Thermo Quickies(6 points)

Ta) State at least 2 equivalent conditions for thermal equilibrium between 2 objects.

1. Same temp

2. In most probable config g is max, S is max

Tb) Calculate the thermal average velocity for a helium atom at room temperature. If the earth's escape velocity is about 11.2 Km/s, why isn't there much He in our atomosphere?

$$\frac{3}{2}k_BT = \frac{1}{2}mv^2 \quad v_{avg} = \sqrt{\frac{3k_BT}{m}} = c\sqrt{\frac{3(8.625\times10^{-5}eV/K)(293K)}{4(938\times10^6 eV)}}$$

V_{escape} ?

$E = KE + PE$

$= \frac{1}{2}mV_{es}^2 - \left(\frac{GM}{r_e}\right)m = 0$

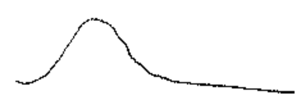
$E_{conserved}$ @ $r=\infty$ $E=0$

$$V_{es} = \sqrt{\frac{2GM_e}{r_e}} = \sqrt{2gr_e}$$

$= 1348.5$ m/s

need about 11.2 km/s to get out

$\Rightarrow$ speed distribution has tails $\Leftarrow$



Tc) Which process requires more heat energy to produce the same temperature change. (Why? 10 words or less)

Constant Volume

Constant Pressure

Thermo (20 points) One mole of an ideal gas

Please derive the expression for the amount of work done during the isothermal expansion of an ideal gas from an initial volume V1 to a final volume V2.(10 points)

$$W = \int_{V_1}^{V_2} P\,dV$$

Isothermal
T is constant
$PV = Nk_BT$
$P = \frac{Nk_BT}{V}$

$$W = \int_{V_1}^{V_2} Nk_BT \frac{dV}{V} = Nk_BT \ln(V)\Big|_{V_1}^{V_2}$$

$$Nk_BT(\ln V_2 - \ln V_1) = Nk_BT \ln\left(\frac{V_2}{V_1}\right)$$

Suppose that this gas doubles in volume at T = 300K, how much work was done by the gas? How much heat energy was absorbed by the gas?
(recall N = 6.02 x 10^{23} for one mole) (5 points each)

$$W = 6.02\times 10^{23} \cdot 1.38\times 10^{-23} \frac{J}{K}\ 300K \cdot \ln(2)$$

$$= 1727.5\ J$$

$Q = W$ Isothermal $\Delta E_{int} = 0$

$$Q = 1727.5\ J$$

Statistics and error analysis (10 points each)

1) Suppose you have a uniform distribution of data between 0 and 1. Compute the average and the sigma of this distribution. In other words, what is the expectation value of x for the distribution f(x)=1 and what is the expectation value of (x–average)^2 for this data in the interval between 0 and 1?

$$\langle x \rangle = \frac{\int x f(x) dx}{\int f(x) dx} \qquad f(x) = 1$$

$$\sigma = \sqrt{\langle (x-\bar{x})^2 \rangle} \qquad \langle x \rangle = \frac{x^2}{2}\Big|_0^1 \Big/ x\Big|_0^1 = \frac{1}{2}$$

$$\langle (x-\bar{x})^2 \rangle = \frac{\int_0^1 (x^2 - 2x\bar{x} + \bar{x}^2) dx}{1} = \frac{1}{3} - \frac{1}{2} = \frac{1}{12} \qquad \sigma = \frac{1}{\sqrt{12}}$$

2) Suppose you are giving a lecture demonstration with a simple pendulum. If you can only measure the length of the pendulum to 10 centimeters and it is 1.5 m long, how much error is there in your estimate for the period. Remember that the period for a simple pendulum is a square root of something and it involves a length and g and 2pi. (the rest is dimensional analysis) Should you worry about it for the demonstration? (why? 10 words or less)

$$T = 2\pi \sqrt{\frac{\ell}{g}} \longrightarrow \left(\frac{T}{\ell}\right)$$

$$dT = \frac{1}{2} 2\pi \frac{1}{\sqrt{\ell}} \frac{1}{\sqrt{g}} d\ell \qquad \text{let } dT = \sigma_T, \; d\ell = \sigma_\ell$$

$$\sigma_T = \frac{1}{2} \frac{T}{\ell} \sigma_\ell$$

$$\frac{\sigma_T}{T} = \frac{1}{2} \frac{\sigma_\ell}{\ell} = \frac{1}{2} \frac{10}{150} = 3.33\%$$

