

## Proportional Tubes

When charged particles traverse matter, the charged particles lose energy. Sometimes through direct collisions with the electrons and nucleons of the material traversed, but usually by ionizing or exciting the molecules of the material. The results of the energy loss, mostly electrons, photons and positively charged ions, (and rarely some more exotic particles if the energy is high enough!) can be used to detect the passage of the charged particle through the matter. The usual methods to detect the passage are to try to detect the electrons and/or positive ions, or to look at the photons. In this lab, we will try to detect the charge created by the passage of energetic particles through a volume of gas. The gas is the key. By placing a potential difference across the gas, we can accelerate the deposited charge and create even more ionizations. For low potential differences, this is tough to see, for high potential differences you get huge signals, but the volume of gas can get saturated with so much charge, that the detector is “dead” for a period of time (this is what a Geiger tube acts like). What we are interested in is creating a detector with just the right potential difference so that the charge we see from the avalanche process is proportional to the energy deposited (i.e. the initial charge created by the energetic particle that lost energy in the gas volume) by the incoming particle.

### The Avalanche Process

The avalanche process for a tube with a potential difference between a central wire and the tube wall is shown diagrammatically in Figure 1. Mathematically, the amplification, or gain, of the incoming charge is expressed as:

$$\ln G = \frac{\ln(2)}{\ln(b/a)} \frac{V}{\Delta V} \ln\left(\frac{V}{\ln(b/a)aE_{min}}\right)$$

$b$  The outer radius of the tube.

$a$  The inner radius of the tube.

$V$  The potential difference between the inner and outer radius.

$\Delta V$  You need  $q\Delta V$  of energy to ionize one gas molecule.

$E_{min}$  The value of the electric field at the point where secondary ionizations occur. Sometimes you'll see this replaced with something like  $S_o$ , the minimum distance from the wire where the same thing happens.

The quantities  $E_{min}$  and  $\Delta V$  are properties of the gas while the geometry of the tube ( $a$  and  $b$ ) and the potential difference are usually set by the experimenter. This expression comes from the following reasoning. Every time the electron has  $q\Delta V$  of energy, it creates a new electron by ionizing a particle. So, now you have 2 electrons. This means that in every  $q\Delta V$ , the number of electrons double. The gain can be thought of as the product of all these doublings:

$$G = 2^{\text{Number of doublings}} \quad \text{or} \quad \ln G = (\text{Number of doublings}) \ln 2$$

To calculate the number of doublings, we have to know where the doublings start,  $S_o$ , where they end,  $a$ , the potential difference between  $S_o$  and  $a$ , and the  $q\Delta V$  needed to produce an ionization in the gas. ( $\ell$  is the length of your tube.)

