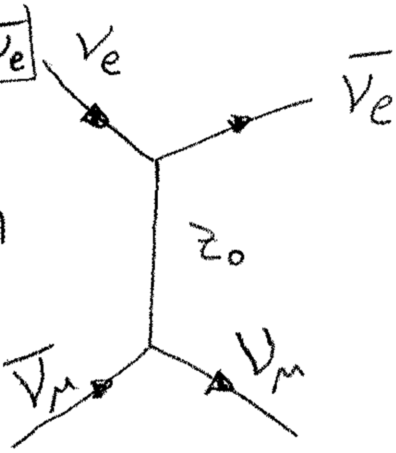


looks OK

$L_e = 0$ $\quad$ $L_\mu = 0$

$L_\mu = 0$ $\quad$ $L_\mu = 0$

$\Rightarrow \boxed{\nu_e + \bar{\nu}_\mu \to \nu_\mu + \bar{\nu}_e}$



$L_e = 1$

$L_\mu = -1$

nope $\quad$ $L_e = -1$

$L_\mu = 1$

$\Rightarrow$ $\Sigma^- \to K^- K^+ K^-$ violates baryon number conservation

$\boxed{\text{nope}}$

$\Rightarrow$ $\bar{p} + \Sigma^- \to K^- K^+ K^- \pi^+$

conserves baryon number

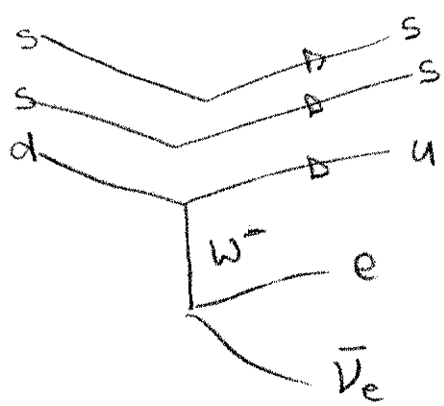
S $\quad -1 \to -1 \; +1 \; -1$

conserves strangeness must be strong

see if quarkness is conserved nope

$\Rightarrow$ charge conservation is violated

$\Rightarrow$ $\Xi^- \to \Xi^0 e^- \bar{\nu}_e$ likely to be <u>very</u> rare



(not much mass for phase space)

Σ s quark transition is more likely (more phase space) c)