Period has an error of 3% or so

so $\Rightarrow$ students won't notice

some constant

$f_{MB}(\mathcal{E}) = X\, e^{-\mathcal{E}/k_B T}$

$K_B = 1.38\times10^{-23}\frac{J}{K}\times\frac{1eV}{1.6\times10^{-19}J}$

$p(\mathcal{E}) = g(\mathcal{E})\, f_{MB}(\mathcal{E})$

Statistical Mechanics

1) The total number of Atomic (H) hydrogen atoms in a room at 273K is 1.34×10^{27}. Using the Maxwell Boltzmann distribution $f_{MB}(E) = e^{-E/k_B T}$ and your knowledge of the number of possible states in the n=1 and n=2 levels of atomic hydrogen, estimate the number of hydrogen atoms in the first excited state (n=2) of hydrogen. (10 points)

about $\frac{1}{40}$ eV

know $(\#\text{ state } \mathcal{E}=-13.6eV)e^{+13.6eV/k_BT} = (\#\text{ state } \mathcal{E}=\frac{+13.6eV}{4})e^{+\frac{13.6eV}{4k_BT}}$

2 ways to get this　　8 ways to get this

$K_BT = 8.625\times10^{-5}\frac{eV}{K}\cdot 273K = 2.35\times10^{-2}eV$

$$P_{rat} = \frac{\text{Prob its @ } \mathcal{E}=-13.6eV/4}{\text{Prob its @ } \mathcal{E}=-13.6eV} = \frac{8}{2}\frac{e^{+\frac{13.6eV}{4}/k_BT}}{e^{+13.6eV/k_BT}} = 4e^{-\left(\frac{3(13.6eV)}{4}\right)/2.35\times10^{-2}} = 4e^{-434.04}$$

atoms in 1st excited state = # Atom · P_{rat}

$= 1.34\times10^{27}\cdot 4e^{-434} = e^{62.5}\cdot 4\cdot e^{-434} = 4e^{-371.5}$

$\boxed{\#\text{atoms} = 10^x \quad x = \frac{\ln(4)-371.5}{\ln(10)} = -160.7}$

tiny = none

2) (5+10 points)The fermi energy for copper is 7.1 eV. How hot would an ideal monatomic gas need to be to have this energy (on average) for a single molecule? Speaking of temperature, how crazy was the estimate that led to the ultraviolet catastrophe before Plank figured out blackbody radiation? Start with Plank's expression and find the high temperature limit for:

$$I(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4}\frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T}-1}$$

as $T\Rightarrow\infty$ $e^{hc/\lambda k_BT} \to 1 + \frac{hc}{\lambda k_BT}$

In case you don't remember, ultraviolet photons are short wavelength photons. What happens to the intensity of emitted photons at short wavelengths using the expression you derived? If the expression you derived were true at all temperatures, what would happen if you heated up something? (20 words or less)

$KE = \frac{3}{2}k_BT = 7.1eV$

$T = 7.1eV / \frac{2}{3}\cdot 8.625\times10^{-5}eV/K$

$= 123{,}478K!$

at high temp

$I(\lambda) = \frac{c}{4}\frac{8\pi}{\lambda^4}\frac{1}{\cancel{\frac{hc}{\lambda k_BT}}}\cancel{\frac{hc}{\lambda}}$

$= \frac{c}{4}\frac{8\pi}{\lambda^4}k_BT$

@ short wavelength the intensity diverges

heat it up - you'd get x-rays (or worse) if this was true at all temps.