$$\text{Number of Doublings} = \frac{V_a - V_{S_o}}{\Delta V}$$

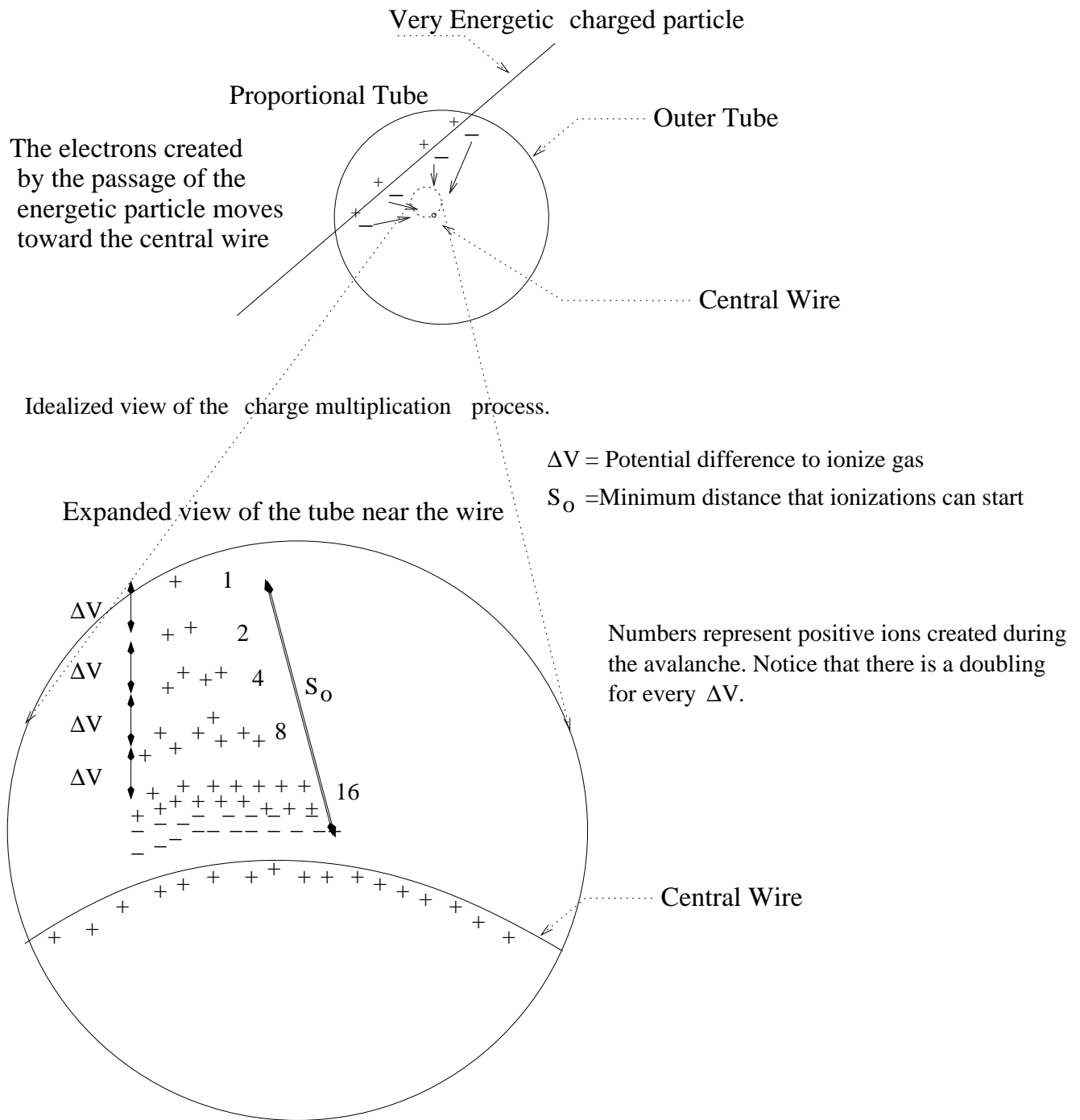


Figure 1: The amplification process in a picture. The positive charges are typically very heavy and move quite slowly at a characteristic drift velocity for the gas. The electrons move quite fast when they get close to the wire, and it is the passage of the electrons to the central wire that creates the bulk of the ionization. (Those positive charges are just too far away and move too slow to have much effect.) Notice the doubling effect of the secondary ionization as the electrons move toward the central wire (represented by the large semi-circle).

$$V_a - V_{S_o} = \int_a^{S_o} E(r) dr$$

$$E(r) = \frac{q}{2\pi\epsilon_o r \ell}$$

$$E_{min} = \frac{q}{2\pi\epsilon_o S_o \ell}$$

And you can compute  $q$  by knowing the potential difference between the wire,  $a$ , and the tube wall,  
**b. For your write up, complete the steps to calculate the gain.**

### The Time Development of the Signal on the Wire

We are going to think of the signal on the wire in terms of the energy needed to create it. Viewed this way, the electrons that reach the wire have a small kinetic energy since they burned it all up creating positive ions. They won't contribute much to the overall signal (typically 2%! ). The positive ions though, get accelerated from very near the tube wall to the outer diameter of the tube, moving with a characteristic velocity depending on the strength of the electric field, very much like the charge moving in a conducting wire. Mathematically, we say the drift velocity of the positive ions behaves similarly to the drift velocity of the charge carriers in a wire:

$$v_D = \mu E, \quad (\text{compare to}) \quad v_D = \frac{Q}{M} E \tau$$

where we have replaced  $\frac{\text{charge}}{\text{mass}} \text{Collision time}$  by the mobility of the gas,  $\mu$ . In a small amount of time, the bulk of the positive ions will move from very near the surface of the wire,  $a$ , to a new point  $R$ :

$$dt = \frac{dr}{\mu E(r)}, \quad \text{or} \quad \int_0^t dt = \int_a^R \frac{dr}{\mu E(r)}$$

$$t = \frac{R^2 - a^2}{2a\mu E(a)}$$

**In your lab write up, please fill in the steps to get the above equation.** Now, we can turn this around and solve for  $R$  as a function of time!

