This is a note on determining errors from many trials of human measurements in the polarization lab.

This arguement assumes that each human measurement is in principle a good measurement of an actual experimental value distributed according to some unknown distribution with a sigma that we will call σ_{hum}. That is to say, that if we made 1000 measurements, our distribution of measurements would be centered at the best experimental value we think we can possibly measure givien our technique, and the distribution would have a width σ_{hum}.

Now, if the error on a single measurement, X_{hum} is σ_{hum}, then the error on the difference of 2 measurements at a point with identical experimental contitions is

$$\delta({}_1X_{hum} -_2 X_{hum}) = \sqrt{2}\sigma_{hum}$$

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This is useful. For many such independent measurements where we take the difference between two measurements, we will expect a distribution of results centered at zero, but now with a width of $\sqrt{2}\sigma_{hum}$

The nice thing about taking a difference is that it eliminates any offset, and it allows us to use many different measurements to determine our σ_{hum}.

So, for instance, in the determination of the Verdet Constant, if we look at the distribution of all the differences we have measured, we find:

$$\sigma_{DIFFERENCES} = \sqrt{\sum_{k=1}^{N}({}_1^kX_{hum} -_2^k X_{hum})^2/N} = \sqrt{2}\sigma_{hum}$$

(Notice we didn't use N-1, this is because we know the center of the distribution should be 0!)

One common technique that we have been using to determine σ_{hum} is to find the average of $|({}_1X_{hum} -_2 X_{hum})|/2$ for several differences. This is the equivalent of finding the expectation value of $|X|$ for our distribution that has $\sqrt{2}\sigma_{hum}$. and then dividing by 2. Theoretically, the expectation value (or expected average value) of $|X|$ for this distribution ends up being $2\sigma_{hum}/\sqrt{\pi}$. So our average of $|({}_1X_{hum} -_2 X_{hum})|/2$ for several differences will actually *underestimate* the σ_{hum} by a factor of $\sqrt{\pi}$.

So, summary, 2 possible methods for finding the error for your measurements in the polarization lab:

$$1)\ \ (1/\sqrt{2})\sqrt{\sum_{k=1}^{N}({}_1^kX_{hum} -_2^k X_{hum})^2/N} = \sigma_{hum}$$

$$2)\ \ (\sqrt{\pi}/2)\sum_{k=1}^{N}|{}_1^kX_{hum} -_2^k X_{hum}|/N = \sigma_{hum}$$

These will likely not be identical with your data, but they should be the same in the limit you took many many points.