$$R(t) = a \sqrt{1 + \frac{2\mu E(a)t}{a}}$$

So that the energy gained by the ions as a function of time looks like:

$$\Delta \text{Energy}(t) = q_{ions}(V_a - V_{R(t)}) = q_{ions} \int_a^{R(t)} \frac{q_{wire}}{2\pi\ell\epsilon_o} \frac{dr}{r}$$

$$\text{and with} \quad \Delta \text{Energy}_{\text{tot}} = q_{ions} V$$

$$\text{you can write} \quad \Delta \text{Energy}(t) = q_{ions} V F(t)$$

$$\text{where} \quad F(t) = \frac{\ln(1 + t/t_o)}{2 \ln(b/a)}$$

**What is  $t_o$ ?**

In order to understand the signal with something we will actually measure, i.e. current or voltage, lets treat the change in energy as a perturbation of the stored energy in the tube:(since this is an energy loss to the tube, we'll have to remember to put in a minus sign)

$$\text{Stored Energy} = \frac{1}{2}C_{tube}V^2$$

$$\Delta \text{Energy} = C_{tube}V\Delta V$$

and finally, lets call this  $\Delta V$ ,  $v_{in}(t)$ , so that we have:

$$v_{in}(t) = -\frac{q_{ions}}{4\pi\epsilon_o\ell} \ln(1 + \frac{t}{t_o})$$

Notice that we put in the minus sign, and used  $C_{tube}$  to cancel out a few things.

### The Time Development of the Signal Seen

What we see on the scope is a function of the time development of the signal on the wire and the components we attach to the tube to look at the signal. To analyze the output signal, we consider the derivative of charges or voltages so that we can disregard the high voltage since it is constant. The actual setup and the idealized version are shown in Figure 2.

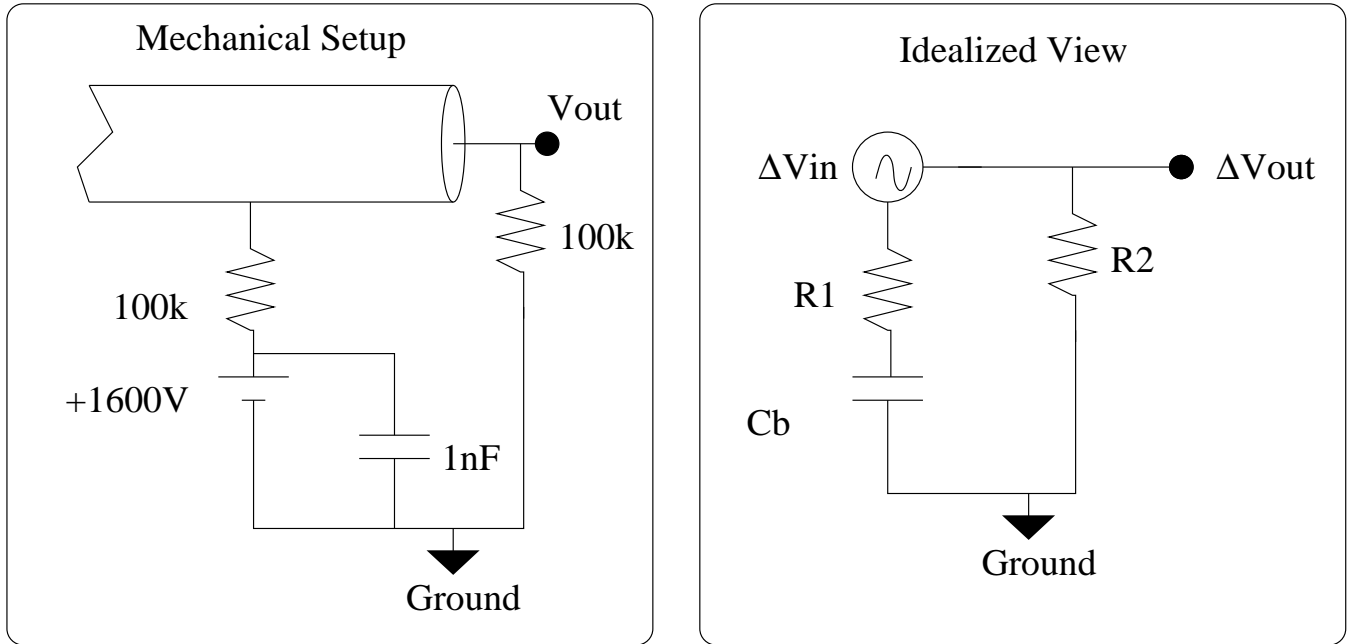


Figure 2: The actual setup showing the connection of the high voltage to the tube via the 100k Ohm resistor,  $R_1$ , the 1nF capacitor to filter noise from the power supply,  $C_B$ , and the 100k output resistor,  $R_2$ . The idealized view removes the high voltage from the diagram and treats the tube as a voltage source,  $\Delta V_{in}$ .

In order to calculate  $\Delta V_{out}$  notice that the sum of the voltages across  $C_B$ ,  $R_1$ , and  $\Delta V_{out}$  add up to be  $V_{out}$ . Now, notice too that the same current must flow through  $R_1$  and  $R_2$ , and  $|\Delta V_{R_1}| = |\Delta V_{R_2}|$ . If we choose the convention that  $\Delta V_{out}$  is positive with respect to ground, then  $\Delta V_{R_2} = -\Delta V_{out}$  and:

$$\Delta V_{out}(t) = \Delta V_{in}(t) - \Delta V_{out} + \Delta q_B/C_B$$

If we take all these deltas with respect to time, we get:

$$\frac{dV_{out}(t)}{dt} = \frac{dV_{in}(t)}{dt} - \frac{dV_{out}(t)}{dt} + \frac{1}{C_B} \frac{dq_B}{dt}$$

You have to perform the usual trick where you ask yourself, for a decrease in  $q_b$  what happens to the voltage? It goes down, but as we've set up the equations, we've defined positive current to be flowing clockwise, so we need to use:

$$\frac{dq_B}{dt} = -i = -V_{out}/R_2$$

and our equation becomes, after a little manipulation:

$$2\frac{dV_{out}(t)}{dt} + \frac{1}{R_1 C_B} V_{out}(t) = \frac{dV_{in}(t)}{dt}$$

In order to solve this equation, we use a trick. We assume that there is some factor,  $\beta$ , that lets us rewrite the equation as:

$$\frac{d(\beta V_{out}(t))}{dt} = \beta \frac{dV_{out}(t)}{dt} + \frac{\beta}{2R_1 C_B} V_{out}(t) = \frac{\beta}{2} \frac{dV_{in}(t)}{dt}$$

This gives us:

$$V_{out}(t) \frac{d\beta}{dt} + \beta \frac{dV_{out}(t)}{dt} = \beta \frac{dV_{out}(t)}{dt} + \frac{\beta}{2R_1 C_B} V_{out}(t)$$

And if we eliminate  $\beta \frac{dV_{out}(t)}{dt}$  from both sides:

$$V_{out}(t) \frac{d\beta}{dt} = \frac{\beta}{2R_1 C_B} V_{out}(t)$$

So that with a little rearranging we get:

$$\frac{d\beta}{\beta} = \frac{dt}{2R_1 C_B}$$

With the usual solution:

$$\beta = M e^{t/(2R_1 C_B)} \text{ (M is a constant)}$$

Now we tackle the  $V_{in}(t)$  part:

$$\int_0^{t'} \frac{d(\beta V_{out}(t))}{dt} dt = \int_0^{t'} \frac{\beta}{2} \frac{dV_{in}(t)}{dt} dt$$

Plugging in for  $\beta$  (note  $\int_0^{t'} d(\beta V_{out}(t)) = \beta V_{out}(t')$ ), and differentiating our expression for  $V_{in}(t)$  from before, we get:

$$Me^{t/(2R_1C_B)}V_{out}(t') = \int_0^{t'} (Me^{t/(2R_1C_B)}) \left(-\frac{q_{ions}}{4\pi\epsilon_o\ell}\right) \frac{1}{t_o} \frac{dt}{(1+t/t_o)}$$

Canceling  $M$ 's and rearranging gives:

$$e^{t'/(2R_1C_B)}V_{out}(t') = \left(-\frac{q_{ions}}{4\pi\epsilon_o\ell}\right) \int_0^{t'} \frac{(e^{t/(2R_1C_B)})}{(t+t_o)} dt$$

And to tidy things up:

$$e^{t'/(2R_1C_B)}V_{out}(t') = \left(-\frac{q_{ions}}{4\pi\epsilon_o\ell}\right) \int_0^{t'} (e^{-t_o/(2R_1C_B)}) \frac{(e^{(t+t_o)/(2R_1C_B)})}{(t+t_o)} dx$$

Now, we taylor expand for  $t+t_o < 2R_1C_B$ :

$$e^{t'/(2R_1C_B)}V_{out}(t') = \left(-\frac{q_{ions}}{4\pi\epsilon_o\ell}\right) (e^{-t_o/(2R_1C_B)}) \int_0^{t'} \frac{(1+(t+t_o)/(2R_1C_B)+...)}{(t+t_o)} dt$$

After integrating we get:

$$e^{t'/(2R_1C_B)}V_{out}(t') = \left(-\frac{q_{ions}}{4\pi\epsilon_o\ell}\right) (e^{-t_o/(2R_1C_B)}) (\ln[(t'+t_o)/t_o] + t'/(2R_1C_B) + ...)$$

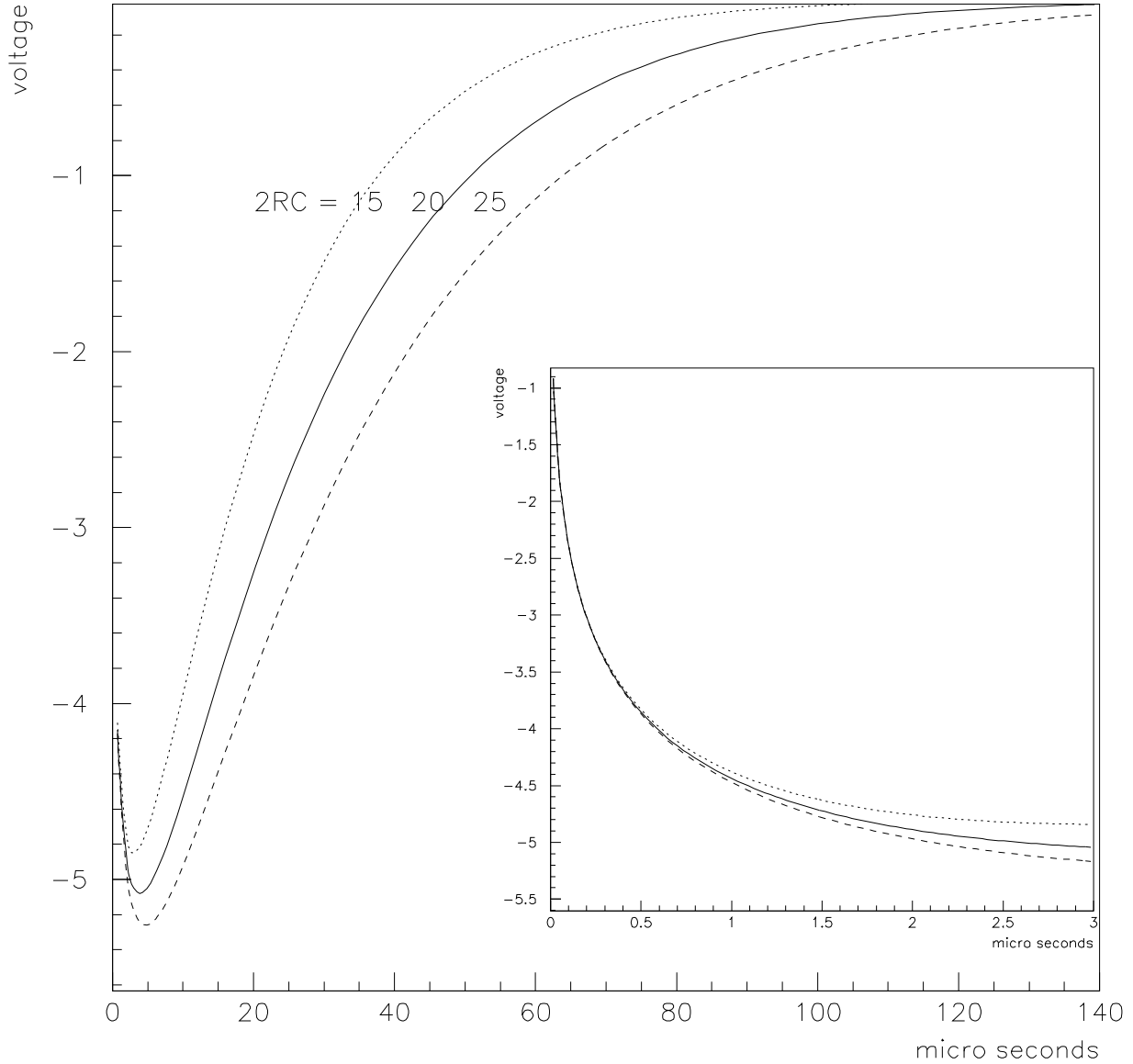
And finally,

$$V_{out}(t) = \left(-\frac{q_{ions}}{4\pi\epsilon_o\ell}\right) (e^{-(t+t_o)/(2R_1C_B)}) (\ln[(t+t_o)/t_o] + t/(2R_1C_B) + ...)$$

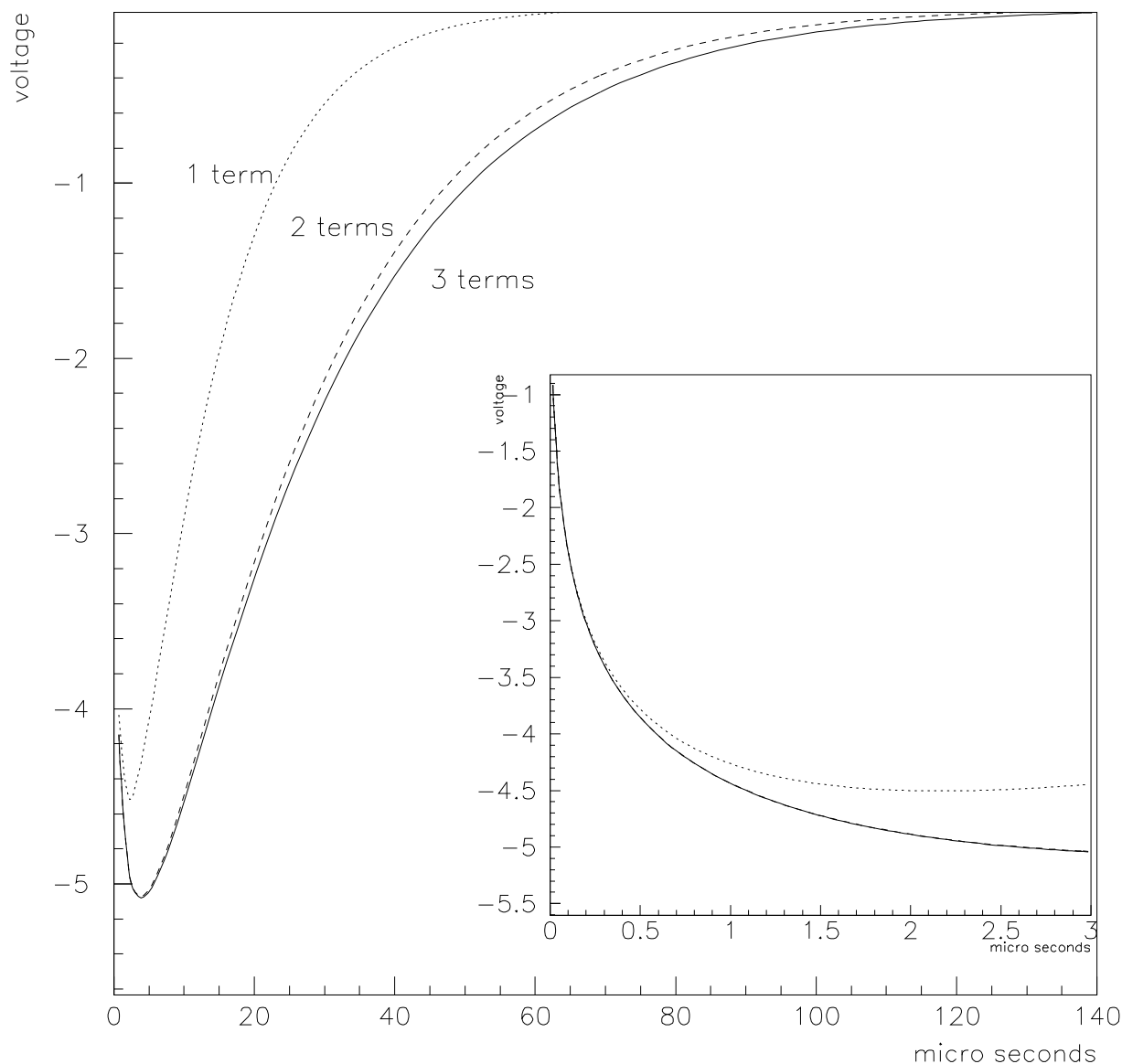
For higher terms note:

$$\int \frac{e^{ax}}{x} dx = \ln x + \frac{ax}{1!} + \frac{a^2x^2}{2 \cdot 2!} + \frac{a^3x^3}{3 \cdot 3!} + ...$$

In the figure below, we have kept the first three terms of the expansion and plotted the shape of the response for  $2R_1C_B = 15, 20$ , and  $25\mu s$ .



In the next figure, we motivate dropping higher order terms in the integration. Notice that there is not much difference between 2 and three terms in the expansion, but keeping only one term should underestimate the signal a bit. A value of  $2R_1C_B = 20ns$  was used to create the plot.



We need one more piece to put everything together. We need to know how much initial ionization was caused by our incoming particle. It helps if we can choose a radioactive source that leaves a well defined amount of charge in our detector. One such source is  $^{55}\text{Fe}$ . The low energy x-rays emitted by this source will typically create 233 electrons in Argon as they lose their energy. So, now you can calculate the gain, and compare it to what you see. You can look at the signal on the scope and compare it to the theoretical time response. This is quite a *tour De force* for this particle detector!

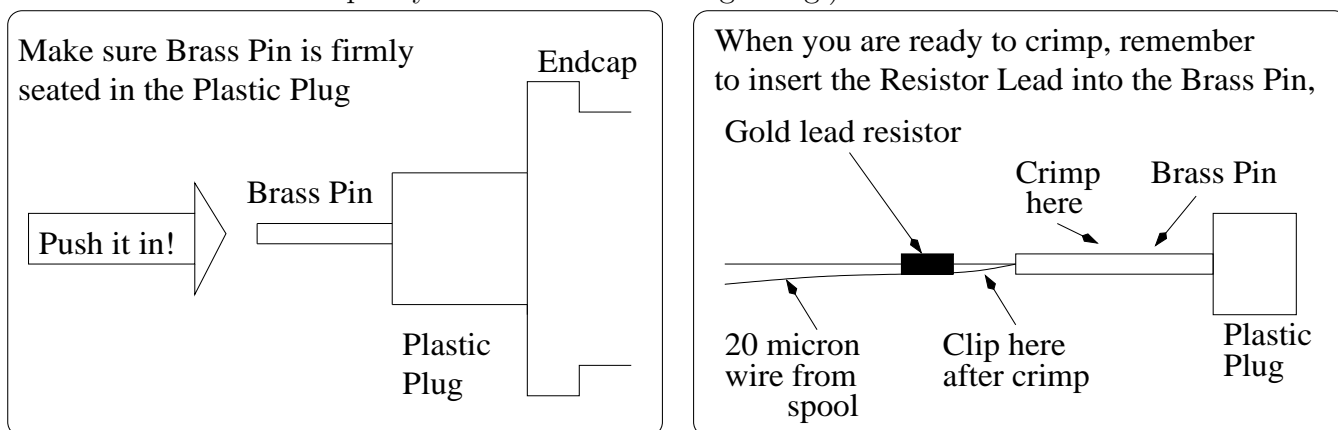
For your calculations, it is useful to note that the gas you are going to be using is called P10. It is a mixture of Ar(90%) and  $\text{CH}_4$ (10%). For P10 at atmospheric pressure  $E_{\min} = 48 \pm 3 \text{ kV/cm}$  and  $\Delta V = 23.6 \pm 5.4 \text{ V}$ . The methane is placed in the gas to absorb low energy photons that would otherwise give many more secondary ionizations and render the tube dead for a long period of time. The wire that you are using has a diameter of 20 microns. You should measure the inner diameter of the copper tube you use.



## Procedure

Assemble your tube. Hopefully there is a tube already assembled and you can learn a lot as you take it apart.

1. Make sure your parts are relatively clean. You don't have to be a psycho about it since the tube is a fairly robust little detector, but things shouldn't be grimy.
2. If you are taking it apart. Disconnect the high voltage (red cable) from the tube. Unwrap the aluminum foil from the tube (**why is it there?**). Make sure any little bits on the end (like resistors and tape) are removed from the brass pins.
3. Remove the endcaps from the tube. On each of the endcaps there is a little brass tube inserted into a white plastic button. Remove the brass pins and insert fresh ones.
4. String the fine wire all the way through the assembly. (This will take some patience!) Once you have the wire through one endcap, through the tube, and through the other endcap, you can replace the endcaps. Sometimes it helps to rotate the piece you are trying to thread into so the wire doesn't get stuck. Some groups find that using another thicker wire as a guide helps too.
5. Pull a little extra wire through the tube. About 4ft worth is good. Next, take one of the resistors with the gold leads and place the lead in the brass tube closest to the spool of wire. Take the crimp tool and squish the brass tube around the wire. (It is useful to practice a couple times on the brass pins you removed at the beginning.)



6. Before you crimp the other end, you need to tension the wire with a 50 gram weight. I won't derive the conditions for electrostatic equilibrium on a wire in an electric field that is sagging due to gravity. Just take my word for it that you need to have some tension on the wire or it may snap to the wall during the application of high voltage.
7. After you have inserted a gold lead into the brass pin and crimped the other end. Take an ohm meter and measure the resistance between the 2 brass pins (should be on the order of 50 Ohms) and between one of the pins and the inside pin of the high voltage connector (should be 0). If all is fine, hook up the gas.
8. Insulate one end of the tube with tape or a little piece of tubing, and connect the resistor to the other end. (Only one end has a resistor.) Attach the high voltage cable and wrap the assembly with aluminum foil. You will need to clamp the foil to the outside of the high voltage connector with one of the clamps provided.
9. Attach the scope lead to the pin with the resistor, and the scope ground to the other end of the resistor. Begin flowing gas into the tube. (**How long do you need to wait?**) After enough gas has flowed, increase the High Voltage to 1600 V. (Note: we are actually biasing the outer

wall of the tube to -1600V and the tube is at 0V, so no licking that tube!) You may notice a few negative signals. Ask your instructor for a radioactive source.

10. For several voltages, find the average pulse height of the output pulse by eye and make a note of it. The Iron-55 source will give you a very good line to do this. (Note at higher voltages, the tube may not be able to get rid of all the charge before another x-ray is present. This Space-Charge effect will lower the tube gain and make your nice line disappear.)
11. For one of your best looking voltages, sketch (or print out a representative pulse by pressing the run/stop button until you get one you like.) the output pulse so that you can integrate the voltage to find the total charge going through the resistor at the end of the tube. Be sure you note the voltage.
12. Remove the scope probe and hook up the buffer unit. Make sure the 9V batteries are plugged in and that the buffer is hooked up to the single channel analyzer(SCA). The SCA has a low and a high window set by the knobs on the unit. If a pulse arrives with a peak voltage between the low and high window, a logic pulse is sent to the scaler. If you think you have set reasonable levels for the lower and upper window of the SCA, try taking a few 30 second runs with the scaler counting and the Fe-55 source in place, recording the number of counts in each run. Move the Fe-55 source to the side of the hole and repeat. Now have your instructor remove the source and repeat your measurement. Ask your instructor for a beta source and a gamma source. Repeat the sequence of measurements again. Notice how the signal behaves relative to the position of the source. (There is a thin plastic window into the tube interior that helps you see these effects. You need to have this in order for the Iron-55 x-rays to penetrate.)

**In your write up, be sure to include the derivation steps left out in the introduction and any other questions that were asked. Plot voltage versus gain and see if you get the behavior you expect theoretically. Calculate the gain you got for the voltage pulse that you sketched in item 11. How well does your pulse agree with the calculation? Did you get the gain you calculated? How did the three different sources behave. Did you make a note of the activity of each source? Discuss your results.**